Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351401_OC**
Name of Paper	: **C8 Partial Differential Equations**
Semester	: **IV**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1 (i) Derive the damped wave equation and telegraph equation of a string.

(ii) Reduce the equation $u_{xx} + xyu_{yy} = 0 \quad , x, y < 0$ to canonical form.

(iii) Prove the uniqueness of the solution of the initial boundary-value problem:

$$u_{tt} = 25\, u_{xx}, \quad 0 < x < \pi, t > 0,$$
$$u(x,0) = f(x), \quad 0 \le x \le \pi,$$
$$u_t(x,0) = g(x), \quad 0 \le x \le \pi,$$
$$u_x(0,t) = 0, \quad u_x(\pi,t) = 0, \quad t > 0$$

2 (i) Find the integral surfaces of the equation $uu_x + u_y = 1$ for the initial data $x(s,0) = \frac{s^2}{2},\ y(s,0) = 2s,\ u(s,0) = s$

(ii) Apply $e^u = v$ and then $v(x,y) = f(x) + g(y)$ to solve the equation $x^4 u_x^2 + y^2 u_y^2 = e^{-2u}$.

(iii) Use a separable solution $v(x,y) = g(x) + f(y)$ to solve the equation $v_x^2 + v_y^2 = v$.

3 (i) Find the characteristics and characteristic coordinates, and reduce the equation

$$u_{xx} - (2cosx)u_{xy} + (1 + cos^2x)u_{yy} + u = 0$$

to canonical form.

(ii) Determine the general solutions of the equation

$$u_{xx} + 4u_{xy} + 4u_{yy} = 0$$

(iii) Transform the equation

$$u = -2u_{xx} - 9u_{xy} - 3u_{yy} - u_y$$

to the form $v_{mn} = cv$, where c = constant, by introducing the new variables $v = ue^{-(am+bn)}$, where 'a' and 'b' are undetermined coefficients.

4 (i) Determine the solution of the Cauchy problem given by

$$u_{tt} = 9u_{xx}, \;\; u(x,0) = x^3, u_t(x,0) = x$$

(ii) Determine the solution of the initial boundary value problem

$$u_{tt} = u_{xx}, \quad 0 < x < 1, t > 0$$
$$u(x,0) = \sin \pi x, \quad 0 \le x \le 1$$
$$u_t(x,0) = 0, \qquad 0 \le x \le 1$$
$$u(0,t) = 0, \; u(1,t) = 0, \; t \ge 0$$

(iii) Determine the solution of characteristic initial value problem

$$xy^3 u_{xx} = x^3 \; y\, u_{yy} + y^3 u_x - x^3 u_y,$$
$$u(x,y) = f(x), \;\; y^{\,2} + x^2 = 4, \;\; 0 \le x \le 2$$
$$u(x,y) = g(y), \qquad x = 0, \;\; 0 \le y \le 2,$$
$$f(0) = g(2)$$

5. (i) Solve using method of separation of variables

$$u_{tt} = 9u_{xx}, 0 < x < 2\,, t > 0$$
$$u(x,0) = x(2-x), 0 \le x \le 2$$
$$u_t(x,0) = 0, \;\; 0 \le x \le 2$$
$$u(0,t) = t, \;\; u(2,t) = 0, t > 0$$

(ii) Determine the solution of

$$u_{tt} = u_{xx} + x^2, 0 < x < 1\,, t > 0$$
$$u(x,0) = 0, 0 \le x \le 1$$
$$u_t(x,0) = 0, \;\; 0 \le x \le 1$$
$$u(0,t) = 0, \;\; u(1,t) = 0, t \ge 0$$

(iii) Solve the heat conduction problem

$$u_t = 9u_{xx}, 0 < x < 1\,, t > 0$$
$$u(x,0) = x^2(\,1 - x\,), 0 \le x \le 1$$
$$u(0,t) = 0, t > 0$$
$$u(1,t) = 0, t > 0$$

6 (i) Find the partial differential equation arising from the surface

$$2z = (\alpha x + y)^2 + \beta$$

Find the solution of the Cauchy problem:

$$uu_x - uu_y = u^2 + (x+y)^2, \text{ with u = 1 on y = 0}$$

(ii) Determine the solution of the initial boundary value problem

$$u_{tt} = 9u_{xx}, 0 < x < \infty, t > 0,$$
$$u(x,0) = 0, \quad 0 \le x < \infty,$$
$$u_t(x,0) = x^3, \quad 0 \le x < \infty,$$
$$u_x(0,t) = 0, \qquad t \ge 0$$

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(iii) Solve the characteristic initial-value problem

$$xu_{xx} - x^3u_{yy} - u_x = 0, x \neq 0,$$
$$u(x,y) = f(y) \text{ on } y - \frac{x^2}{2} = 1 \quad \text{for } 0 \le y \le 2,$$
$$u(x,y) = g(x) \text{ on } y + \frac{x^2}{2} = 3 \quad \text{for } 2 \le y \le 4,$$

with f(2) = g(2).