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[This question paper contains 47 printed pages]

**Your Roll No.** : .....

**Sl. No. of Q. Paper** : **9134** **IC**

**Unique Paper Code** : 12271403

**Name of the Course** : **B.A. (Hons.)**  
**Economics - CBCS**  
**Core**

**Name of the Paper** : **Introductory**  
**Econometrics**

**Semester** : **IV**

**Time : 3 Hours** **Maximum Marks : 75**

**Instructions for Candidates :**

परीक्षार्थियों के लिए निर्देश :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.

इस प्रश्न-पत्र के प्राप्त होने पर तुरंत शीर्ष पर अपना रोल नंबर लिखें।

- (b) Answer may be written either in **English** or in **Hindi**; but the same medium should be used throughout the paper.

इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तर एक ही भाषा में होने चाहिए।

P.T.O.

(c) Answer any **five** questions out of **Seven**.

सात में से किन्हीं पाँच प्रश्नों के उत्तर दीजिए।

(d) **All** questions carry equal marks.

सभी प्रश्नों के अंक समान हैं।

(e) Use of simple non-programmable calculator is allowed. Statistical tables are attached for your reference.

सरल गैरप्रोग्राम कैल्कुलेटर के उपयोग की अनुमति दी जाती है। आपके संदर्भ के लिए सांख्यिकी टेबल प्रश्न-पत्र के अंत में दी गयी है।

1. State whether the following statements are **True** or **False**. Give reasons for your answer.

5×3=15

बताइए कि निम्नलिखित कथन सत्य हैं या असत्य। अपने उत्तर हेतु कारण भी दीजिए।

(a) In a regression model  $\ln Y_i = \beta_1 + \beta_2 X_i + u_i$ , if  $\hat{\beta}_2$  is multiplied by 100 we obtain the growth rate estimate of  $Y_i$ .

समाश्रयण (regression) मॉडल  $\ln Y_i = \beta_1 + \beta_2 X_i + u_i$ , में, यदि  $\hat{\beta}_2$  को 100 से गुणा किया जाता है तो हमें  $Y_i$  की वृद्धि दर का आकलन (estimate) प्राप्त होता है।

(b) In regression through origin models the conventionally computed  $r^2$  may not be meaningful.

मूल बिन्दु (origin) से समाश्रयण वाले मॉडलों में परम्परपरागत रूप से गणित  $r^2$  निरर्थक हो सकता है।

(c) In simple regression model  $Y_i = \beta_1 + \beta_2 X_i + u_i$ , the OLS estimators  $\hat{\beta}_1$  and  $\hat{\beta}_2$  each follow normal distribution only if  $u_i$  follows normal distribution.

सरल समाश्रयण मॉडल  $Y_i = \beta_1 + \beta_2 X_i + u_i$ , में OLS आकलकों (estimators)  $\hat{\beta}_1$  व  $\hat{\beta}_2$  में से प्रत्येक का बंटन (distribution) प्रसामान्य (normal) तभी होता है यदि  $u_i$  का बंटन प्रसामान्य हो।

(d) P-value of a test statistic is equal to the level of significance.

किसी जाँच प्रतिदर्शज (test statistic) का P-मान सार्थकता स्तर (level of significance) के बराबर होता है।

- (e) If the estimate of slope coefficient in a bivariate regression is zero, the measure of coefficient of determination is also zero.

यदि एक द्वि-चर समाश्रयण में ढाल गुणांक (slope coefficient) का आकलन शून्य हो, तो निर्धारण गुणांक (coefficient of determination) का मान भी शून्य होगा।

2. (a) You have the following information :

आपको निम्नलिखित सूचनाएँ दी गई हैं :

$$\sum X = 1680, \sum Y = 1110, \sum XY = 204200,$$

$$\sum X^2 = 315400, \sum Y^2 = 133300, n = 10. \text{ Assume}$$

all assumptions of CLRM are fulfilled. Obtain

मान लीजिए कि CLRM की सभी मान्यताएँ सन्तुष्ट होती हैं। निम्नलिखित को ज्ञात कीजिए :

(i)  $\hat{\beta}_1$  and  $\hat{\beta}_2$

$\hat{\beta}_1$  व  $\hat{\beta}_2$

- (ii) Establish 95% interval for the population slope coefficient  $\beta_2$

समष्टि (population) ढाल गुणांक  $\beta_2$  हेतु 95% विश्वास्त्यता अन्तराल (confidence interval)

(iii)  $R^2$  7

- (b) Average score of students in a certain exam are known to be normally distributed with mean value 75 and standard deviation 9. Some coaching classes claim that it is possible to increase the average score of students with an additional use of their study material. It is believed that score with additional study material would remain normally distributed with  $\sigma = 9$ . Let  $\mu$  denote the true average score of students when additional material is used. 5

यह ज्ञात है कि किसी परीक्षा में विद्यार्थियों के औसत अंकों (score) का बंटन प्रसामान्य है जिसका माध्य (mean) 75 व मानक विचलन (standard deviation) 9 है। कुछ कोचिंग केन्द्रों का दावा है कि उनकी पाठ्य-सामग्री के उपयोग से इन औसत अंकों को बढ़ाया जा सकता है। यह माना जाता है कि अतिरिक्त पाठ्य-सामग्री के साथ अंकों का बंटन प्रसामान्य ही रहेगा जिसका मानक विचलन  $\sigma = 9$  होगा। मान लीजिए कि  $\mu$  अतिरिक्त पाठ्य सामग्री के उपयोग के साथ विद्यार्थियों के वास्तविक अंक (true score) हैं।

- (i) What are the appropriate null and alternative hypothesis ?

उपयुक्त शून्य (null) व वैकल्पिक (alternate) परिकल्पनाएँ (hypotheses) क्या हैं ?

- (ii) Let  $\bar{X}$  denote the sample average score for 25 randomly selected students. Consider the test procedure with test statistic  $\bar{X}$  and rejection region  $\bar{x} \geq 77.9$ . What is the probability distribution of the statistic when  $H_0$  is true ? What is the probability of Type I error ?

मान लीजिए कि  $\bar{X}$  यादृच्छिक तौर पर (randomly) चयनित 25 विद्यार्थियों के समूह हेतु प्रतिदर्श औसत अंक (sample average score) हैं। जाँच प्रतिदर्शज  $\bar{X}$  व अस्वीकृति-क्षेत्र (rejection region)  $\bar{x} \geq 77.9$  वाली जाँच प्रक्रिया पर विचार कीजिए। यदि  $H_0$  सत्य हो ता प्रतिदर्शज का प्रायिकता बंटन (probability distribution) क्या होगा ? I प्रकार की त्रुटि (error) की प्रायिकता (probability) क्या होगी ?

