This question paper contains 4 printed pages] 7/12/19 M

Roll No. | | | | | | | | | | |

S. No. of Question Paper : 8597

Unique Paper Code : 32351101 J

Name of the Paper : Calculus

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : I

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

Attempt any *four* questions from Section I.

1. State Leibnitz's theorem for finding nth derivative of product of two functions. If $y = a\cos(\ln x) + b\sin(\ln x)$, prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

P.T.O.

2. Evaluate the following limit :

$$\lim_{x \to 0^+} x^{\sin x}.$$

3. Find the intervals of increase and decrease of the following function, discuss its concavity and then sketch its graph $y = (x+1)^2(x-5)$.

4. Sketch the graph of the polar curve $r = 3\cos 2\theta$.

5. A manufacturer estimates that when 'x' units of a particular commodity are produced each month, the total cost (in dollars) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that corresponds to the maximum profit.

Section II

Attempt any *four* questions from Section II.

6. Find a reduction formula for $\int \operatorname{cosec}^n x\,dx$, $n \geq 2$ is an integer. Evaluate $\int \operatorname{cosec}^4 x\,dx$.

7. Find the volume of the solid generated when the region bounded by $y = \sqrt{25 - x^2}$, $y = 3$, is revolved about the x-axis.

8. The base of a certain solid is enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross-section perpendicular to the x-axis is a semicircle with its diameter across the base. Find the volume of the solid.

9. Find the arc length of the parametric curve :

$$x = (1 + t)^2,\ y = (1 + t)^3,\ 0 \leq t \leq 1.$$

10. Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, about the x-axis.

Section III

Attempt any *three* questions from Section III.

11. Find the equation of the parabola whose focus is (–1, 4) and directrix is $x = 5$.

12. Find the equation of the hyperbola whose foci are (1, 8) and (1, –12) and vertices are 4 units apart.

13. Describe the graph of the equation :

$$9x^2 + 4y^2 + 18x - 24y + 9 = 0.$$

14. Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

Section IV

Attempt any *four* questions from Section IV.

15. Evaluate :

$$\lim_{t->0^+} [\frac{\sin 3t}{\sin 2t}\hat{i} + \frac{\log(\sin t)}{\log(\tan t)}\hat{j} + (t\log t)\hat{k}].$$

16. The acceleration of a moving particle is $\vec{A}(t) = 24t^2\hat{i} + 4\hat{j}$. Find the particle's position as a function of t if $\vec{R}(0) = \hat{i} + 2\hat{j}$ and $\vec{v}(0) = 0$.

17. If a shot putter throws a shot from a height of 5 ft with an angle of 46° and initial speed of 25 ft/sec, what is the horizontal distance of the throw ?

18. Find $\vec{T}(t)$, $\vec{N}(t)$ and $\vec{B}(t)$ for $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + \hat{k}$ at $t = \frac{\pi}{4}$.

19. Show that the curvature of the polar curve $r = e^{\alpha\theta}$ is inversely proportional to r.