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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8525**

Unique Paper Code : **32355101** **J**

Name of the Paper : **Calculus**

Name of the Course : **Mathematics : G.E. for Honours**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) Find the open interval on which $f(x) = x^3 - 3x + 3$ is concave up and concave down. Also determine points of inflection, if any.

(*b*) Find the interval in which the function $f(x)$ is (*i*) increasing (*ii*) decreasing $f(x) = 2x^3 - 9x^2 + 12x$.

(*c*) Evaluate :

$$\lim_{x \to -\infty} \frac{3x^2 - 5x + 7}{7x^2 + 2x - 3}.$$

6+6

P.T.O.

2. (*a*) Find the volume of the solid that results when the region enclosed by $y = x^2, x = 0, x = 2, y = 0$, is revolved about x-axis.

(*b*) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1, x = 4, y = 0$ is revolved about y-axis.

(*c*) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about y-axis. 6½+6½

3. (*a*) Evaluate :

$$\lim_{x \to 0} \frac{10(\sin x - x)}{x^3}.$$

(*b*) Describe the graph of the equation :

$$y^2 - 8x - 6y - 23 = 0.$$

(*c*) Find the asymptotes of the graph of the function :

$$f(x) = -\frac{8}{x^2 - 4}.$$

6+6

4. (*a*) Identify the symmetries of the curve $r^2 = \cos\theta$ and then sketch the curve.

(*b*) Solve the initial value problem and find $\vec{r}$ as a vector valued function of t.

$$\frac{d\vec{r}}{dt} = \frac{3}{2}(t+1)^{\frac{1}{2}}\,\hat{i} + e^{-1}\hat{j} + \frac{1}{t+1}\hat{k}, \quad \hat{r}(0) = \hat{k}.$$

(*c*) Find a unit tangent and unit normal vector for space curve :

$$\vec{r}(t) = (\cos t + t\sin t)\hat{i} + (\sin t - t\cos t)\hat{j} + 3\hat{k}.$$

6+6

5. (*a*) Write acceleration $\vec{a}$ in the form $a_T\,T + a_N\,N$ without finding T and N for :

$$\vec{r}(t) = (t+1)\hat{i} + 2t\,\hat{j} + t^2\,\hat{k} \quad \text{at } t = 1.$$

(*b*) Show that :

$$f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & \end{cases}$$

is continuous at every point except at origin.

(c) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$, where $w = x + 2y + z^2$, $x =$ [illegible],

$y = r^2 + \ln s$, $z = 2r$. 6½+6½

6. (*a*) Find the direction in which $f(x, y) = xe^y + z^2$

(*i*) increases most rapidly at $P\left(1, \ln 2, \frac{1}{2}\right)$

(*ii*) decreases most rapidly $P\left(1, \ln 2, \frac{1}{2}\right)$.

(*b*) Find equations of tangent plane and normal lines for the curve $z^2 - 2x^2 - 2y^2 - 12 = 0$ at $P(1, -1, 4)$.

(*c*) Find all the local maxima, local minima and saddle points of the curve :

$$f(x, y) = 4xy - x^4 - y^4.$$

6½+6½