- (iii) Using the testing procedure in (ii) what is the probability of type II error when in fact  $\mu = 80$ .

(ii) में दी गई जाँच प्रक्रिया की सहायता से बताइए कि II प्रकार की त्रुटि की प्रायिकता क्या होगी यदि वास्तव में  $\mu = 80$ .

- (c) In a regression model,  $Y_i = \beta_1 + \beta_2 X_i + u_i$ , show that the mean of actual  $Y_i$  is equal to the mean of estimated  $\hat{Y}_i$ . 3

समाश्रयण मॉडल  $Y_i = \beta_1 + \beta_2 X_i + u_i$ , में दर्शाइए कि वास्तविक  $Y_i$  का माध्य (mean) आकलित  $\hat{Y}_i$  के माध्य के बराबर होता है।

3. (a) Consider the following simple regression model

$$\text{price} = \beta_0 + \beta_1 \text{ assess} + u$$

where price is the housing price assess and is the assessment of housing prices. The estimated equation is :

निम्नलिखित सरल समाश्रयण मॉडल पर विचार कीजिए

$$\text{price} = \beta_0 + \beta_1 \text{ assess} + u$$

जहाँ price आवासों की कीमत है तथा assess आवासों की कीमतों का आकलन है। आकलित समीकरण निम्न प्रकार है :

$$\widehat{\text{price}} = -14.47 + 0.976 \text{ assess}$$

$$t = (16.27) \quad (0.049)$$

$$n = 88, \text{SSR} = 165644.51, r^2 = 0.820$$

- (i) How will you test the constraints  $\beta_1=1$  and  $\beta_0=0$  in the above regression if you are given the SSR in the restricted model as 209448.99 ? Conduct the necessary test(s) at 1% level of significance and give your conclusion. 3
- उपरोक्त समाश्रयण में आप प्रतिबन्धों (restrictions)  $\beta_1=1$  व  $\beta_0=0$  का परीक्षण किस प्रकार करेंगे यदि आपको दिया हुआ है कि प्रतिबन्धित समाश्रयण (restricted regression) में SSR का मान 209448.99 है ? आवश्यक परीक्षण (परीक्षणों) को 1% सार्थकता स्तर पर कीजिए तथा अपना निष्कर्ष दीजिए।

- (ii) Suppose now that the estimated model is :

अब मान लीजिए कि आकलित मॉडल निम्न प्रकार है :

$$\text{price} = \beta_0 + \beta_1 \text{ assess} + \beta_2 \text{ lotsize} + \beta_3 \text{ sqrft} + \beta_4 \text{ bdrms} + u$$

where

जहाँ

lotsize = the size of the lot

= समूह का आकार

sqrft = the square footage

= क्षेत्रफल वर्गफुट में

bdrms = the number of bedrooms

= शयनकक्षों की संख्या

The  $R^2$  from estimating this model using the same 88 houses is 0.829. Test at 1% level of significance that all partial slope coefficients are equal to zero. 2

उन्हीं 88 की सहायता से इस मॉडल हेतु  $R^2=0.829$  है। 1% सार्थकता स्तर पर इस बात का परीक्षण कीजिए कि सभी आंशिक (partial) ढाल गुणांकों के मान शून्य के बराबर हैं।

- (b) Let  $X \sim N(\mu, \sigma^2)$ , Consider two independent random samples of observations on X. The samples are of size  $n_1$  and  $n_2$  with means  $\bar{X}_1$  and  $\bar{X}_2$  respectively. Two estimators of the population mean are proposed :

मान लीजिए  $X \sim N(\mu, \sigma^2)$ , X पर प्रेक्षणों के दो स्वतन्त्र यादृच्छिक (random) प्रतिदर्शों (samples) पर विचार कीजिए। इन प्रतिदर्शों के आकार क्रमशः  $n_1$  व  $n_2$  तथा माध्य  $\bar{X}_1$  व  $\bar{X}_2$  हैं। समष्टि माध्य के दो आकलक (estimators) प्रस्तावित किए जाते हैं :

$$\hat{\mu} = \frac{\bar{X}_1 + \bar{X}_2}{2}, \tilde{\mu} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$$

Check whether these estimators are unbiased and calculate their variance.

इन आकलकों की अनभिनतता (unbiasedness) हेतु जाँच कीजिए तथा इनके प्रसरणों (variances) को ज्ञात कीजिए।

- (c) For each of the following pairs of dependent (Y) and independent variables (X), pick the most appropriate functional form. Explain the reason for your answer :

निम्नलिखित में से निर्भर (dependent) (Y) व स्वतन्त्र (independent) (X) चरों के प्रत्येक युग्म हेतु, सर्वाधिक उपयुक्त फलनीय रूप (functional form) का चयन कीजिए। अपने उत्तर हेतु कारण समझाइए।

- (i) Y = demand for food X = price of food

Y = भोजन की माँग X = भोजन की कीमत

- (ii) Y = AFC of production X = output

Y = उत्पादन की AFC X = उत्पाद

- (iii) Y = Population in India X = time

Y = भारत में जनसंख्या X = समय

4. (a) In a regression of average wages (W) on the number of employees (N) for a random sample of 30 firms, the following results were obtained :

30 फर्मों के एक यादृच्छिक प्रतिदर्श हेतु औसत मजदूरी (W) के कर्मचारियों की संख्या (N) पर समाश्रयण हेतु निम्नलिखित परिणाम प्राप्त हुए :

Regression 1:  $\bar{W} = 7.5 + 0.009 N$   
समाश्रयण 1:

$$t = (16.10) \quad R^2 = 0.90$$

Regression 2 :  $\frac{\bar{W}}{N} = 0.008 + 7.8 \frac{1}{N}$

समाश्रयण 2 :

$$t = (14.43) \quad (76.58) \quad R^2 = 0.99$$

(i) How would you interpret the two regressions ?

आप इन दोनों समाश्रयणों की व्याख्या किस प्रकार करेंगे ?

(ii) What might be the reason for transforming Regression 1 into Regression 2 ? What assumption has been made about the error variance in going from Regression 1 to Regression 2 ?

समाश्रयण 1 को समाश्रयण 2 में रूपान्तरित करने के पीछे क्या कारण हो सकता है ? समाश्रयण 1 से समाश्रयण 2 पर जाने में त्रुटि पद (error term) के प्रसरण के बारे में क्या मान्यता ली गई है ?

