Name of Course	: **Generic Elective**
Unique Paper Code	: **32355301**
Name of Paper	: **GE-3 Differential Equations**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Determine the constant A such that the differential equation

$$(Ax^2y + 2y^2)dx + (x^3 + 4xy)dy = 0$$

is exact and solve the resulting exact equation.

Solve the following initial value problems:

i) $\frac{dy}{dx} + \frac{4}{x}y = x^3y^2, \quad y(2) = -1, \ x > 0.$

ii) $(3xy + y^2)dx + (x^2 + xy)dy = 0, \ y(1) = -1$.

2. Find the orthogonal trajectory of the family $y = c\sin x$ that passes through the point $(2\pi, 2)$. Also find the family of oblique trajectory that intersects the family of circles $x^2 + y^2 = c^2$ at an angle $\frac{\pi}{4}$.

Show that the family of confocal conics $\frac{x^2}{\lambda+a^2} + \frac{y^2}{\lambda+b^2} = 1$, where a and b are fixed constants and λ is the parameter, is self orthogonal.

3. Show that the set $\{1, x, x^2\}$ of functions forms a basis for the solution set of a differential equation. Also, find such a differential equation.

Find the general solution of the second order equation $t^2y'' + 2ty' - 2y = 0$ given that $y_1(t) = t$ is a solution. Also solve the initial value problem

$$y''' + 3y'' - 4y = 0, \ y(0) = 1, \ y'(0) = 0, \ y''(0) = \frac{1}{2}.$$

4. Find the general solution of the following differential equations:

i) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + 3e^{2x}.$

ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x.$

iii) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x.$

5. Find the partial differential equation arising from the equation $ax^2 + by^2 + z^2 = 1$, where $z = z(x, y)$.

Find the general solution of the linear partial differential equation

$$(y + xu)p - (x + yu)q = x^2 - y^2.$$

Using $v = \ln u$ and $v = f(x) + g(y)$, solve the equation

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2, \; u(x, 0) = e^{x^2}.$$

6. Reduce the following equations to canonical form and hence find their solutions

i) $u_x - yu_y = 1 + u.$

ii) $y\, u_{xx} + (x + y)u_{xy} + x\, u_{yy} = 0, \; y \neq x.$

iii) $u_{xx} - 4u_{xy} + 4\, u_{yy} = 0.$