

Name of Course : CBCS (LOCF) B.A. (Prog.)
 Unique Paper Code : 62351101
 Name of Paper : Calculus
 Semester : I
 Duration : 3 hours
 Maximum Marks : 75 Marks

Attempt any four questions. All questions carry equal marks.

1. Find all the points of discontinuity of the function f given by

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ \frac{1}{2} - x & \text{if } 0 < x < \frac{1}{2} \\ \frac{1}{2} & \text{if } x = \frac{1}{2} \\ \frac{3}{2} - x & \text{if } \frac{1}{2} < x < 1 \\ 1 & \text{if } x = 1 \end{cases}$$

Also, examine the nature of discontinuity in each case.

Show that the function

$$g(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is differentiable when $\sin^{-1} x = 0$.

Find the points at which the function h is not differentiable, where

$$h(x) = |x + 1| + 12|2x + 5|.$$

2. Find the n^{th} differential coefficient of $\tan^{-1} x$.

If $y = e^{a \sin^{-1} x}$, then show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0.$$

If $z = \sin^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$, then show that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{1}{2} \tan z.$$

3. Find the equation of tangents and normal to the curve

$$x = 2a \cos \theta - a \cos 2\theta, \quad y = 2a \sin \theta - a \sin 2\theta \quad \text{at } \theta = \frac{\pi}{2}.$$

For the curve

$$y^2(a^2 + x^2) = x^2(a^2 - x^2)$$

show that origin is a node and that the nodal tangents bisect the angles made by the axes. Also, trace this curve.

4. Find the radius of curvature at the origin for

$$2x^4 + 3y^4 + 4x^2y + xy - y^2 + 2x = 0.$$

Trace the curve $y = x^3 - (x + 1)^2$. Also find the asymptotes of the following curve

$$(x^2 - y^2)^2 - 4x^2 + x = 0.$$

5. Show that there is no real number p for which the equation $x^2 - 3x + p = 0$ has two distinct roots in $[0,1]$.

Show that e^{-x} lies between $1 - x$ and $1 - x + \frac{x^2}{2}$ for all $x \in \mathbb{R}$.

In the Cauchy's Mean Value Theorem on $[a, b]$ ($0 < a < b$)

- (i) If $f(x) = \sqrt{x}$ and $g(x) = 1/\sqrt{x}$, then c is the geometric mean between a and b .
(ii) If $f(x) = 1/x^2$ and $g(x) = 1/x$, then c is the harmonic mean between a and b .

6. Find maxima and minima of the function

$$f(x) = \sin x + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} \quad \text{on } [0, \pi].$$

Show that the maximum value of

$$g(x) = \left(\frac{1}{x}\right)^x \quad \text{is } e^{1/e}.$$

Evaluate the function h for maximum and minimum values and separate the intervals of increasing and decreasing, where h is given by

$$h(x) = 2x^5 - 10x^4 + 10x^3 - 15, \quad x \in \mathbb{R}$$