Name of the Course : **B.Sc. (Prog.) Mathematical Sciences**

Semester : **IV**

Unique Paper Code : **42354401**

Name of the Paper : **Real Analysis**

Duration: **2 Hours** Maximum Marks: **75**

Attempt any four questions. All questions carry equal marks. All symbols have usual meaning.

1. Find the infimum and supremum, if they exist, of each of the following sets
(i) $A = \left\{1 - \frac{1}{2n} : n \in \mathbb{N}\right\}$ (ii) $B = \left\{\frac{1}{n} + \frac{1}{m} : m, n \in \mathbb{N}\right\}$ (iii) $C = \left\{\sum_{i=1}^{n} \frac{1}{3^{i-1}} : n \in \mathbb{N}\right\}$.

2. Let $x_1 = 5/2$ and $x_{n+1} = 1 + \sqrt{x_n - 1}$, $n \in \mathbb{N}$. Show that the sequence (x_n) is monotonic decreasing and bounded below. Find the limit of the sequence.

3. For $n \geq 1$, define $f_n, f : [0,1] \to \mathbb{R}$ by $f_n(x) = x^n$ and $f(x) = \begin{cases} 0, & \text{if } 0 \leq x < 1 \\ 1, & \text{if } x = 1 \end{cases}$.

Prove that the sequence (f_n) converges to f in $[0, 1]$. Also examine the uniform convergence of the sequence (f_n) to f in $[0, 1]$.

4. Discuss the convergence of the series

$$\sum_{n=1}^{\infty}\left[\left(\frac{n+1}{n}\right)^{n+1} - \left(\frac{n+1}{n}\right)\right]^{-n}.$$

Find the radius of convergence and exact interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(n+1)}{(n+2)(n+3)} x^n.$$

5. Define exponential function as the sum of a power series and determine its domain. Prove that $E(x+y) = E(x)E(y)$, for all $x, y \in \mathbb{R}$.
If e denotes $E(1)$, prove that $E(x) = e^x$ for all real x.

6. State necessary and sufficient condition for a function to be Reimann integrable. Show that a function f defined as

$$f(x) = \begin{cases} 2^{-n}, & \text{when } x \in \left(2^{-n-1}, 2^{-n}\right], n = 0,1,2,3,\ldots \\ 0, & \text{when } x = 0 \end{cases}$$

is integrable on [0, 1].

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