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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7168**

Unique Paper Code : **62357502** **J**

Name of the Paper : **Differential Equations**

Name of the Course : **B.A. (Prog.) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Solve the initial value problem : 6

$$(y + \sqrt{x^2 + y^2})\, dx - x dy = 0; \quad y(1) = 0 .$$

(*b*) Solve : 6

$$(xy^2 - x^2) dx + (3x^2y^2 + x^2y - 2x^3 + y^2)\, dy = 0$$

(*c*) Solve : 6

$$y + px = x^4 p^2$$

2. (*a*) Solve : 6.5

$$\frac{d^2y}{dx^2} + 4 = \cos 2x + \sin 2x$$

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(*b*) Solve : 6.5

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\,(x + \frac{1}{x})$$

(*c*) Consider the differential equation : 6.5

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

(*i*) Verify that $y_1 = e^x$ and $y_2 = e^{2x}$ are the solutions of the above differential equation.

(*ii*) Find a particular solution of the form

$$y = c_1 y_1 + c_2 y_2$$

that satisfies the initial condition $y(0) = 1$, $y'(0) = 0$.

3. (*a*) Using the method of variation of parameters, solve : 6

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\,e^x \log x, \quad (x > 0)$$

(*b*) Given that $y = x + 1$ is a solution of differential equation : 6

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0.$$

Find a linearly independent solution by reducing the order and write the general solution.

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(c) Solve : 6

$$x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 2y = x^2 \log x + 3x$$

4. (a) Solve the following system of equations : 6.5

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0,$$

$$\frac{dy}{dt} + 3y + 5x = 0.$$

(b) Solve : 6.5

$$\frac{dx}{1} = \frac{dy}{2} = \frac{dz}{5z + \tan(y - 2x)}.$$

(c) Solve : 6.5

$$(ydx + xdy)(a - z) + xydz = 0.$$

5. (a) Eliminate the arbitrary function f from the equation : 6

$$z = e^{ax+by} f(ax - by)$$

to find the corresponding partial differential equation.

(b) Find the general solution of the differential equation : 6

$$x(y^2 - z^2)q - y(x^2 + z^2)p = (x^2 + y^2)z.$$

(c) Find the complete integral of the partial differential equation : 6

$$16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0.$$

6. (*a*) (*i*) Classify the following partial differential equation into elliptic, parabolic or hyperbolic : 2.5

$$x(xy-1)r-(x^2y^2-1)s+y(xy-1)t+(x-1)p$$

$$+(y-1)q=0$$

where $r=\dfrac{\partial^2 z}{\partial x^2}, s=\dfrac{\partial^2 z}{\partial x \partial y}, t=\dfrac{\partial^2 z}{\partial y^2}, p=\dfrac{\partial z}{\partial x}, q=\dfrac{\partial z}{\partial y}$.

(*ii*) Form a partial differential equation by eliminating constants *a*, *b* from the relation : 4

$$z=ax+by+cxy.$$

(*b*) Find the general solution of the differential equation : 6.5

$$x^2(y-x)q+y^2(x-y)p=z(x^2+y^2).$$

(*c*) Find the complete integral of 6.5

$$px+qy=pq.$$

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