Name of the Course : **B.Sc. (Physical Sciences)**

Semester : **II**

Unique Paper Code : 4**2351201_OC**

Name of the Paper : **Calculus and Geometry**

Duration: **2 Hours** Maximum Marks: **75**

Attempt any four questions. All questions carry equal marks. All symbols have usual meaning.

1. Given $\varepsilon = 0.02, a = -2, f(x) = 2x - 2, L = -6,$ find $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$.
Examine the differentiability of the function

$$g(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

at $x = 0$ and continuity of g and g' at $x = 0$.

2. For $a = -1$ and $b = 8$, check the differentiability and continuity of the function $f(x) = x^{2/3}$ in $[a, b]$.
Also show that there is no point 'c' in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. Does this contradict the mean valve theorem? Justify.

3. Find the asymptotes of the curve $y^3 - x^2y + 2xy^2 - 2x^3 + 7xy + 3y^2 + 2x^2 - 2x + 2y + 1 = 0$.

For the function $f(x) = \frac{1}{3}x^3 - 9x + 2$, find all the critical points, intervals in which the function increases and decreases and identify relative extrema. Find all inflexion points and intervals in which the function is concave up, concave down. Sketch the graph of the given function.

4. Find the area of the surface that is generated by revolving the portion of the curve $y = 2x^3$ between $x = 0$ and $x = 2$ about the *x*-axis. And find the volume of the given curve lying between $x = 0$ and $x = 4$ about the *x*-axis.

Evaluate $\int (r^2 - x^2)^{5/2} dx$ using reduction formula.

5. Find the equation for the ellipse with foci $(1, \pm 3)$ and major axis with end-points $(1, \pm 6)$.

Determine a rotation angle θ that will eliminate the *xy*-term from the equation $2x^2 + 5xy + 2y^2 + 9\sqrt{2}x + 9\sqrt{2}y + 9 = 0$. Identify the conic, sketch its graph and label it.

6. Sketch the ellipsoid $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{16} = 1$.

A particle moves along the curve $\boldsymbol{r}(t) = \sin t\, i + \cosh t\, j + (\tan^{-1} t) k$, where t is the time. Find $\lim_{t \to 0}(\boldsymbol{r}(t).\boldsymbol{r}'(\mathrm{t}))$. Also for $\boldsymbol{F} = \sin x\ i + \cos(x - y)\, j + z\, k$, find grad $\boldsymbol{F}$, div $\boldsymbol{F}$, curl $\boldsymbol{F}$ and $\nabla \times (\nabla \times \boldsymbol{F})$.