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[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 7277 **J**

Unique Paper Code : 42354302

Name of the Paper : Algebra

Name of the Course : **B.Sc. (Prog.) / Mathematical Sciences**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.
5. Marks are indicated.

UNIT – I

1. (a) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix}; a \in \mathbb{R};\ a \neq 0 \right\}$

Show that G is an abelian group under matrix multiplication. (6)

(b) Describe the symmetries of a non-square rectangle. Construct the corresponding Cayley table. (6)

(c) Let $H = \{x \in U(20) : x \equiv 1 \bmod 3\}$. List all elements of H. Prove or disprove that H is a subgroup of U(20). (6)

2. (a) Prove that an abelian group with two elements of order 2 must have a subgroup of order 4. (6)

(b) Define Cyclic Group. Is U(8) with the operation of multiplication modulo 8 a cyclic group? Justify. (6)

(c) Let $a, b \in S_n$. Prove that $a^{-1}b^{-a}ab$ is an even permutation. (6)

3. (a) State Lagrange's theorem for finite groups. Prove that in a finite group, the order of each element of the group divides the order of the group. (6)

(b) Let H be a subgroup of G and $a, b \in G$. Prove that either $aH = bH$ or $aH \cap bH = \phi$. (6)

(c) Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} ; a, b, d \in \mathbb{R}; ad \neq 0 \right\}$. Is H a normal subgroup of $GL(2, \mathbb{R})$? Justify. (6)

UNIT – II

4. (a) Define center of a ring R. Prove that center of a ring is a subring of R. (6.5)

(b) Define field and an integral domain. Prove that every field is an integral domain. Is the converse true? Justify. (6.5)

(c) Find all zero divisors in $\mathbb{Z}_{20}$. What is the relationship between the zero divisors and the units of $\mathbb{Z}_{20}$? (6.5)

UNIT – III

5. (a) Determine whether or not the set

$$\left\{ \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

is linearly independent over $\mathbb{Z}_5$. (6.5)

(b) Let $V = \left\{ \begin{pmatrix} a & b \\ b & c \end{pmatrix} ; a, b, c \in \mathbb{Q} \right\}$ be a vector space over $\mathbb{Q}$. Find a basis of V over $\mathbb{Q}$. (6.5)

(c) Which of the following is a subspace of $\mathbb{R}^3$? Justify.

(i) $S=\{(a,b,c)\in\mathbb{R}^3 : 2a+3b=4c\}$

(ii) $T=\{(a,b,c)\in\mathbb{R}^3 : a^2+b^2=c^2\}$ (6.5)

6. (a) Which of the following function T from $\mathbb{R}^2$ into $\mathbb{R}^2$ is a linear transformation? Justify

(i) $T(a, b) = (a-b, 0)$

(ii) $T(a, b) = (a^2, b)$ (6.5)

(b) Let $T: \mathbb{C}^3 \rightarrow \mathbb{C}^3$ be a linear transformation defined by

$T(x, y, z) = (x-y+2z,\ 2x+y,\ -x-2y+2z)$.

If $(a, b, c) \in \mathbb{C}^3$, what are the conditions on a, b and c so that the vector be in range of T? What is the rank of T? (6.5)

(c) Let $T:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T(1,2) = (2,3)$ and $T(0,1) = (1,1)$. Find $T(a,b)$ for any $(a,b) \in \mathbb{R}^2$. (6.5)