Name of the Course : **B.Sc. (Hons.) Mathematics CBCS**

Semester : **VI**

Unique Paper Code : **32357607**

Name of the Paper : **DSE-3: Probability Theory and Statistics**

Duration: **2 Hours** Maximum Marks: **75**

*Attempt any four questions. All questions carry equal marks. All symbols have usual meaning.*

**1.** Let the probability density function of a continuous random variable $X$ be given by $f(x) = cx(1-x)$ for $0 < x < 1$ and $f(x) = 0$ elsewhere. Find the value of the constant $c$. Further, find the value of mean, variance, moment generating function of $Y = 3X + 1$ and the probability that $Y$ lies within 2 standard deviations of the mean of $Y$. Compute the cumulative distribution function of $Y$ and use it to find $P[Y > 2.5]$.

**2.** Consider a random experiment of shooting a bullet inside a circular disc of radius $r$. Let a random variable $X$ denote the distance of the bullet mark from the center of the disc along the radius of the disc passing through the bullet mark. Find the average distance $X$, median distance $X$, standard deviation of $X$ and the moment generating function of $X$. Also find the probability that the bullet will hit (i) exactly at the center of the disc (ii) exactly within a concentric circular disc of radius $r/4$ and (iii) exactly on the boundary of the concentric circular disc of radius $r/4$.

**3.** Let $X_1$ and $X_2$ have the joint probability mass function $p(x_1, x_2)$ given by

| $(x_1, x_2)$ | (0,0) | (0,1) | (1,0) | (1,1) | (2,0) | (2,1) |
|---|---|---|---|---|---|---|
| $p(x_1, x_2)$ | 1/18 | 3/18 | 4/18 | 3/18 | 6/18 | 1/18 |

and $p(x_1, x_2) = 0$ elsewhere. Find the marginal probability mass functions $p_{X_1}(x_1)$ and $p_{X_2}(x_2)$ and the conditional means $E(X_2/x_1)$ and $E(X_1/x_2)$.

Further, give example of two random variables $X$ and $Y$ such that $X$ and $Y$ are dependent but the covariance between $X$ and $Y$ is 0.

**4.** Let $X$ and $Y$ have the joint probability density function given by

$$f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Find the joint moment generating function of $X$ and $Y$ and hence compute the coefficient of correlation between $X$ and $Y$.

**5.** State Markov and Chebyshev's inequalities and explain their significance. Let $X$ be a random variable with $E(X) = 3$ and $E(X^2) = 13$. Use the Chebyshev's inequality to determine a lower bound for $P(-2 < X < 8)$. Further, obtain an upper bound for $P[\,|X| \geq 1]$ using Chebyshev's inequality if $X$ is a random variable having probability mass function given by

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $p(x)$ | $1/8$ | $6/8$ | $1/8$ |

**6.** Let $X_1, X_2, \dots X_n$ be a sequence of independent Bernoulli variates such that

$$P(X_i = 1) = p, P(X_i = 0) = q,\ p + q = 1.$$

Let $X = (\sum Xi)/n,\ 1 \leq i \leq n.$ Verify Central Limit Theorem by showing that $X$ tends to be Normal as $n \to \infty$. A customer care center is operating in such a manner that the number of customers attended on a particular day is a random variable with mean of 250 persons and a standard deviation of 20 persons. Use the Central Limit Theorem or otherwise, find the probability that the average (mean) number of customers attended in a random sample of 45 days is at least 255?