(iii) Can you relate the slopes and intercepts of the two models ?

क्या आप इन दो मॉडलों के ढालों (slopes) व अन्तःखण्डों (intercepts) के मध्य सम्बन्ध बता सकते हैं ?

(iv) Can you compare the  $R^2$  of the two models ? Give reasons.

क्या आप इन दो मॉडलों के  $R^2$  की तुलना कर सकते हैं ? कारण दीजिए।

(b) The thickness of the graph paper (measured in GSM) used during examinations should be such that it does not tear off easily while plotting a graph. Let  $\mu$  denote the true average thickness of the new type of graph paper under consideration. The true average thickness of the graph paper should be greater than or equal to 20 GSM for it to be acceptable for all practical uses. A random sample of size  $n$  is drawn from a population with normal distribution. What conclusion is appropriate in each case ?

परीक्षाओं के दौरान उपयोग किए जाने वाले ग्राफ पेपर की मोटाई (GSM में) इतनी होनी चाहिए कि यह ग्राफ बनाते समय आसानी से फटे नहीं। मान लीजिए कि  $\mu$  एक नए प्रकार के विचाराधीन ग्राफ पेपर की वास्तविक (true) औसत मोटाई है। इस पेपर के सभी प्रायोगिक उपयोगों हेतु स्वीकार्य होने हेतु इसकी वास्तविक औसत मोटाई 20 GSM से अधिक या बराबर होना चाहिए। प्रसामान्य बंटन (normal distribution) वाली एक समष्टि से आकार  $n$  का एक यादृच्छिक प्रतिदर्श लिया जाता है। निम्नलिखित में से प्रत्येक स्थिति में क्या निष्कर्ष उपयुक्त है ?

(i)  $n = 15, t = 3.2, \alpha = .05$

(ii)  $n = 9, t = 1.8, \alpha = .01$

(iii)  $n = 24, t = -0.2$

- (c) Suppose that earnings of individuals are dependent on whether they are skilled workers and their work experience over the years. 6

मान लीजिए कि व्यक्तियों की मजदूरी इस बात पर निर्भर करती है कि क्या वे कुशल (skilled) हैं, तथा उनका कार्यानुभव (work experience) कितना है।

- (i) Define dummy variables to capture whether workers are skilled or not. Take workers being unskilled as the reference category.

श्रमिक कुशल हैं या नहीं, इस हेतु मूक चर (dummy variable) परिभाषित कीजिए। अकुशल (unskilled) श्रमिकों को सन्दर्भ श्रेणी (reference category) में लीजिए।

- (ii) Develop a model which is linear in parameters that shows earnings of an individual as a function of work experience and whether they are skilled. Interpret your model.

प्राचलों (parameters) में रेखीय (linear) एक ऐसा मॉडल विकसित कीजिए जो कि व्यक्ति की मजदूरी को कार्यानुभव व क्या वह कुशल है या नहीं, इस बात के फलन के तौर पर दर्शाता है। अपने मॉडल की व्याख्या कीजिए।

- (iii) Now assume that there is an interaction between skill of the workers and their work experience. How would the model in (ii) change. Interpret the new model.

अब मान लीजिए कि इस मॉडल में श्रमिकों के कौशल व उनके कार्यानुभव का एक परस्पर सम्बन्ध व्यक्त करने वाला (interaction) पद भी है। उपरोक्त भाग (ii) में आपका मॉडल किस प्रकार परिवर्तित हो जाएगा ? नए मॉडल की व्याख्या कीजिए।

5. (a) The results of a logarithmic regression of demand for food on price and personal disposable income is given as : 5

भोजन की माँग के कीमत व व्यक्तिगत प्रयोज्य आय (personal disposable income) पर एक लघुगणकीय (logarithmic) समाश्रयण के परिणाम निम्न प्रकार हैं :

$$\log Q_t = 2.34 - 0.31 \log P_t + 0.45 \log Y_t + 0.65 \log Q_{t-1}$$

$$Se = (0.05) \quad (0.20) \quad (0.14)$$

$$n = 50 \quad R^2 = 0.90 \quad d = 1.8$$

where  $Q$  = food consumption per capita

जहाँ  $Q$  = प्रति व्यक्ति भोजन का उपभोग

$P$  = food price

$P$  = भोजन की कीमत

$Y$  = real per capita disposable income

$Y$  = वास्तविक प्रति व्यक्ति प्रयोज्य आय

- (i) Just by looking at the estimated regression, do you suspect serial correlation in it ?

क्या इस मॉडल को देखने मात्र से आपको इसमें स्व-सहसम्बन्ध (serial correlation) का संदेह होता है ?

- (ii) Which test do you use to confirm your suspicion and why ?

अपने संदेह की पुष्टि करने हेतु आप कौन-से परीक्षण का उपयोग करेंगे व क्यों ?

- (iii) Outline the steps of the above mentioned test and provide a conclusion on the basis of your calculations.

उपरोक्त परीक्षण के चरणों की रूपरेखा दीजिए तथा अपनी गणनाओं के आधार पर निष्कर्ष दीजिए।

- (b) Suppose you are given the following regression : 5

मान लीजिए कि आपको निम्नलिखित समाश्रयण दिया गया है :

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_t^2 + \mu_t$$

Do you think the model suffers from multicollinearity ? If yes then what are the possible remedies of the problem ?

क्या आपको लगता है कि यह मॉडल बहुसंरेखता (multicollinearity) से ग्रस्त है ? यदि हैं तो इस समस्या के संभव उपचार (remedies) क्या हैं ?

- (c) State and prove the minimum variance property of the slope coefficient in a two variable regression model. 5

एक द्विचर समाश्रयण मॉडल में ढाल गुणांक के न्यूनतम प्रसरण गुणधर्म (minimum variance property) को लिखिए व सिद्ध कीजिए।

6. (a) Consider the following models : 5

निम्नलिखित मॉडलों पर विचार कीजिए :

$$\text{Model I : } \ln Y_i^* = \alpha_1 + \alpha_2 \ln X_i^* + u_i^*$$

मॉडल I :

$$\text{Model II : } \ln Y_i = \beta_1 + \beta_2 \ln X_i + u_i$$

मॉडल II :

where  $Y_i^* = w_1 Y_i$  and  $X_i^* = w_2 X_i$ , the  $w$ 's being constants.

जहाँ  $Y_i^* = w_1 Y_i$  व  $X_i^* = w_2 X_i$ , दोनों  $w$  अचर (constants) हैं।

- (i) Establish the relationships between the two sets of regression coefficients and their standard errors.

