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This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper	:	**8562**
Unique Paper Code	:	**42351101** **J**
Name of the Paper	:	**Calculus and Matrices**
Name of the Course	:	**B.Sc. Mathematical Sciences/B.Sc. (Prog.)**
Semester	:	**I**

Duration : **3 Hours**   Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* questions from each section.

Section I

1. (*a*) Examine the existence of the limit of the function :

$$g(t)=\begin{cases} t-2 & : \quad t<0 \\ t^2 & : \quad 0\le t\le 2 \\ 2t & : \quad t>2 \end{cases}$$

at $t = 0, 2$.

(*b*) Find a value of the constant k, if possible, that will make the function continuous everywhere.

$$f(x)=\begin{cases} 7x-2 & x\le 1 \\ kx^2 & x>1 \end{cases}.$$

P.T.O.

(c) If $y = x^2 \sin x$, then prove that :

$$\frac{d^n y}{dx^n} = (x^2 - n^2 + n)\sin\left(x + \frac{n\pi}{2}\right) - 2nx\cos\left(x + \frac{n\pi}{2}\right).$$ 4+4+4

2. (a) Show that the function $y = |x|$ is differentiable on $(-\infty, 0)$ and $(0, \infty)$ but has no derivative at $x = 0$.

(b) If $y = a\cos(\log x) + b\sin(\log x)$, then show that :

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

(c) Find the Maclaurin series for the function $f(x) = \frac{1}{x+1}$ assuming the validity of expansion. 4+4+4

3. (a) State and prove Lagrange's mean value theorem. Also discuss its geometrical significance.

(b) Find the value of c for the following function that satisfies the hypotheses of the Lagrange's mean value theorem :

$$f(x) = x^2 + 2x - 1,\ a = 0,\ \ b = 1.$$

(c) Prove that :

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0.$$ 5+4+3

4. (a) Sketch the graphs of the following functions (any *two*) :

(i) $y = 1 + \sqrt{x-1}$

(ii) $y = \sin 2x$ in $[0, 2\pi]$

(iii) $y = e^{-|x|} - 1$.

(b) Given the function $f(x) = |x|$. The graph of the function $f(x)$ is shifted vertically down 3 units and horizontally right 2 units followed by a reflection across x-axis. Sketch the original function $f(x)$ along with the new graph. Also write the equation for the new graph. 6+6

Section II

5. (a) Sketch the contour plot of $f(x) = x^2 + y^2$ using the level curves at heights $k = 0, 3, 5$.

(b) Let $f(x, y) = x^2 + y^2 - 2$. Find an equation of the level curve that passes through the point $(1, -2, 0)$.

(c) Let. $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. Find the eigenvalues and the corresponding eigenvectors of the matrix A.

4+4+5

6. (*a*) Verify that $u(x, t) = \sin(x - 4t)$ is a solution of the wave equation.

(*b*) Row reduce the matrix A to reduced row echelon form. Circle the pivot positions in the final matrix and hence determine its rank :

$$A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}.$$

(*c*) For what value of λ and μ do the following system of linear equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have :

(*i*) a unique solution

(*ii*) no solution

(*iii*) an infinite number of solutions. 4+4+5

7. (*a*) Let

$$f(x, y) = x^2 y + 5y^3.$$

Find the slope of the surface $z = f(x, y)$ in *x*-direction at the point $(1, -2)$.

(*b*) Check whether the set $\{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$ is linear independent or not.

(*c*) Check whether the transformation $T : R^2 \to R^2$ defined as $T(x, y) = (x + 4y, y)$ is linear Sketch the image of the unit square with vertices (0, 0), (0, 1), (1, 1), (1, 0) under the given transformation. 4+4+5

8. (*a*) Find the standard matrix of the reflection about *xz* plane.

(*b*) Find the polar representation of the following numbers :

(*i*) $z_1 = -1 - i$.

(*ii*) $z_2 = 1 - i\sqrt{3}$.

(*c*) If $z_1 = 1 - i$ and $z_2 = \sqrt{3} + i$, then find $Arg(z_1 z_2)$ and $|z_1 z_2|$. 5+4+4

9. (*a*) Find the equation of the circle whose radius is 3 and whose center has affix $1-i$.

(*b*) Find the equation of the straight line joining the points whose affixes are $z_1 = 1-i$ and $z_2 = 2-5i$.

(*c*) Compute $(1+i)^{1000}$.

(*d*) Solve the equation using De Moivre's theorem $z^7 + z = 0$. 3+3+3+4