11/12/19 E

This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	**: 8227**	**J**
Unique Paper Code	**: 32355101**	
Name of Paper	**: Calculus**	
Name of Course	**: Mathematics : G.E. (OC)**	
Semester	**: I**	
Duration	**: 3 hours**	
Maximum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions carry equal marks, 5 each.

Attempt any five questions from each Section.

SECTION I

1. Use $\in-\delta$ definition to show that $\lim_{x\to 0} x\sin\frac{1}{x}=0$.

2. Find the horizontal and vertical asymptotes of the curve $f(x)=\dfrac{x-1}{x^2+2}$.

3. Find the linearization of $f(x) = \cos x$ at $x=-\frac{\pi}{2}$.

4. For $f(x) = 4x^3 - x^4$:

   (i) Find the intervals on which f is increasing and the intervals on which f is decreasing.

P.T.O.

(ii) Find where the graph of f is concave up and where it is concave down.

5. Use L' Hôpital's rule to find $\lim_{t\to 0}\frac{10(\sin t - t)}{t^3}$.

6. Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1$, $x=4$ about the line $y=1$.

7. The radius r of a circle increases from $a = 10$ m to 10.1 m. Use dA to estimate the increase in the circle's area A. Estimate the area of the enlarged circle and compare your estimate to the true area.

SECTION II

8. The region bounded by the parabola $y=x^2$ and the line $y=2x$ in the first quadrant is revolved about the y-axis to generate a solid. Use washer method to find the volume of the solid.

9. Find the length of the curve $y=\left(\frac{x}{2}\right)^{2/3}$ from $x=0$ to $x=2$.

10. Is the area under the curve $y=\frac{(\ell n\, x)}{x^2}$ from $x=1$ to $x=\infty$ finite? If so, what is it?

11. Use direct comparison test to determine whether $\int_1^\infty \frac{dx}{\sqrt{x}+e^{3x}}$ converges.

12. Sketch the graph of the curve $r=\cos 2\theta$ in polar coordinates.

13. If $r(t)$ is a differentiable vector-valued function of t of constant length, then show that $r(t)$ is orthogonal to $\frac{dr(t)}{dt}$ for all t. Verify this result for the function $r(t) = 3\sin 5t\mathbf{i} + 9\mathbf{j} - 3\cos 5t\mathbf{k}$.

14. Find the arc length parametrization for the helix $r(t) = 4\cos t\mathbf{i} + 4\sin t\mathbf{j} + 3t\mathbf{k}$, $0 \le t \le \frac{\pi}{2}$.

SECTION III

15. If $r(t)=\left(\frac{t^3}{3}\right)\mathbf{i}+\left(\frac{t^2}{2}\right)\mathbf{j}$, $t>0$, find binomial vector and torsion.

16. Find the limit of f as $(x, y)\to(0, 0)$ and show that limit does not exist for the function $f(x, y)=\frac{x-y}{x+y}$.

17. If $f(x, y) = \cos^2(3x - y^2)$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

18. Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of u and v if $z = 4e^x \ell n\, y$, $x = \ell n\,(u\cos v)$, $y = u\sin v$.

19. Find the directional derivative of the function f at P_0 in the direction of v where $f(x, y, z) = \cos xy + e^{yz} + \ell n\, zx$, $P_0\left(1, 0, \frac{1}{2}\right)$ and $v = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

20. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point :

Surfaces : $x^2 + y^2 = 4$, $x^2 + y^2 - z = 0$, Point : $\left(\sqrt{2}, \sqrt{2}, 4\right)$.

21. A delivery company accepts only rectangular boxes the sum of whose length and girth (perimeter of cross-section) does not exceed 108 inches. Find the dimensions of the acceptable box of largest volume.