Unique Paper Code	**3221401 OC**
Name of Paper	Mathematical Physics III
Name of Course	**B.Sc.(Hons.)Physics-CBCS**
Semester	I**V**

Duration: **3 Hours** Maximum Marks: **75**

All questions carry equal marks. **Attempt four questions**

Use of Scientific calculator is allowed

1. Prove that $\cos^{-1} z = \frac{1}{i}\ln(z + \sqrt{z^2 - 1})$ for the principal branch and then find all the values of $\cos^{-1} i$.

   Locate and name the singularities of the following functions

   $f(z) = cosec(\frac{1}{z^2})$

   $f(z) = \sqrt{\frac{z}{(1-z)}}$

2. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent's series valid for

   $1 < |z| < 2$

   $|z - 1| > 2$

   Find the residue of $f(z) = \exp\left(\frac{3}{z}\right) at\ z = 0$

3. Using the method of contour integration prove the following

   $\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$

   $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$

4. Determine the Fourier transform of xe^{-x^2}

   If $F(\omega)$ is the Fourier transform of $f(x)$, determine the Fourier transform of $f(x)sin\ px$

   Where $p > 0$

5. Solve the following simultaneous differential equations using Laplace Transforms

   $2x(t) - y(t) - y'(t) = 4(1 - e^{-t})$

   $2x'(t) + y(t) = 2(1 + 3e^{-2t})$

   subject to the conditions $x(0) = y(0) = 0$

   $y'(t) = \frac{dy}{dt};\ \ x'(t) = \frac{dx}{dt}$

Determine the inverse Laplace transform of the following function

$F(s) = \frac{1}{s(s^2+1)}$

6. Evaluate the Laplace transform of $f(t) = \left\{\frac{1-e^{-t}}{t}\right\}$

Given$f(x) = 1 - x^2$ for $|x| < 1$
$f(x) = 0$ for $|x| > 1$
Determine the Fourier transform of $f(x)$

Evaluate the following integral $\int_0^3 \delta(x+1)(3x-5)dx$

18.75