समाश्रयण गुणांकों व इनकी मानक त्रुटियों के इन दो समूहों के मध्य सम्बन्ध स्थापित कीजिए।

- (ii) Is the  $R^2$  different between the two models ?

क्या इन दो मॉडलों के  $R^2$  भिन्न होंगे ?

- (b) Suppose the CLRM applies to  $Y_i = \beta_2 X_i + \varepsilon_i$ .

मान लीजिए कि  $Y_i = \beta_2 X_i + \varepsilon_i$  पर CLRM लागू होता है।

- (i) Find the slope coefficient in the regression of Y on X

Y के X पर समाश्रयण में ढाल गुणांक ज्ञात कीजिए।

- (ii) Suppose now we have a regression of X on Y,  $X_i = \gamma_2 Y_i + v_i$ . Is slope coefficient of regression on X on Y an inverse of slope of regression of Y on X. 4

अब मान लीजिए कि हमारे पास X का Y पर समाश्रयण,  $X_i = \gamma_2 Y_i + v_i$  है। क्या X के Y पर समाश्रयण में ढाल गुणांक Y के X पर ढाल गुणांक का व्युत्क्रम (inverse) होता है ?

(c) Using data on compensation per employee in thousands of dollars (COMP) and average productivity in thousands of dollars (PROD) for a cross section of 50 firms for the year 1958, the following regression results were obtained (t ratios in parentheses) :

50 फर्मों के एक अनुप्रस्थ (cross section) हेतु वर्ष 1958 में प्रति व्यक्ति कर्मचारी वेतन (हजार डॉलरों में) (COMP) व औसत उत्पादकता (हजार डॉलरों में) (PROD) के आँकड़ों की सहायता से निम्नलिखित समाश्रयण परिणाम प्राप्त हुए (कोष्ठकों में t अनुपात हैं) :

$$= 1992.35 + 0.233\text{PROD}_i$$

$$t = (2.1275) \quad (2.333) \quad R^2 = 0.5891$$

Since the cross-sectional data included heterogeneous units, heteroscedasticity was likely to be present. The Park test was performed and the following results of auxiliary regression were obtained :

चूँकि अनुप्रस्थ आँकड़ों में विजातीय (heterogeneous) इकाइयाँ सम्मिलित थीं, प्रसरण-विषमता (heteroskedasticity) के विद्यमान होने की सम्भावना थी। पार्क का परीक्षण (Park's test) किया गया तथा सहायक (auxiliary) समाश्रयण से निम्नलिखित परिणाम प्राप्त हुए :

$$\widehat{\ln e_i} = 35.817 - 2.8099\text{PROD}_i$$

$$t = (0.934) \quad (-0.667) \quad R^2 = 0.0595$$

(i) Use the result of auxiliary regression to check if the model indeed suffers from heteroscedasticity. Perform the test at 5% level of significance.

सहायक समाश्रयण के परिणामों की सहायता से जाँच कीजिए कि क्या यह मॉडल वास्तव में प्रसरण-विषमता से ग्रस्त है। 5% सार्थकता स्तर पर परीक्षण कीजिए।

(ii) What could be the possible remedies of heteroscedasticity ?

प्रसरण-विषमता हेतु क्या सम्भव उपचार हो सकते हैं ?

7. (a) The following model was estimated for United States from 1958 to 1977 : 5

निम्नलिखित मॉडल को 1958 से 1977 हेतु संयुक्त राज्य अमेरिका हेतु आकलित किया गया था :

$$\hat{Y}_t = 10.078 - 10.337 D_t - 17.549 \left( \frac{1}{X_t} \right) +$$

$$38.173 D_t \left( \frac{1}{X_t} \right)$$

$$se = (1.4204) (1.6859) (8.3373) (9.399)$$

$$R^2 = 0.8787$$

where Y = year-to-year percentage change in the index of hourly earnings

जहाँ Y = प्रति घण्टा मजदूरी के सूचकांक में वर्ष-दर-वर्ष प्रतिशत परिवर्तन

X = percent unemployment rate

X = प्रतिशत बेरोजगारी की दर

D = 1 for 1958-1969

= 0 if otherwise

D = 1, 1958-1969 हेतु

= 0 अन्यथा

- (i) Show the Phillips curve for two periods separately.

दोनों अवधियों हेतु फिलिप्स वक्र को अलग-अलग दर्शाइए।

- (ii) Are differential intercept and slope coefficients statistically significant ? What does this suggest ?

क्या विभेदक (differential) अन्तःखण्ड व ढाल गुणांक सांख्यिकीय तौर पर सार्थक हैं ? यह क्या बताता है ?

- (iii) Interpret the regression.

इस समाश्रयण की व्याख्या कीजिए।

- (b) Two models for Engel expenditure function are estimated. 5

एंजेल व्यय फलन (Engel expenditure function) हेतु दो मॉडल आकलित किए गए हैं.

$$\text{Model I : } Y_t = 1087.930 + 0.077X_t$$

मॉडल I :

$$t = (25.58) (21.64) \quad R^2 = 0.350 \quad F = 468.645$$

Model II :  $Y_t = 4005.077 + 0.3381/X_t$

मॉडल II :

$$t = (19.259) \quad (-20.816) \quad R^2 = 0.333 \quad F = 433.310$$

where  $Y_t$  = expenditure on food in rupees

जहाँ  $Y_t$  = भोजन पर व्यय, रुपयों में

= total expenditure in rupees

= कुल व्यय, रुपयों में

- (i) Interpret all coefficient value of the two models.

इन दो मॉडलों के सभी गुणांकों के मानों की व्याख्या कीजिए।

- (ii) Are the sign of the coefficients in the two models contradictory ?

क्या इन दो मॉडलों में गुणांकों के चिन्ह परस्पर विरोधी (contradictory) हैं ?

- (iii) Can we compare the results of the two models ?

क्या हम इन दो मॉडलों के परिणामों की तुलना कर सकते हैं ?

- (iv) Diagrammatically show the sample regression function in the above model.

उपरोक्त मॉडल में प्रतिदर्श समाश्रयण फलन को रेखाचित्र की सहायता से दर्शाइए।

- (c) Consider the following fitted regression model. Standard error is given in parenthesis :

5

निम्नलिखित आकलित समाश्रयण मॉडल पर विचार कीजिए। मानक त्रुटियाँ कोष्ठकों में दी हुई हैं :

$$Y_t = -9.6 + 2.1X_1 + 0.45X_2 \quad R^2 = 0.92$$

$$se = (8.3) \quad (1.98) \quad (1.77)$$

- (i) Do you see any problem with this regression ?

