Name of the Course : **CBCS B.Sc. Mathematical Sciences / B.Sc. (Prog.)**

Unique Paper Code : **42357618**

Name of the Paper : **DSE- Numerical Methods**

Semester : **VI**

Duration : **3 Hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks. All symbols have usual meanings.

1. Let $f(x) = x^4 - 18x^2 + 45$ and $g(x) = x^3 + x^2 - 3x - 3$.
   (i) Verify that both the equations $f(x) = 0$ and $g(x) = 0$ have a root on the interval (1, 2).
   (ii) By performing three iterations of Newton-Raphson method, with $x_0 = 1$, find an approximation of the root of $f(x) = 0$.
   (iii) By performing three iterations of Bisection method, find an approximation of the root of $g(x) = 0$.
   (iv) Given that the exact value of the root in both cases is $x = \sqrt{3}$, compute the absolute error in the approximations obtained.

2. Find the inverse of A, using Gauss-Jordan elimination

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 13 \\ 3 & 11 & 22 \end{bmatrix}.$$

   Using Gauss-Seidel iteration method, solve the system of equations given by

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.2 \\ 1.3 \end{bmatrix}.$$

3. Use the Lagrange's interpolation to find a polynomial that passes through the points $(0, 2), (1, 3), (2, 12)$ and $(5, 147)$.

   Using the following data

x	1.0	1.5	2.0	2.5
$f(x)$	2.7183	4.4817	7.3891	12.1825

   estimate the value of $f(2.15)$ using
   (i) Newton's forward difference interpolation
   (ii) Newton's backward difference interpolation.

Compare the errors and find which of the methods gave a better approximation of $f(2.15)$.

4. The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the initial acceleration using the entire data:

Time t (sec)	0	5	10	15	20
Velocity v(m/sec)	0	2	13	68	227

A boundary value problem is defined by

$$\frac{d^2y}{dx^2} + y + 1 = 0, \quad 0 \le x \le 1$$

where $y(0) = 0$ and $y(1) = 0$ with $h = 0.5$. Use the finite difference method to determine the value of $y(0.5)$. Its exact solution is

$$y(x) = \cos x + \frac{1 - \cos 1}{\sin 1} \sin x - 1.$$

Calculate the error.

5. Use the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

to approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ taking $h = 1, 0.1, 0.01$.

Use Euler's method to approximate the solution of the initial value problem

$$\frac{dy}{dt} = \frac{(1+y)^2}{t}, \qquad y(1) = 0, 1 \le t \le 4$$

taking 5 steps.

6. Find approximate value of the integral $I = \int_0^2 e^x dx$ using
   (i) Trapezoidal Rule;
   (ii) Simpson's 1/3 rule;
   (iii) Simpson's 3/8 rule.