(E) lib. 17-12-19

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8649**

Unique Paper Code : **62351101** **J**

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Programme) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

Marks are indicated against each question.

1. (*a*) Evaluate : $\lim_{x\to 0} \frac{e^{1/x}-1}{e^{1/x}+1}$. 6

(*b*) Examine for points of discontinuity of the function f defined on [0, 1] as follows :

$$f(x) = \begin{cases} 0, \text{ if } x = 0 \\ \frac{1}{2} - x, \text{ if } 0 < x < \frac{1}{2} \\ \frac{1}{2}, \text{ if } x = \frac{1}{2} \\ \frac{3}{2} - x, \text{ if } \frac{1}{2} < x < 1 \\ 1, \text{ if } x = 1 \end{cases}$$

State the types of discontinuity also. 6

(c) Discuss the derivability of the function :

$$f(x) = \begin{cases} -x^2, & \text{if } x \le 0 \\ 5x-4, & \text{if } 0 < x \le 1 \\ 4x^2-3x, & \text{if } 1<x<2 \\ 3x+4, & \text{if } x \ge 2 \end{cases}$$

at $x = 0, 1, 2.$ 6

2. (a) Show that $y = x + \tan x$ satisfies the differential equation

$$\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0.$$ 6.5

(b) If $y = \cos(m\sin^{-1} x)$, show that : 6,5

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-m^2)y_n = 0.$$

(c) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, use Euler's theorem to show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2} \tan u.$$ 6,5

3. (a) Find the point on the curve :

$$x = a(\theta + \sin\theta), y = a(1-\cos\theta),$$

where the tangent is perpendicular to x-axis. 6

(b) Find the equation of the normal at (a, a) to the curve : 6

$$x^2y^3 = a^5.$$

(c) Find the radius of curvature at any point of the curve : 6

$$x = a(\cot t + t\sin t), y = a(\sin t - t\cos t)$$

4. (a) Find the asymptotes of the following curve : 6,5

$$xy^3 - x^3 = a(x^2+y^2).$$

(b) Find the position and nature of the double points on the curve : 6,5

$$y^2 = 2x^2y + x^4y - 2x^4.$$

(c) Trace the curve : 6,5

$$9ay^2 = x(x-3a)^2.$$

5. (a) Show that there is no real number 't' for which the equation $x^2 - 3x + t = 0$ has two distinct roots in $[0, 1]$. 6

(b) Show that : 6

$$\frac{x}{1+x} < \log(1+x) < x \text{ for all } x > 0.$$

(c) State Lagrange's mean value theorem. Explain why Lagrange's mean value theorem is not applicable to the following function :

$$f(x) = \begin{cases} x, & \text{if } x \ne 0 \\ 0, & \text{if } x = 0 \end{cases}$$

for $x \in [-1, 1]$. 6

6. (*a*) Assuming the validity of expansion, find the Maclaurin's series expansion of $e^x \cos x$. 6.5

(*b*) Derermine the values of p and q for which

$$\lim_{x \to 0} \frac{x(1 + p\cos x) - q\sin x}{x^3}$$ exists and equals 1. 6.5

(*c*) Show that $x^4 - 4x^3 + 6x^2 - 4x + 1$ has a maximum at $x = 1$. 6.5