क्या आपको इस समाश्रयण में कोई समस्या नजर आती है ?

(ii) If yes, what is the problem ?

यदि हाँ, तो वह समस्या क्या है ?

(iii) Outline the steps for performing an auxiliary regression to detect the presence of problem in the regression.

इस समाश्रयण में समस्या का पता लगाने हेतु सहायक समाश्रयण करने हेतु प्रयुक्त होने वाले चरणों की रूपरेखा दीजिए।

## Appendix

### STATISTICAL TABLES

TABLE I  
LOGARITHMS

|    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | Mean Differences |   |    |    |    |    |    |    |    |
|----|------|------|------|------|------|------|------|------|------|------|------------------|---|----|----|----|----|----|----|----|
|    |      |      |      |      |      |      |      |      |      |      | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4                | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0756 | 4                | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3                | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3                | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1461 | 1492 | 1522 | 1553 | 1584 | 1614 | 1644 | 1675 | 1702 | 1732 | 3                | 6 | 9  | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3                | 6 | 8  | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3                | 5 | 8  | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2294 | 2320 | 2345 | 2370 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2                | 5 | 7  | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2555 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2                | 5 | 7  | 9  | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2                | 4 | 7  | 9  | 11 | 13 | 15 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2                | 4 | 6  | 8  | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2                | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 18 |
| 22 | 3425 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2                | 4 | 6  | 8  | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3656 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2                | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2                | 4 | 5  | 7  | 9  | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2                | 3 | 5  | 7  | 9  | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2                | 3 | 5  | 7  | 8  | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2                | 3 | 5  | 6  | 8  | 9  | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2                | 3 | 5  | 6  | 8  | 9  | 11 | 12 | 14 |
| 29 | 4625 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1                | 3 | 4  | 6  | 7  | 9  | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1                | 3 | 4  | 6  | 7  | 9  | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1                | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1                | 3 | 4  | 5  | 7  | 8  | 9  | 11 | 12 |
| 33 | 5186 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1                | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1                | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1                | 2 | 4  | 5  | 6  | 7  | 9  | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1                | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1                | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1                | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1                | 2 | 3  | 4  | 5  | 7  | 8  | 9  | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1                | 2 | 3  | 4  | 5  | 6  | 8  | 9  | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 7  | 7  |
| 53 | 7243 | 7252 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 6  | 7  |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 6  | 7  |

|    | 0 1 2 3 4 5 6 7 8 9 |      |      |      |      |      |      |      |      |      | Mean Differences |   |   |   |   |   |   |   |   |  |
|----|---------------------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|---|---|---|---|---|--|
|    |                     |      |      |      |      |      |      |      |      |      | 1                | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| 55 | 7404                | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |  |
| 56 | 7482                | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |  |
| 57 | 7559                | 7566 | 7574 | 7582 | 7590 | 7597 | 7604 | 7612 | 7619 | 7627 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |  |
| 58 | 7634                | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 59 | 7709                | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 60 | 7782                | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 61 | 7853                | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 62 | 7924                | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 63 | 7993                | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 64 | 8062                | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 65 | 8129                | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 66 | 8195                | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 67 | 8261                | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 68 | 8325                | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 69 | 8388                | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 70 | 8451                | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 71 | 8513                | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 72 | 8573                | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 73 | 8633                | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 74 | 8692                | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 75 | 8751                | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 76 | 8808                | 8814 | 8820 | 8825 | 8831 | 8837 | 8843 | 8848 | 8854 | 8859 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 77 | 8865                | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 78 | 8921                | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 79 | 8976                | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 80 | 9031                | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 81 | 9085                | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 82 | 9138                | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9185 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 83 | 9191                | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 84 | 9243                | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 85 | 9294                | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 86 | 9345                | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 87 | 9395                | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 88 | 9445                | 9450 | 9455 | 9460 | 9465 | 9470 | 9475 | 9480 | 9485 | 9490 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 89 | 9494                | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 90 | 9542                | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 91 | 9590                | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 92 | 9638                | 9642 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 93 | 9681                | 9685 | 9689 | 9693 | 9697 | 9701 | 9705 | 9709 | 9713 | 9717 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 94 | 9721                | 9725 | 9729 | 9733 | 9737 | 9741 | 9745 | 9749 | 9753 | 9757 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 95 | 9761                | 9765 | 9769 | 9773 | 9777 | 9781 | 9785 | 9789 | 9793 | 9797 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 96 | 9801                | 9805 | 9809 | 9813 | 9817 | 9821 | 9825 | 9829 | 9833 | 9837 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 97 | 9841                | 9845 | 9849 | 9853 | 9857 | 9861 | 9865 | 9869 | 9873 | 9877 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 98 | 9881                | 9885 | 9889 | 9893 | 9897 | 9901 | 9905 | 9909 | 9913 | 9917 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |
| 99 | 9921                | 9925 | 9929 | 9933 | 9937 | 9941 | 9945 | 9949 | 9953 | 9957 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |  |

