This question paper contains 4+2 printed pages] 4/11/19 M

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7941**

Unique Paper Code : **32357501** **J**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics : DSE-1**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the *six* questions are compulsory.

Attempt any *two* parts from each question.

Marks are indicated against each question.

Use of Non-Programmable Scientific Calculator is allowed.

1. (*a*) Find a root of the equation $x \sin(x) + \cos(x) = 0$ using Newton-Raphson Method for starting approximation $x_0 = \pi$. 6.5

P.T.O.

(*b*) A real root of the equation $x^3 - 5x + 1 = 0$ lies in]0, 1[. Perform three iterations of Bisection Method to obtain the root. 6.5

(*c*) Prove that fixed point method converges at a linear rate using $g(x) \equiv x^2 - 2x - 3 = 0$ and starting approximation $x_0 = 4$. 6.5

2. (*a*) Using Secant Method, find a real root of the equation $xe^x - 1 = 0$ correct upto four decimal places. Given the root lies between 0 and 1, perform three iterations. 6

(*b*) Consider the function $g(x) = 1 + x - \frac{x^3}{8}$. Analytically verify that this function has unique fixed point on the real line. Perform three iterations starting at $x_0 = 0$ to locate the fixed point. 6

(c) Evaluate order of convergence of Newton Method numerically using $f(x) \equiv x^5 + 2x - 1 = 0$ and initial approximation $x_0 = \frac{1}{2}$. 6

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \quad 4 \quad 1]^T$. 6.5

(b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$4x_1 + x_2 + 2x_3 = 4$$

$$3x_1 + 5x_2 + x_3 = 7$$

$$x_1 + x_2 + 3x_3 = 3$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. 6.5

(*c*) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. 6.5

4. (*a*) Construct the Lagrange form of the interpolating polynomial from the following data : 6

x	-1	0	1
$f(x) = e^x$	e^{-1}	e^0	e^1

(*b*) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial :

x	-1	0	1	2
y	5	1	1	11

Hence, estimate the value of $f(0.5)$. 6

(c) Find the maximum value of the step size h that can be used in the tabulation of $f(x) = e^x$ on the interval [0, 1] so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4}. 6

5. (a) Define the forward difference operator Δ, the central difference operator δ and the averaging operator μ. Prove that :

(i) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{1}{4}\delta^2}$

(ii) $\mu = \left(1+\frac{1}{2}\Delta\right)(1+\Delta)^{-\frac{1}{2}}$. 6.5

(b) Use the formula

$$f'(x_0) \approx \frac{f(x_0)-f(x_0-h)}{h}$$

to approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ taking $h = 1, 0.1, 0.01$. What is the order of approximation ? 6.5

(c) Derive the following forward difference approximation for the second derivative : 6.5

$$f''(x_0) \approx \frac{f(x_0) - 2f(x_0 + h) + f(x_0 + 2h)}{h^2}.$$

6. (a) Define degree of precision of a quadrature rule. Show that the degree of precision of the Trapezoidal rule is 1. 6

(b) Approximate the value of the integral $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's 1/3rd rule. 6

(c) Use Euler's method to approximate the solution of the initial value problem.

$x' = tx^3 - x$, $x(0) = 1$, $0 \le t \le 1$ taking 4 steps. 6