12/12/19 M

[This question paper contains 4 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **7467** **J**

Unique Paper Code : 32351502

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Group Theory - II

Semester : V

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

(c) All questions carry equal marks.

1. (a) Let Inn (D_8) denotes the group of inner automorphisms on the dihedral group D_8 of order 8. Find Inn (D_8). 6

(b) Define inner automorphism of a group G induced by $g \in G$. Then prove that the set Inn(G) of all inner automorphism of a group G is a normal subgroup of the group Aut(G) of all automorphisms of G. 2+4

P.T.O.

(c) Let G be a cyclic group of order n. Then prove that Aut(G) is isomorphic to U(n). Here Aut(G) denotes the group of automorphisms on G and $U(n) = \{m \in N : m < n \text{ and } \gcd(m, n) = 1\}$ is a group under multiplication modulo n. 6

2. (a) Prove that every characteristic subgroup of a group G is a normal subgroup of G. Is the converse true ? Justify. 4+2

(b) Let G_1 and G_2 be finite groups. If $(g_1,g_2) \in G_1 \oplus G_2$, then prove that

$$|(g_1,g_2)| = \text{lcm}(|g_1|, |g_2|)$$

where $|g|$ denotes order of an element g in a group G. 6

(c) Prove that D_8 and S_3 cannot be expressed as an internal direct product of two of its proper subgroups. Here D_8 and S_3 denote the dihedral group of order 8 and the symmetric group on the set {1, 2, 3} respectively. 3+3

3. (a) State Fundamental Theorem for Finite Abelian Groups. Find all Abelian groups (upto isomorphism) of order 1176. 2+4

(b) Let G be an Abelian group of order 120 and G has exactly three elements of order 2. Determine the isomorphism class of G. 6

(c) For a group G, let the mapping from $G \times G \to G$ be defined by $(g, a) \to gag^{-1}$. Then prove that this mapping is a group action of G on itself. Also, find kernel of this action and the stabilizer G_x of an element $x \in G$. 2+2+2

4. (a) Let $G=\{1,a,b,c\}$ be the Klein 4-group. Label the group elements 1,a,b,c as integers 1,2,3,4 respectively. Compute the permutation σ_a, σ_b and σ_c induced by the group element a, b, c respectively under the group action of G on itself by left multiplication. 6.5

(b) Let G act on a set A. If $a,b \in A$ and $b=g.a$ for some $g \in G$, then prove that $G_b = gG_ag^{-1}$ where G_a is the stabilizer of a . Deduce that if G acts transitively on A then kernel of the action is $\bigcap_{g \in G} g\, G_a\, g^{-1}$. 3+3.5

(c) Let G be a group acting on a non empty set A and $a \in A$. Then prove that the number of elements in orbit containing a is equal to index of the stabilizer of a. 6.5

5. (a) State the class equation for finite groups. Find conjugacy classes of the quaternion group Q_8 and hence verify the class equation for Q_8. 2+3+1.5

(b) Let p be a prime and P be a group of prime power order p^α for some $\alpha \geq 1$. Then prove that P has a non trivial centre. Deduce that a group of order p^2 is an Abelian group. 4+2.5

(c) Let G be a non-Abelian group of order 231. Then prove that a Sylow 11-subgroup is normal and is contained in the centre of G. 2.5+4

6. (a) Let G be a group of order pq such that $p < q$ and p does not divide (q -1). Then prove that G is a cyclic group. Hence deduce that a group of order 33 is cyclic. 4.5+2

(b) Define a simple group. Prove that groups of order 72 and 56 are not simple. 1 + 2.5 + 3

(c) Let G be a group such that $|G|=2n$, where $n \geq 3$ is an odd integer. Then prove that G is not simple. 6.5