TABLE II  
ANTI-LOGARITHMS

|    | 0 1 2 3 4 5 6 7 8 9 |      |      |      |      |      |      |      |      |      | Mean Differences |   |   |   |    |    |    |    |    |  |
|----|---------------------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|----|----|----|----|----|--|
|    |                     |      |      |      |      |      |      |      |      |      | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |  |
| 50 | 3162                | 3170 | 3177 | 3184 | 3192 | 3199 | 3206 | 3214 | 3221 | 3228 | 1                | 1 | 2 | 3 | 4  | 4  | 5  | 6  | 7  |  |
| 51 | 3236                | 3243 | 3251 | 3258 | 3266 | 3273 | 3281 | 3289 | 3296 | 3304 | 1                | 2 | 2 | 3 | 4  | 5  | 5  | 6  | 7  |  |
| 52 | 3311                | 3319 | 3327 | 3334 | 3342 | 3350 | 3357 | 3365 | 3373 | 3381 | 1                | 2 | 2 | 3 | 4  | 5  | 5  | 6  | 7  |  |
| 53 | 3388                | 3396 | 3404 | 3412 | 3420 | 3428 | 3436 | 3443 | 3451 | 3459 | 1                | 2 | 2 | 3 | 4  | 5  | 6  | 6  | 7  |  |
| 54 | 3467                | 3475 | 3483 | 3491 | 3499 | 3508 | 3516 | 3524 | 3532 | 3540 | 1                | 2 | 2 | 3 | 4  | 5  | 6  | 6  | 7  |  |
| 55 | 3548                | 3558 | 3565 | 3572 | 3581 | 3589 | 3597 | 3600 | 3614 | 3622 | 1                | 2 | 2 | 3 | 4  | 5  | 6  | 7  | 7  |  |
| 56 | 3631                | 3639 | 3648 | 3656 | 3664 | 3673 | 3681 | 3690 | 3698 | 3707 | 1                | 2 | 3 | 3 | 4  | 5  | 6  | 7  | 8  |  |
| 57 | 3715                | 3724 | 3733 | 3741 | 3750 | 3758 | 3767 | 3776 | 3784 | 3793 | 1                | 2 | 3 | 3 | 4  | 5  | 6  | 7  | 8  |  |
| 58 | 3802                | 3811 | 3819 | 3828 | 3837 | 3846 | 3855 | 3864 | 3873 | 3882 | 1                | 2 | 3 | 4 | 4  | 5  | 6  | 7  | 8  |  |
| 59 | 3891                | 3899 | 3908 | 3917 | 3926 | 3935 | 3944 | 3954 | 3963 | 3972 | 1                | 2 | 3 | 4 | 5  | 5  | 6  | 7  | 8  |  |
| 60 | 3981                | 3990 | 3999 | 4009 | 4018 | 4027 | 4036 | 4046 | 4055 | 4064 | 1                | 2 | 3 | 4 | 5  | 6  | 6  | 7  | 8  |  |
| 61 | 4074                | 4083 | 4093 | 4102 | 4111 | 4121 | 4130 | 4140 | 4150 | 4159 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |  |
| 62 | 4169                | 4178 | 4188 | 4196 | 4207 | 4217 | 4227 | 4236 | 4246 | 4256 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |  |
| 63 | 4266                | 4276 | 4285 | 4295 | 4305 | 4315 | 4325 | 4335 | 4345 | 4355 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |  |
| 64 | 4365                | 4375 | 4385 | 4395 | 4406 | 4416 | 4426 | 4436 | 4446 | 4457 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |  |
| 65 | 4467                | 4477 | 4487 | 4498 | 4508 | 4519 | 4529 | 4539 | 4550 | 4560 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 8  | 9  |  |
| 66 | 4576                | 4581 | 4592 | 4603 | 4613 | 4624 | 4634 | 4645 | 4656 | 4667 | 1                | 2 | 3 | 4 | 5  | 6  | 7  | 9  | 10 |  |
| 67 | 4677                | 4688 | 4699 | 4710 | 4721 | 4732 | 4742 | 4753 | 4764 | 4775 | 1                | 2 | 3 | 4 | 5  | 7  | 8  | 9  | 10 |  |
| 68 | 4786                | 4797 | 4808 | 4819 | 4831 | 4842 | 4853 | 4864 | 4875 | 4887 | 1                | 2 | 3 | 4 | 6  | 7  | 8  | 9  | 10 |  |
| 69 | 4898                | 4909 | 4920 | 4932 | 4943 | 4955 | 4966 | 4977 | 4989 | 5000 | 1                | 2 | 3 | 5 | 6  | 7  | 8  | 9  | 10 |  |
| 70 | 5012                | 5023 | 5035 | 5047 | 5058 | 5070 | 5082 | 5093 | 5105 | 5117 | 1                | 2 | 4 | 5 | 6  | 7  | 8  | 9  | 11 |  |
| 71 | 5129                | 5140 | 5152 | 5164 | 5176 | 5188 | 5200 | 5212 | 5224 | 5236 | 1                | 2 | 4 | 5 | 6  | 7  | 8  | 10 | 11 |  |
| 72 | 5248                | 5260 | 5272 | 5284 | 5297 | 5309 | 5321 | 5333 | 5346 | 5358 | 1                | 2 | 4 | 5 | 6  | 7  | 9  | 10 | 11 |  |
| 73 | 5370                | 5383 | 5395 | 5408 | 5420 | 5433 | 5445 | 5458 | 5470 | 5483 | 1                | 3 | 4 | 5 | 6  | 8  | 9  | 10 | 11 |  |
| 74 | 5495                | 5508 | 5521 | 5534 | 5546 | 5559 | 5572 | 5585 | 5598 | 5610 | 1                | 3 | 4 | 5 | 6  | 8  | 9  | 10 | 12 |  |
| 75 | 5623                | 5636 | 5649 | 5662 | 5675 | 5688 | 5702 | 5715 | 5728 | 5741 | 1                | 3 | 4 | 5 | 7  | 8  | 9  | 10 | 12 |  |
| 76 | 5754                | 5768 | 5781 | 5794 | 5808 | 5821 | 5834 | 5848 | 5861 | 5875 | 1                | 3 | 4 | 5 | 7  | 8  | 9  | 11 | 12 |  |
| 77 | 5888                | 5902 | 5916 | 5929 | 5943 | 5957 | 5970 | 5984 | 5998 | 6012 | 1                | 3 | 4 | 5 | 7  | 8  | 10 | 11 | 12 |  |
| 78 | 6026                | 6039 | 6053 | 6067 | 6081 | 6095 | 6109 | 6124 | 6138 | 6152 | 1                | 3 | 4 | 6 | 7  | 8  | 10 | 11 | 12 |  |
| 79 | 6166                | 6180 | 6194 | 6209 | 6223 | 6237 | 6252 | 6266 | 6281 | 6295 | 1                | 3 | 4 | 6 | 7  | 9  | 10 | 11 | 13 |  |
| 80 | 6310                | 6324 | 6339 | 6353 | 6368 | 6383 | 6397 | 6412 | 6427 | 9442 | 1                | 3 | 4 | 6 | 7  | 9  | 10 | 12 | 13 |  |
| 81 | 6457                | 6471 | 6486 | 6501 | 6516 | 6531 | 6546 | 6561 | 6577 | 6592 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 12 | 14 |  |
| 82 | 6607                | 6622 | 6637 | 6653 | 6668 | 6683 | 6699 | 6714 | 6730 | 6745 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 12 | 14 |  |
| 83 | 6761                | 6776 | 6792 | 6808 | 6823 | 6839 | 6855 | 6871 | 6887 | 6902 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 13 | 14 |  |
| 84 | 6915                | 6934 | 6952 | 6969 | 6986 | 7003 | 7021 | 7037 | 7053 | 7069 | 2                | 3 | 5 | 6 | 8  | 10 | 11 | 13 | 15 |  |
| 85 | 7079                | 7096 | 7112 | 7129 | 7145 | 7161 | 7178 | 7194 | 7211 | 7228 | 2                | 3 | 5 | 7 | 8  | 10 | 12 | 13 | 15 |  |
| 86 | 7214                | 7261 | 7278 | 7295 | 7311 | 7328 | 7345 | 7362 | 7379 | 7396 | 2                | 3 | 5 | 7 | 8  | 10 | 12 | 13 | 15 |  |
| 87 | 7412                | 7430 | 7417 | 7464 | 7482 | 7499 | 7516 | 7534 | 7551 | 7568 | 2                | 3 | 5 | 7 | 9  | 10 | 12 | 14 | 16 |  |
| 88 | 7580                | 7603 | 7620 | 7636 | 7654 | 7671 | 7691 | 7709 | 7727 | 7745 | 2                | 4 | 5 | 7 | 9  | 11 | 12 | 14 | 16 |  |
| 89 | 7762                | 7780 | 7798 | 7816 | 7834 | 7852 | 7870 | 7889 | 7907 | 7925 | 2                | 4 | 5 | 7 | 9  | 11 | 13 | 14 | 16 |  |
| 90 | 7943                | 7982 | 7980 | 7998 | 8017 | 8035 | 8054 | 8072 | 8091 | 8110 | 2                | 4 | 6 | 7 | 9  | 11 | 13 | 15 | 17 |  |
| 91 | 8128                | 8147 | 8166 | 8185 | 8204 | 8222 | 8241 | 8260 | 8279 | 8290 | 2                | 4 | 6 | 8 | 9  | 11 | 13 | 15 | 17 |  |
| 92 | 8316                | 8337 | 8356 | 8375 | 8395 | 8411 | 8433 | 8453 | 8472 | 8492 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |  |
| 93 | 8511                | 8531 | 8551 | 8570 | 8590 | 8610 | 8630 | 8650 | 8670 | 8690 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |  |
| 94 | 8716                | 8720 | 8750 | 8770 | 8790 | 8810 | 8831 | 8851 | 8872 | 8892 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |  |
| 95 | 8913                | 8933 | 8954 | 8974 | 8995 | 9016 | 9036 | 9057 | 9078 | 9099 | 2                | 4 | 6 | 8 | 10 | 12 | 15 | 17 | 19 |  |
| 96 | 9129                | 9141 | 9163 | 9188 | 9204 | 9226 | 9247 | 9268 | 9290 | 9311 | 2                | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |  |
| 97 | 9323                | 9354 | 9376 | 9397 | 9415 | 9441 | 9482 | 9484 | 9406 | 9538 | 2                | 4 | 7 | 9 | 11 | 13 | 15 | 17 | 20 |  |
| 98 | 9550                | 9522 | 9524 | 9516 | 9535 | 9561 | 9588 | 9705 | 9727 | 9750 | 2                | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |  |
| 99 | 9773                | 9795 | 9817 | 9849 | 9808 | 9888 | 9956 | 9931 | 9951 | 9977 | 2                | 5 | 7 | 9 | 11 | 14 | 18 | 20 |    |  |

