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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8617** 13/12/19 M

Unique Paper Code : **32351102** **J**

Name of the Paper : **Algebra**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *six* questions are compulsory.

Do any *two* parts from each question.

1. (*a*) Solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetical progression. 5

(*b*) Find all the rational roots of $96y^3 - 16y^2 - 6y + 1 = 0$. 5

(*c*) (*i*) Find the geometric image of the complex numbers z, such that $|z + i| \geq 2$.

(*ii*) Find the polar representation of the complex number $z = -4i$ and find $\text{Arg } z$. 2,3

2. (*a*) Find all complex numbers z such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$. 5

P.T.O.

(*b*) Solve the equation : 5

$$z^4 = 5(z-1)(z^2-z+1).$$

(*c*) Show that : 5

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

3. (*a*) For (x, y) and (u, v) in R^2, define $(x, y) \sim (u, v)$ if $x^2+y^2 = u^2+v^2$.

Prove that $\sim$ defines an equivalence relation on R^2. Find equivalence classes of (1, 0) and (1, 1). 6

(*b*) Suppose $f : A \to B$ and $g : B \to C$ are functions :

(*i*) If *gof* is one-to-one and *f* is onto, prove that *g* is one-to-one.

(*ii*) If *gof* is onto and *g* is one-to-one, prove that *f* is onto. 3,3

(c) Prove that the intervals (0, 1) and (0, ∞) have the same cardinality. 6

4. (*a*) (*i*) Suppose *a* and *b* are integers and *p* is a prime such that $p|ab$. Then prove that $p|a$ or $p|b$.

(*ii*) Find the quotient *q* and the remainder *r* as defined in division algorithm. If $a = -517$ and $b = 35$. 3½,3

(*b*) Using Euclid's Algorithm, find integers *x*, *y* such that $150x + 284y = 4$. 6½

(*c*) Using Principle of Mathematical Induction prove that for any $x \in R$, $x > -1$, $(1+x)^n \geq 1+nx$, $\forall n \in N$. 6½

5. (*a*) Consider the following system of linear equations :

$$x_1 + 3x_2 + x_3 = 1$$
$$-4x_1 - 9x_2 + 2x_3 = -1$$
$$-3x_2 - 6x_3 = -3$$

Write the matrix equation and the vector equation of the above system of equations. Find the general solution in parametric vector form by reducing the augmented matrix to echelon form. 7½

(*b*) Let $T : R^2 \to R^3$ be a linear transformation such that :

$$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$$

(*i*) Find standard matrix of T.

(*ii*) Is T one-to-one ? Is T onto ? Justify your answers.

(*iii*) Find X such that T(X) = (–1, 4, 9). 7½

(*c*) (*i*) Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find an eigenvector corresponding to an eigenvalue $\lambda = 3$.

(*ii*) Show that if λ is an eigenvalue of A and $p(t) = c_0 + c_1 t + c_2 t^2 + ... + c_n t^n$, then one eigenvalue of $p(A)$ is $p(\lambda)$. 5,2½

6. (*a*) (*i*) Using homogeneous coordinates, find the 3×3 matrix that produce the following composite transformation : Reflect points through the x-axis, and then rotate 30° about the origin.

(*ii*) Show that $H = \{(a, b, c) \in R^3 \mid b = 2a + 3c\}$ is a subspace of R^3. 5,2½

(*b*) Let $S = \{v_1, v_2, v_3, v_4\}$ where $v_1 = (1, 2, 2)$, $v_2 = (3, 2, 1)$, $v_3 = (11, 10, 7)$, $v_4 = (7, 6, 4)$. Find a basis for the subspace W = span S of R^3. What is dim W ? 7½

(*c*) Compute the rank and nullity of the matrix A. Show that rank A + nullity A = number of columns of A.

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & -1 & 3 \\ 7 & -8 & 3 \\ 5 & -7 & 0 \end{bmatrix}.$$ 7½