Sr. No. of Question Paper	:	1
Unique Paper Code	:	32227502
Name of the Paper	:	Advanced Mathematical Physics (DSE Paper)
Name of the Course	:	B. Sc. (Hons) Physics (CBCS)
Semester	:	V

Duration : 3 Hours   Maximum Marks: 75

Attempt any ***four*** questions. All questions carry equal marks.

1. (a) Consider the set S = $\left\{1, 2, 4, \frac{1}{2}, \frac{1}{4}\right\}$,

determine whether S forms a group w. r. t. multiplication. (3.75)

(b) If V is a vector space of all 2 × 2 matrices over real field **R**, determine whether W is a subspace of V where W consists of all matrices with zero determinant. (5)

(c) Let T: $\mathbf{R}^3 \rightarrow \mathbf{R}^3$ be the linear transformation defined by

$$T(x, y, z) = (x + y - 2z, x + 2y + z, 2x + 2y - 3z)$$

Find the matrix representation of (5, 5)

(i) T^{-1} w. r. t. the basis $\{e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)\}$

(ii) T w. r. t. the basis $\{a_1 = (1, 1, 1), a_2 = (1, 2, 3), a_3 = (2, -1, 1)\}$

2. (a) Determine whether the set of vectors $\{b_1 = (1, 2, -1), b_2 = (2, 3, 4), b_3 = (1, 5, -3)\}$ form a basis of $\mathbf{R}^3$. (3.75)

(b) If H is a Hermitian matrix and I is a Unit matrix, determine whether

$$P = (I - iH)(I + iH)^{-1}$$

is a Unitary matrix. $[i = \sqrt{-1}]$ (5)

(c) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

hence, diagonalize A. (10)

3. (a) State Cayley-Hamilton theorem, using it find B^{-1}, where

$$B = \begin{bmatrix} 1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7 \end{bmatrix}.$$ (6.75)

(b) If $C = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, then prove that

$$\exp(i\theta C) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$ (12)

[here, $i = \sqrt{-1}$]

4. (a) Given the components of second order tensors

$$a_{kp} = \begin{bmatrix} 5 & 1 & 0 \\ 6 & 4 & 2 \\ 7 & 8 & 3 \end{bmatrix} \quad \text{and} \quad b_{kp} = \begin{bmatrix} 2 & 3 & 9 \\ 0 & 5 & 8 \\ 1 & 7 & 4 \end{bmatrix}$$

find $a_{km}\, b_{pm}$ and $a_{km}\, b_{mp}$, where $m = 1, 2, 3$

(3, 3)

(b) Show that $\in_{hku} = \begin{vmatrix} \delta_{h1} & \delta_{h2} & \delta_{h3} \\ \delta_{k1} & \delta_{k2} & \delta_{k3} \\ \delta_{u1} & \delta_{u2} & \delta_{u3} \end{vmatrix}$ (6)

and hence prove that

$$\in_{hku} \in_{pcm} = \begin{vmatrix} \delta_{hp} & \delta_{hc} & \delta_{hm} \\ \delta_{kp} & \delta_{kc} & \delta_{km} \\ \delta_{up} & \delta_{uc} & \delta_{um} \end{vmatrix}$$

(6.75)

5. (a) If C_{kmphu} is a tensor of order 5, prove that C_{kmpmu} is a tensor of order 3. (4.75)

(b) Using tensors, prove that

(i) $(A \times B) \times (C \times D) = C\,(A \bullet B \times D) - D\,(A \bullet B \times C)$ (6)

(ii) $\nabla (A \bullet B) = A \times (\nabla \times B) + B \times (\nabla \times A) + (B \bullet \nabla) A + (A \bullet \nabla) B$ (8)

6. (a) If $R_{pk} = \begin{bmatrix} 1 & 3 & 8 \\ 5 & 4 & 7 \\ 2 & 0 & 9 \end{bmatrix}$

find symmetric tensor S_{pk} and skew-symmetric tensor A_{pk} such that

$$R_{pk} = S_{pk} + A_{pk}$$ (3, 3)

(b) Prove that

$$g_{\mu\nu}\, g^{\nu\gamma} = \delta_{\mu}^{\ \gamma}$$

$$g_{\mu\nu}\, C^{\nu\gamma} = g^{\gamma\nu}\, C_{\mu\nu}$$

$$A^{\mu\nu}\, B_{\mu\nu} = A_{\mu\nu}\, B^{\mu\nu} \qquad (4.75, 4, 4)$$