Set B

Name of the Department: Department of Physics and Astrophysics
Name of Course: B.Sc. Hons. Physics – CBCS (Core)
Name of the Paper: Statistical Mechanics
Semester - VI
Unique Paper Code: 32221602
Question paper Set number: Set B

Maximum Marks: 75

Attempt any four questions. All questions carry equal marks.

Q1. Explain the concept of microstate, macrostate and the most probable macrostate on the basis of the results obtained when throwing two identical 6-sided dice, each with the numbers 1-6 written on their faces. Identify the macrostates having zero entropy.

An isolated system consists of 10 particles distributed among three two-fold degenerate energy levels labelled 1, 2, 3, such that the energies of the levels are ϵ_1= 2 eV, ϵ_2= 3 eV, ϵ_3= 4 eV, and their occupation numbers are $N_1 = 4$, $N_2 = 3$, $N_3 = 3$. (a) If the occupation number of level 1 decreases by 2, find the new occupation numbers of levels 2 and 3. (b) Find the thermodynamic probabilities and entropies of the new macrostate, if the particles are (i) distinguishable, (ii) indistinguishable.

Q2. Define partition function. Discuss its significance.

Consider a system consisting of two particles, each of which can be in any one of four single particle states of respective energies 0, ϵ, 2ϵ and 3ϵ. The system is in contact with a heat reservoir at temperature $T = (k\beta)^{-1}$. Determine the partition function Z if the particles obey (a) MB statistics, (b) BE statistics, (c) FD statistics.

A system of N particles exists in a phase space of two cells, with degeneracies $g_1 = g_2 = 1$. If $\varepsilon_1 = 0$ and $\varepsilon_2 = \varepsilon$, find the number of particles in each cell and the total energy of the system in equilibrium. Find the values of the total energy when $T = 0$ and $T = \infty$.

Q3. A system consisting of N diatomic molecules each of mass m and moment of inertia I is confined in a volume V and at temperature T. The partition function of the system is given by

$$Z = V\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\left(\frac{2\pi}{h}\right)^2 2IkT$$

Find the (a) Helmholtz function F, (b) Pressure P, (c) Internal energy U, (d) Entropy S, (e) Enthalpy H, and (f) Gibb's function G, of the system.

Q4. Obtain Planck's law for blackbody radiation treating it as a collection of oscillators. Use Planck's radiation formula to obtain Wien's constant and Stefan's constant.
A body at 1000 K emits maximum energy at a wavelength 30,000 Å. If a star emits maximum energy at wavelength 5000 Å, what would be the temperature of the star?

Q5. Consider a system of N indistinguishable spinless bosons, each of mass m, confined in a cubical box of volume $V = L^3$ at temperature $T > 0$.

Obtain expressions for (i) density of states in the neighbourhood of energy ϵ, (ii) number of bosons $n(\epsilon)$ having energy between ϵ and $\epsilon + d\epsilon$, in terms of mass m, volume V, energy ϵ, temperature T, chemical potential μ.

Show that in the limit of the Bose-Einstein distribution becoming equal to the classical (Boltzmann) distribution with $e^{-\mu/kT} >> 1$, the average distance d between the particles becomes very large compared to the de Broglie wavelength λ associated with their thermal motion.

Evaluate the first order difference in internal energy between the system of N indistinguishable spinless bosons when $d >> \lambda$ and the system of N distinguishable spinless classical particles, both systems having the same volume $V = L^3$ and particle mass m.

Q6. Show that the electron gas in a white dwarf star of mass $M = 10^{30}$ kg and density $\rho = 10^{10}$ kg m^{-3} is highly degenerate and relativistic. The temperature of the star is of the order of 10^7 K.

Obtain an expression for the mass-radius relationship for a white dwarf star. What is the physical significance of Chandrasekhar mass limit?

Calculate the temperature at which a state with energy 0.4 eV above the Fermi energy has 2% probability of being occupied by an electron.