Sr. No. of Question Paper	:	2
Unique Paper Code	:	32221301_OC
Name of the Paper	:	Mathematical Physics II
Name of the Course	:	B. Sc. (Hons) Physics (CBCS)
Semester	:	III

Duration : 3 Hours **Maximum Marks: 75**

Attempt any ***four*** questions. All questions carry equal marks.

1. Given, $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \frac{\pi x}{4}, & 0 < x < \pi \end{cases}$ with $f(x+2\pi) = f(x)$

(a) Find Fourier Series and hence prove that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots = \frac{\pi^2}{8}$$

(10.75, 3)

(b) Using Parseval's Identity, prove that:

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots\ldots = \frac{\pi^4}{96} \quad . \text{ Given } \sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

(5)

2. (a) Find Fourier Cosine Series for the function

$$f(x) = x(\pi - x), \quad 0 < x < \pi \quad .$$

Hence, prove that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{6}$$

(10.75)

(b) Using Beta and Gamma functions, evaluate:

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta \quad \text{and} \quad \int_0^{\infty} \frac{x\, dx}{x^6 + 1}$$

(4, 4)

3. (a) Identify and name the singularities of the following differential equation:

$$x^2(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 4y = 0$$

(6)

(b) Solve the following differential equation using Frobenius method:

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0$$

(12.75)

4. (a) Prove the Recurrence Relation for Bessel Polynomial:

$$n\, J_n(x) + x\, J_n'(x) = x\, J_{n-1}(x)$$

(6.75)

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(b) Show that: $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x - x \cos x}{x} \right)$ (6)

(c) Using Generating Function, show that:

$$J_n(u+v) = \sum_{s=-\infty}^{\infty} J_s(u)\, J_{n-s}(v)$$ (6)

5. (a) Expand $f(x) = x^3$ in a series of Legendre Polynomial,

and hence evaluate $\int_{-1}^{1} x^3\, P_3(x)\, dx$ (5, 6)

(b) Write the Rodrigue's formula for Legendre polynomials and hence find

$P_1(x)$, $P_2(x)$ and $P_3(x)$ (7.75)

6. (a) Using the method of separation of variables, solve the following differential equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ (9)

subject to the conditions $u(0,y)=u(l,y)=u(x,0)=0$ and $u(x,a) = F(x)$.

(b) Solve the one dimensional wave equation under the following conditions:

$u(0,t) = u(l,t) = 0$

$u(x,0) = \begin{cases} x & \text{for } 0 \le x < l/2 \\ l - x & \text{for } l/2 < x \le l \end{cases}$, (9.75)

and $\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$, where $u(x,t)$ is the displacement of the vibrating string.

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