13/12/19 E

[This question paper contains 4 printed pages]

Your Roll No. : .....

Sl. No. of Q. Paper : **7099** **J**

Unique Paper Code : 62354343

Name of the Course : **B.A. (Prog.) Mathematics**

Name of the Paper : Analytical Geometry and Applied Algebra

Semester : III

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(i) Write your Roll No. on the top immediately on receipt of this question paper.

(ii) **All** questions are compulsory.

(iii) Attempt any **two** parts from each question.

1. (a) Identify and sketch the graph

$y = 4x^2 + 8x + 5$

Also label the focus, vertex and directrix. 6.5

(b) Describe the graph of the curve

$x^2 + 9y^2 + 2x - 18y + 1 = 0$

Find its foci, vertices and the ends of the minor axis. 6.5

P.T.O.

(c) Sketch the hyberbola

$16x^2 - y^2 - 32x - 6y = 57$

Also find its vertices, foci and asymptotes. 6.5

2. (a) Find an equation for the parabola that has vertex (5, –3), axis parallel to the y-axis and passes through (9,5). 6

(b) Find an equation for the ellipse that has ends of major axis (±6,0) and passes through(2,3). 6

(c) Find an equation for a hyperbola that satisfies the given conditions :

Asymptotes $y = 2x + 1$ and $y = -2x + 3$ passes through the origin. 6

3. (a) Consider the equation $xy = -9$. Rotate the coordinate axes to remove xy-term.Then identify and sketch the curve. 6.5

(b) Let an x'y' coordinate system be obtained by rotating an xy - coordinate system through an angle of $\theta = 60^0$.

(i) Find the x'y'- coordinates of the point whose xy – coordinates are (–2,6)

(ii) Find an equation of the curve $\sqrt{3}xy + y^2 = 6$ in x'y' - coordinates. 6.5

(c) (i) Describe the surface whose equation is given as

$x^2 + y^2 + z^2 + 2y - 6z + 25 = 0.$

(ii) Determine and sketch the surface represented by the equation $x^2 + y^2 = 25$ in 3-space. 6.5

4. (a) Find u and v if $5u + 2v = 6i - 5j + 4k$ and $3u - 4v = i + 2j + 9k$. Also find a vector of length 3 and oppositely directed to v. 6

(b) (i) Show that direction cosines of a vector satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(ii) Determine if $u = \langle 6, 1, 3 \rangle$ and $v = \langle 4, -6, -7 \rangle$ make an acute angle, an obtuse angle or are orthogonal ? Justify your answer. 6

(c) Find the volume of the parallelepiped with adjacent edges $u = 3i + 2j + k$, $v = i + j + 2k$ and $w = i + 3j + 3k$. Also find the area of the face determined by u and v. 6

5. (a) Find the distance from the point P(1,4,-3) to the line 6.5

$L: x = 2 + t,\ y = -1 - t,\ z = 3t.$

(b) Find the equation of the plane through the points P_1 (–2,1,4), P_2 (1,0,3) that is perpendicular to the plane $4x - y + 3z = 2$. 6.5

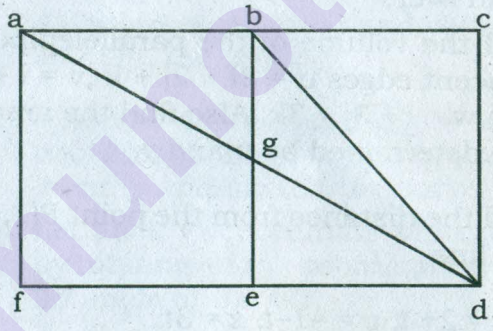
(c) Find the distance between the skew lines

$L_1: x = 1 + 4t,\ y = 5 - 4t,\ z = -1 + 5t;$

$L_2: x = 2 + 8t,\ y = 4 - 3t,\ z = 5 + t.$ 6.5

6. (a) Suppose there are three pitchers of water with capacity 8L,5L and 3L. Initially, the 8 L pitcher is full and the other two are empty. Is there a way to pour among pitchers to obtain exactly 4 litres in 5L pitcher or 3L pitcher ? If so, find the minimal sequence of pourings to get 4 litres in either 5L pitcher or 3L pitcher. 6

(b) For the following graph find a minimal edge cover and a maximal independent set of vertices 6



(c) A supermarket wishes to test the effect of putting cereals on five shelves at different heights. Show how to design such an experiment lasting five weeks and using five brands of the cereal. 6