Name of the Course : **B.Sc. (Prog.) Mathematical Sciences**
/B.Sc. (Prog.) Physical Sciences

Semester : **VI**

Unique Paper Code : **42357618**

Name of the Paper : **Numerical Methods**

Duration: **2 Hours** Maximum Marks: **75**

Attempt any four questions. All questions carry equal marks. All symbols have usual meaning.

1. Determine the roots of the function

$$f(x) = -12 - 21x + 18x^2 - 2.75x^3,$$

using bisection method, with a stopping criterion of 1% and initial guess values $x_l = -1$ and $x_u = 0$.

2. Using Newton-Raphson method for nonlinear systems, solve the nonlinear systems

$$x^2 + y^2 = 4, \quad x^2 - y^2 = 1.$$

The true solutions are easily determined to be $(\pm\sqrt{2.5}, \pm\sqrt{1.5})$. As an initial guess, use $(x_0, y_0) = (1.6, 1.2)$.

3. From the data in the following table find by Lagrange's Interpolation formulas the value of y when $x = 102$ and the value of x when $y = 13.5$.

x	93	96.2	100	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

4. The upward velocity of a rocket is given at three different times in the following table

Time, $t(s)$	5	8	12
Velocity, $v(s)$	106.8	177.2	279.2

The velocity data is approximated by a polynomial as

$$v(t) = a_1 t^2 + a_2 t + a_3, \quad 5 \le t \le 12.$$

Using the Gauss-Seidel method, find the values of a_1, a_2 and a_3. Assume an initial guess of the solution as $(a_1, a_2, a_3) = (1, 2, 5)$ and conduct two iterations.

5. Consider the second order initial value problem

$$\frac{d^2y}{dt^2} + 4y = 0,$$

with $y(0) = 1$ and $\frac{dy}{dt}(0) = 0$. Using the fourth order Runge-Kutta method, solve the initial value problem from $t = 0$ to 4 with step size $h = 0.1$.

6. An $11\,m$ beam is subjected to a load, and the shear force follows the equation

$$V(x) = 5 + 0.25x^2,$$

where V is the shear force, x is the length in distance (metre) along the beam. We know that $V = dM/dx$, and M is the bending moment. Integration yields the relationship

$$M = M_0 + \int_0^x V dx.$$

If M_0 is zero and $x = 11$, calculate M using Simpson's 1/3 rule by taking $1\,m$ increment.