Set A

Name of the Department: Department of Physics and Astrophysics
Name of Course: B.Sc. Hons. Physics-CBCS_Core
Name of the Paper: Statistical Mechanics
Semester-VI
Unique Paper Code: 32221602
Question paper Set number: Set A
Maximum Marks: 75
Time: 2 hours
Attempt any four questions. All questions carry equal marks.

Q1. Differentiate between macrostates and microstates. Consider a system of six distinguishable particles with accessible 3-fold degenerate particle energy states with energies *0, ε, 2ε, 3ε, ….* If the total energy of the system is *5ε*, find all the macrostates of the system, their thermodynamic probabilities, and the total thermodynamic probability of the system. Find the same if the particles are bosons or fermions.

Q2. Two non-identical ideal gases at different temperatures, say T_1 and T_2, be mixed. Calculate the entropy of mixing. Is it different if the two gases were identical?
A mole of Argon (mass number 40) and a mole of Helium (mass number 4) are contained in vessels of equal volume. If Argon is at *T=300 K*, what should be the temperature of Helium so that the two have the same entropy?

Q3. Prove that if radiation of a given wavelength inside a cavity is altered to another wavelength, the temperature changes in inverse ratio.
Calculate average energy of an oscillator of frequency 1.5×10^{18} *Hz* at *T=1800 K*, treating it as a (i) Classical oscillator (ii) Planck's Oscillator.

Q4. For a cloud of atomic Hydrogen at temperature *T*, show that

$$\frac{x^2}{1-x} = \frac{1}{n\lambda_T^3} e^{-\beta I}$$

where *x* is the degree of ionization, *I* is the ionization energy of Hydrogen (*13.6 eV*), *n* is the number density of Hydrogen atoms (both neutral and ionized), and λ_T is the thermal de-Broglie wavelength of the electrons.

Q5. Why is Fermi-Dirac gas alive at *0 K*? Explain. Derive density of states *g(ε)* as a function of energy ε for a free electron gas in 1-dimension and hence calculate its Fermi energy.
Calculate electron density for a strongly degenerate non-relativistic electron gas for which Fermi momentum is $0.1\ m_e c$ where m_e is electron rest mass and *c* is the velocity of light. Also calculate pressure of degenerate gas under these conditions.
Show that the Fermi energy of a relativistic electron gas with the dispersion relation $\varepsilon = pc$ is

$$E_F = hc\left(\frac{3n}{8\pi}\right)^{1/3}$$

where c is the speed of light and n is the electron density. Show that the zero-point energy of such a gas is given by $E_0 = \frac{3}{4} N E_F$.

Q 6. Consider a gas of bosons having spin one and zero rest mass such that N_1 bosons have energy $h\nu_1$, N_2 bosons have energy $h\nu_2$, N_3 bosons have energy $h\nu_3$ … at any given instant of time moving with the speed of light and with total energy $h\nu$. Find the number density and energy density relation for such a gas lying between frequency range ν and $\nu+d\nu$. Compare the distribution function of these particles with bosons of rest mass m each.

Value of Constants:
Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K
Planck's Constant $h = 6.626 \times 10^{-34}$ J-sec.