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This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8352**

Unique Paper Code : **32355301** **J**

Name of the Paper : **Differential Equations**

Name of the Course : **Generic Elective : Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions by selecting any *two* parts from each question.

1. (*a*) Show that the following first order ordinary differential equation :

$$\left(2x\cos y+3x^{2}y\right)dx+\left(x^{3}-y-x^{2}\sin y\right)dy=0,$$

is exact and hence solve the equation with initial condition : $dy = 0$. 6.5

P.T.O.

(*b*) By finding an integrating factor, solve the initial value problem : 6.5

$$(2x^2 + y)dx + (x^2y - x)dy = 0, \; y(1) = 2.$$

(*c*) Solve the following Bernoulli equation : 6.5

$$\frac{dy}{dx} + (x+1)y = e^{x^2} y^3, \; y(0) = 0 \cdot 5.$$

2. (*a*) Find the orthogonal trajectories of the family of parabolas $y^2 = 2cx + c^2$. Is the orthogonal trajectories also a family of parabolas ? 6

(*b*) Solve the initial value problem :

$$y'' + 2y' + 2y = 0, \; y(0) = 1, \; y'(0) = -1. \quad 6$$

(*c*) Find a basis of the following differential equation $(xD^2 + 4D)y = 0$, where $D \equiv d/dx$. Also find the solution satisfying :

$$y(1) = 12, \; y'(1) = -6. \quad 6$$

3. (*a*) Solve by the method of variation of parameters :

$$y'' + 6y' + 9y = x^{-3} e^{-3x}, \; x > 0. \quad 6.5$$

(*b*) Solve the initial-value problem by the method of undetermined coefficients : 6.5

$$y'' + 3y' + 2 \cdot 25y = -10e^{-1.5x}, \; y(0) = 1, \; y'(0) = -1.$$

(*c*) Find a homogeneous linear ordinary differential equation for which two functions $e^{-x}\cos x$ and $e^{-x}\sin x$ are solutions. Show also linear independence by considering their Wronskian. 6.5

4. (*a*) Solve the linear system that satisfies the stated initial conditions : 6

$$\frac{dy_1}{dt} = -3y_1 + 2y_2, \; y_1(0) = 1$$

$$\frac{dy_2}{dt} = y_1 - 3y_2, \; y_2(0) = -2.$$

(*b*) (*i*) Find the partial differential equation arising from the surface : 3

$$z = xy + f(x^2 + y^2).$$

(*ii*) Find the characteristics of the equation : 3

$$u_x - u_y = 1.$$

(c) Obtain the solution of the quasi-linear partial differential equation : 6

$$(y-u)u_x+(u-x)u_y = x-y,$$

with the condition $u = 0$ on $xy = 1$.

5\. (a) Find a power series solution of the following differential equation : 6.5

$$(1-x^2)y'' - 2xy' + 2y = 0.$$

(b) Find the general solution of the linear partial differential equation : 6.5

$$x(y-z)u_x + y(z-x)u_y + z(x-y)u_z = 0.$$

(c) Reduce the linear partial differential equation $u_x - yu_y - u = 1$ to canonical form, and obtain the general solution. 6.5

6\. (a) Apply the method of separation of variables by taking $\log u(x,y) = f(x)+g(y)$, to solve the initial-value problem : 6

$$y^2u_x^2 + x^2u_y^2 = (xyu)^2,\ u(x,0) = 3\exp(x^2/4).$$

(b) Determine the region in which the partial differential equation :

$$u_{xx} + xyu_{yy} + u_x + u_y + u = 1,$$

is hyperbolic, parabolic or elliptic, and transform the equation into canonical form for the parabolic region. 6

(c) Reduce the following partial differential equation with constant coefficients,

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$$

into canonical form and hence find the general solution. 6