Name of the Course : **B.Sc. (Hons.) Mathematics CBCS**

Semester : **IV**

Unique Paper Code : **32351401**

Name of the Paper : **C8 - Partial Differential Equations**

Duration: **2 Hours** Maximum Marks: **75**

Attempt any four questions. All questions carry equal marks. All symbols have usual meaning.

1. Find the general solution of the equation

$$[my\,(x+y) - nu^2]u_x - [lx(x+y) - nu^2]u_y = (lx - my)u$$

2. Reduce the following equation into canonical form and obtain the solution

$$xu_x + yu_y + x^2u = x^2 \text{ with } u(x,y) = \sin(x) \text{ on } y = x^2.$$

3. Derive the one-dimensional heat equation

$$u_t = \kappa u_{xx}, \text{ where } \kappa \text{ is a constant.}$$

4. Transform the equation

$$u_{xx} + 6u_{xy} + 9u_{yy} = 0$$

into the canonical form and then find the general solution.

5. Solve the Goursat problem

$$xy^3u_{xx} - x^3yu_{yy} - y^3u_x + x^3u_y = 0,$$

$$u(x,y) = f(y) \quad \text{on } x = 0 \text{ for } 0 \le y \le 4,$$

$$u(x,y) = g(x) \quad \text{on } x^2 + y^2 = 16 \text{ for } 0 \le x \le 4,$$

$$\text{where } f(4) = g(0).$$

6. Find the solution of the following problem

$$u_t = ku_{xx} - 2h\,,\ 0 < x < 1,\ t > 0, \text{ where } h \text{ is a constant,}$$

$$u(x,0) = u_0(1 - \cos(\pi x)),\ 0 \le x \le 1, \text{ where } u_0 \text{ is a constant}$$

$$u(0,t) = 0,\ u(1,t) = 2u_0\,,\ t \ge 0.$$