Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351102**
Name of Paper	: **BMATH102-Algebra**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Find all the rational roots of the equation $224y^3 - 344y^2 + 22y - 15 = 0$ and also solve the equation $16y^4 - 96y^3 + 56y^2 + 264y - 135 = 0$ given that the roots form an arithmetical progression.
2. Draw a rough sketch of the region corresponding to the inequality $\frac{1}{\sqrt{2}} < |z - 1 - i| < \sqrt{2}$. Use De Movire's theorem to find the square root of $-3 + 4i$. Find the extended argument $Arg\ z$ of the complex number $z = (-\sqrt{3} - i)(1 + i)$.
3. Let $A = \{1,2,3,4,5,6,7\}$. Define a relation R_1 on the set A which is an equivalence relation. Define a relation R_2 on the set A which is not an equivalence relation. Let a be an integer, prove that there exists an integer k such that $a^2 = 5k$ or $a^2 = 5k + 1$. Evaluate (5.6+8.11+19.23)(mod 9).
4. Show that the function $f: \left(\frac{2}{5}, \infty\right) \to \mathbb{R}$ and $g: \mathbb{R} \to \left(\frac{2}{5}, \infty\right)$ defined by $f(x) = log_5(5x - 2)$ and $g(x) = \frac{5^x + 2}{5}$ are the inverse of each other. Prove that the interval (3, 7) and (1, ∞) have the same cardinality. Show that 314 and 159 are relatively prime integers.
5. Describe the solutions of the following system in parametric vector form. Give a geometrical description of the solution set.

$$4x_1 - 2x_2 + 6x_3 = 8$$
$$x_1 + x_2 - 3x_3 = -1$$
$$15x_1 - 3x_2 + 9x_3 = 21$$

Let $T: R^2 \to R^2$ be a linear transformation which first reflects points through the line $x_1 = x_2$ and then rotates points (about the origin) through π/4 radians. Find the standard matrix of *T*.

6. Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$. Find a basis for

(i) Row Space of A.

(ii) Null Space of A.

Also find Rank A and Nullity A.