This question paper contains 7 printed pages]

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8082**

Unique Paper Code : **32357502** **J**

Name of the Paper : **Mathematical Modelling and Graph Theory**

Name of the Course : **B.Sc. (H) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *ten* parts from Question No. **1**.

Attempt any *two* parts from Question Nos. **2** to **5**.

1. (*a*) Draw a simple connected graph with degree sequence (1, 1, 2, 3, 3, 4, 4, 6) 2

(*b*) What is the number of edges in Q_3 ? 2

(*c*) What is the sum of degrees of vertices of K_{30} ? 2

(*d*) State Ore's Theorem. 2

(*e*) How many 2017- regular graphs with 2019 vertices exist ? Justify. 2

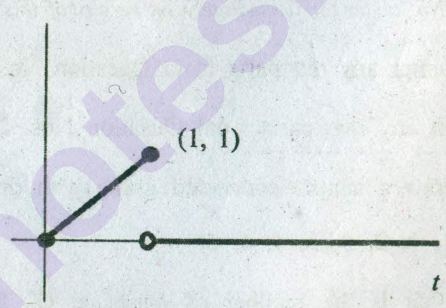
(*f*) Determine whether $x = 0$ is an ordinary point, a regular singular point or an irregular singular point of the differential equation $x^2y'' + (6\sin x)y' + 6y = 0$. 2

P.T.O.

(*g*) Find the Laplace transform of the function $f(t) = (1+t)^3$. 2

(*h*) Find the inverse Laplace transform of the function $F(s) = 2s^{-1}e^{-3s}$. 2

(*i*) Apply the definition to find directly the Laplace transform of the function described in the following figure : 2



(*j*) Many computers use the Linear Congruence Method for generating pseudorandom numbers.The method is given by the rule :

$$x_{n+1} = (ax_n + b) \bmod c$$

What value of c is usually taken by computers to avoid cycling ? 2

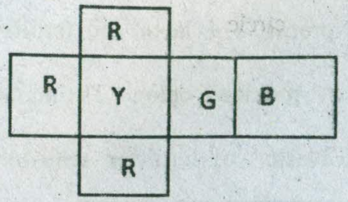
(*k*) A Montana farmer owns 45 acres of land. She is planning to plant each acre with wheat or corn. Each acre of wheat yields $200 in profits, whereas each acre of corn yields $300 in profits. The labor and fertilizer requirements for each are provided below. The farmer has 100 workers and 120 tons of fertilizer available.

	Wheat	**Corn**
Labor (workers)	3	2
Fertilizer (tons)	2	4

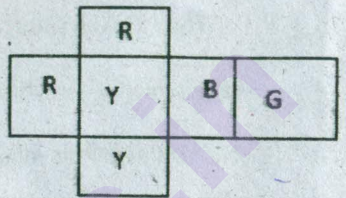
The farmer wants to plant wheat and corn such that her profit maximizes. Formulate the linear programming problem, clearly specifying the objective function and the constraints. 2

2. (*a*) Let G be a simple connected graph with n vertices, where $n \geq 3$ and deg $v \geq n/2$ for each vertex v. Use Ore's Theorem to show that G is Hamiltonian. Give an example of a Hamiltonian graph that does not satisfy the conditions of Ore's Theorem. 6.5

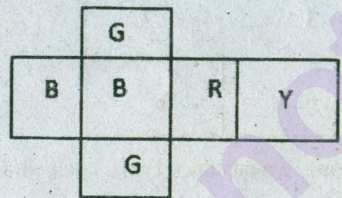
(*b*) Find solution to the four-cubes problem for the following set of cubes : 6.5



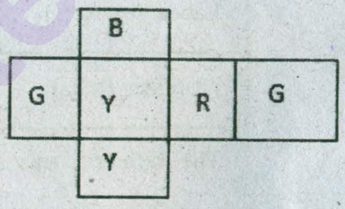
cube 1



cube 2



cube 3



cube 4

(*c*) By finding an Eulerian trail in K_5, arrange a set of fifteen dominoes (from 0–0 to 4–4) in a ring. 6.5

3. (*a*) Use Laplace transforms to solve the initial value problem : 7

$$x'' + 6x' + 18x = \cos 2t; \quad x(0) = 1, \ x'(0) = -1.$$

(*b*) Find two linearly independent Frobenius series solutions of : 7

$$3x^2 y'' + 2xy' + x^2 y = 0 .$$

(*c*) Solve the initial value problem : 7

$$(x^2 + 6x)y'' + (3x+9)y' - 3y = 0; \ y(-3) = 0, \ y'(-3) = 2 .$$

4. (*a*) A small harbor has unloading facilities for ships. Only one ship can be unloaded at any one time. Ships arrive for unloading of cargo at the harbor, and the time between the arrival of successive ships varies from 15 to 145 minutes. The unloading time for ships varies from 45 to 90 minutes. For 5 ships, the data is as given :

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5
Time between successive ships	20	30	15	120	25
Unloading Time	55	45	60	75	80

Draw time-line for each ship and hence answer the following questions :

(*i*) What is the average waiting time for the ships ?

(*ii*) For how much time the harbor remains idle ? 7

(*b*) Using Monte-Carlo simulation, write an algorithm to calculate an approximation to π by considering the number of random points selected inside the quarter circle :

$$Q : x^2 + y^2 = 1,\ x \geq 0,\ y > 0$$

where the quarter circle is taken to be inside the square

$$S : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1.$$ 7

(*c*) Solve using Simplex Method :

Maximize $10x + 35y$

Subject to

$$4x + 3y \leq 24$$

$$4x + y \leq 20$$

$$x, y \geq 0.$$ 7

5. (*a*) Show that there is no knight's tour on 3×6 chessboard. 7

(*b*) Use the factorization :

$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

to derive the inverse Laplace transform to show that :

$$L^{-1}\left\{\frac{s^2}{s^4 + 4a^4}\right\} = \frac{1}{2a}(\cosh at \sin at + \sinh at \cos at).$$ 7

(*c*) Solve the problem :

Maximize $25\ x_1 + 30\ x_2$

subject to

$$20x_1 + 30x_2 \leq 690$$

$$5x_1 + 4x_2 \leq 120$$

$$x_1, x_2 \geq 0$$

Suppose the second constraint is changed to :

$$5x_1 + 4x_2 \leq b_2$$

What is the change in the value of the objective function as b_2 increases by one unit in the range $92 \leq b_2 \leq 172.5$? 7