[This question paper contains 5 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 7273   J

Unique Paper Code   : 42351101 – OC

Name of the Paper   : Calculus and Matrices

Name of the Course   : **B.Sc. (Mathematical Sciences) / B.Sc. (Prog.)**

Semester   : I

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1.   Write your Roll No. on the top immediately on receipt of this question paper.

2.   Attempt any **two** questions from each section.

SECTION – I

1.   (a) Verify that the set $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a

basis of R^4.   (6)

P.T.O.

(b) Is $W = \left\{ \begin{bmatrix} x \\ y \\ 2x \end{bmatrix} : xy > 0 \right\}$ a subspace of R^3? Justify your answer. (6)

2. (a) Let $T : R^3 \to R^3$ be defined by

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y \\ y-z \\ x-z \end{bmatrix}$$

Show that T is a linear transformation. Also find a matrix representation for T. (6)

(b) Find the eigenvalues of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}.$$

(6)

3. (a) Determine the unique solution of the following system of equations

$x + y + z = 6$

$2x + 3y + 4z = 20$

$x + y = z.$ (6)

(b) Find the rank of the matrix

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}.$$

(6)

SECTION – II

4. (a) (i) Determine whether the sequence $\left\{ 1 + \dfrac{(-1)^n}{n} \right\}$ is bounded and monotonic.

(ii) Compute $\lim\limits_{n \to \infty} \left\{ \dfrac{\cos n}{n} \right\}$. (6)

(b) Find the nth derivative of $y = e^{3x} \sin(4x+1)$. (6)

(c) If $y = (\sin^{-1} x)^2$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$$

(6)

5. (a) Sketch the graph of $y = e^{-x} + 1$. (6)

P.T.O.

(b) According to Newton's Law of Cooling, the rate at which a substance cools in air is proportional to the difference between the temperature of the substance and that of the surrounding air. If the temperature of the air is 30°C and the substance cools from 100°C to 80°C in 20 minutes, find when the temperature will be 40°C. (6)

(c) Find the Taylor series generated by $f(x) = \dfrac{1}{x}$ at $x = 2$. (6)

6. (a) Draw the level curves for the surface $z = 9x^2 + 25y^2$ at heights $k = 1, 2, 3$. (6)

(b) Find all the second order partial derivatives of $f(x, y) = e^{x-3y}$. (6)

(c) Verify that $z = e^x \sin(y) + e^y \cos(x)$ is a solution of the Laplace equation. (6)

SECTION – III

7. (a) Prove that the product of all the nth roots of unity is $(-1)^{n-1}$. (4)

(b) Represent graphically the set $\{z : |z| \le |z - 1|\}$. (3½)

8. (a) Evaluate $\dfrac{(\cos\alpha + i \sin\alpha)^4}{(\sin\beta + i \cos\beta)^5}$. (4)

(b) State fundamental theorem of algebra. Form an equation in lowest degree with rational coefficients having $\sqrt{3} + 2$ and $\sqrt{5} - 2$ as two of its roots. (3½)

9. (a) Find the equation of the circle described on the join of the points given by $-1 -3i$ and $5+7i$ as extremities of one of its diameters. (4)

(b) Find the equation of the straight line joining the points whose affixes are $2-5i$ and $1-i$. (3½)