| 0 1 2 3 4 5 6 7 8 9 |      |      |      |      |      |      |      |      |      |      |
|---------------------|------|------|------|------|------|------|------|------|------|------|
| Mean Differences    |      |      |      |      |      |      |      |      |      |      |
|                     | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |      |
| 00                  | 1000 | 1002 | 1006 | 1007 | 1009 | 1012 | 1014 | 1016 | 1019 | 1021 |
| 01                  | 1023 | 1026 | 1028 | 1030 | 1033 | 1035 | 1038 | 1040 | 1042 | 1045 |
| 02                  | 1047 | 1050 | 1052 | 1054 | 1057 | 1059 | 1062 | 1064 | 1067 | 1069 |
| 03                  | 1072 | 1074 | 1076 | 1079 | 1081 | 1084 | 1086 | 1089 | 1091 | 1094 |
| 04                  | 1099 | 1099 | 1102 | 1104 | 1107 | 1109 | 1112 | 1114 | 1117 | 1119 |
| 05                  | 1122 | 1125 | 1127 | 1130 | 1132 | 1135 | 1138 | 1140 | 1148 | 1146 |
| 06                  | 1148 | 1151 | 1153 | 1156 | 1159 | 1161 | 1164 | 1167 | 1169 | 1172 |
| 07                  | 1175 | 1178 | 1180 | 1183 | 1186 | 1189 | 1191 | 1194 | 1197 | 1199 |
| 08                  | 1202 | 1205 | 1208 | 1211 | 1213 | 1216 | 1219 | 1222 | 1225 | 1227 |
| 09                  | 1230 | 1233 | 1236 | 1239 | 1242 | 1245 | 1247 | 1250 | 1252 | 1256 |
| 10                  | 1259 | 1262 | 1265 | 1268 | 1271 | 1274 | 1276 | 1279 | 1282 | 1285 |
| 11                  | 1288 | 1291 | 1294 | 1297 | 1300 | 1303 | 1306 | 1309 | 1312 | 1315 |
| 12                  | 1318 | 1321 | 1324 | 1327 | 1330 | 1334 | 1337 | 1340 | 1343 | 1346 |
| 13                  | 1349 | 1352 | 1355 | 1358 | 1361 | 1365 | 1368 | 1371 | 1374 | 1377 |
| 14                  | 1380 | 1384 | 1387 | 1390 | 1393 | 1396 | 1400 | 1403 | 1406 | 1409 |
| 15                  | 1413 | 1416 | 1418 | 1422 | 1426 | 1429 | 1432 | 1435 | 1439 | 1442 |
| 16                  | 1445 | 1449 | 1452 | 1455 | 1459 | 1462 | 1466 | 1469 | 1472 | 1476 |
| 17                  | 1470 | 1483 | 1486 | 1489 | 1493 | 1496 | 1500 | 1503 | 1507 | 1510 |
| 18                  | 1514 | 1517 | 1521 | 1524 | 1528 | 1531 | 1535 | 1538 | 1542 | 1545 |
| 19                  | 1549 | 1552 | 1556 | 1560 | 1563 | 1567 | 1570 | 1574 | 1578 | 1581 |
| 20                  | 1585 | 1589 | 1592 | 1596 | 1600 | 1603 | 1607 | 1611 | 1614 | 1618 |
| 21                  | 1622 | 1626 | 1629 | 1633 | 1637 | 1641 | 1644 | 1648 | 1652 | 1656 |
| 22                  | 1660 | 1663 | 1667 | 1671 | 1675 | 1679 | 1683 | 1687 | 1690 | 1694 |
| 23                  | 1698 | 1702 | 1705 | 1710 | 1714 | 1718 | 1722 | 1726 | 1730 | 1734 |
| 24                  | 1738 | 1742 | 1745 | 1750 | 1754 | 1758 | 1762 | 1766 | 1770 | 1774 |
| 25                  | 1778 | 1782 | 1786 | 1791 | 1795 | 1799 | 1803 | 1807 | 1811 | 1816 |
| 26                  | 1820 | 1824 | 1828 | 1832 | 1837 | 1841 | 1845 | 1849 | 1854 | 1858 |
| 27                  | 1862 | 1866 | 1871 | 1875 | 1879 | 1884 | 1888 | 1892 | 1897 | 1901 |
| 28                  | 1905 | 1910 | 1914 | 1919 | 1923 | 1928 | 1932 | 1936 | 1941 | 1945 |
| 29                  | 1950 | 1954 | 1959 | 1963 | 1968 | 1972 | 1977 | 1982 | 1986 | 1991 |
| 30                  | 1995 | 2000 | 2004 | 2009 | 2014 | 2018 | 2023 | 2028 | 2032 | 2037 |
| 31                  | 2042 | 2046 | 2051 | 2056 | 2061 | 2065 | 2070 | 2075 | 2080 | 2084 |
| 32                  | 2089 | 2094 | 2099 | 2104 | 2109 | 2113 | 2118 | 2123 | 2128 | 2133 |
| 33                  | 2138 | 2143 | 2148 | 2153 | 2158 | 2163 | 2168 | 2173 | 2178 | 2183 |
| 34                  | 2188 | 2193 | 2198 | 2203 | 2208 | 2213 | 2218 | 2223 | 2228 | 2234 |
| 35                  | 2239 | 2244 | 2249 | 2254 | 2259 | 2265 | 2270 | 2275 | 2280 | 2286 |
| 36                  | 2291 | 2296 | 2301 | 2307 | 2312 | 2317 | 2323 | 2328 | 2333 | 2339 |
| 37                  | 2344 | 2350 | 2355 | 2360 | 2366 | 2371 | 2377 | 2382 | 2388 | 2393 |
| 38                  | 2399 | 2404 | 2410 | 2415 | 2421 | 2427 | 2432 | 2438 | 2443 | 2449 |
| 39                  | 2455 | 2460 | 2466 | 2472 | 2477 | 2483 | 2489 | 2495 | 2500 | 2506 |
| 40                  | 2512 | 2518 | 2523 | 2529 | 2536 | 2541 | 2547 | 2553 | 2559 | 2564 |
| 41                  | 2570 | 2576 | 2582 | 2588 | 2594 | 2600 | 2606 | 2612 | 2618 | 2624 |
| 42                  | 2630 | 2636 | 2642 | 2649 | 2655 | 2661 | 2667 | 2673 | 2679 | 2685 |
| 43                  | 2692 | 2698 | 2704 | 2710 | 2716 | 2723 | 2729 | 2735 | 2742 | 2748 |
| 44                  | 2754 | 2761 | 2767 | 2773 | 2779 | 2786 | 2792 | 2798 | 2805 | 2812 |
| 45                  | 2818 | 2825 | 2831 | 2838 | 2844 | 2851 | 2858 | 2864 | 2871 | 2877 |
| 46                  | 2884 | 2891 | 2897 | 2904 | 2911 | 2917 | 2924 | 2931 | 2938 | 2944 |
| 47                  | 2951 | 2958 | 2965 | 2972 | 2979 | 2985 | 2992 | 2999 | 3006 | 3013 |
| 48                  | 3020 | 3027 | 3034 | 3041 | 3048 | 3055 | 3062 | 3069 | 3076 | 3083 |
| 49                  | 3090 | 3097 | 3105 | 3112 | 3119 | 3126 | 3133 | 3141 | 3148 | 3155 |

Table A.3 Standard Normal Curve Areas

| $\Phi(z) = P(Z \leq z)$       |       |       |       |       |       |       |       |       |       |       |
|-------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Standard normal density curve |       |       |       |       |       |       |       |       |       |       |
| Shaded area = $\Phi(z)$       |       |       |       |       |       |       |       |       |       |       |
|                               | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
| z                             | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
| -3.4                          | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3                          | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2                          | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1                          | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0                          | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9                          | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8                          | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7                          | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6                          | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5                          | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4                          | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3                          | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2                          | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1                          | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0                          | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9                          | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8                          | .0359 | .0352 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7                          | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6                          | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5                          | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4                          | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| -1.3                          | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2                          | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1                          | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0                          | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9                          | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8                          | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7                          | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6                          | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5                          | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4                          | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3                          | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3482 |
| -0.2                          | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1                          | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0                          | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

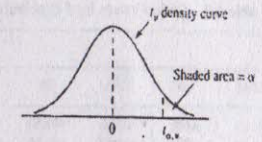
(continued)

Table A.3 Standard Normal Curve Areas (cont.)

$\Phi(z) = P(Z \leq z)$

| z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9278 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Table A.5 Critical Values for t Distributions



| v        | $\alpha$ |       |        |        |        |        |        |
|----------|----------|-------|--------|--------|--------|--------|--------|
|          | .10      | .05   | .025   | .01    | .005   | .001   | .0005  |
| 1        | 3.078    | 6.314 | 12.706 | 31.821 | 63.657 | 318.31 | 636.62 |
| 2        | 1.886    | 2.920 | 4.303  | 6.965  | 9.925  | 22.326 | 31.598 |
| 3        | 1.638    | 2.353 | 3.182  | 4.541  | 5.841  | 10.213 | 12.924 |
| 4        | 1.533    | 2.132 | 2.776  | 3.747  | 4.604  | 7.173  | 8.610  |
| 5        | 1.476    | 2.015 | 2.571  | 3.365  | 4.032  | 5.893  | 6.869  |
| 6        | 1.440    | 1.943 | 2.447  | 3.143  | 3.707  | 5.208  | 5.959  |
| 7        | 1.415    | 1.895 | 2.365  | 2.998  | 3.499  | 4.785  | 5.408  |
| 8        | 1.397    | 1.860 | 2.306  | 2.896  | 3.355  | 4.501  | 5.041  |
| 9        | 1.383    | 1.833 | 2.262  | 2.821  | 3.250  | 4.297  | 4.781  |
| 10       | 1.372    | 1.812 | 2.228  | 2.764  | 3.169  | 4.144  | 4.587  |
| 11       | 1.363    | 1.796 | 2.201  | 2.718  | 3.106  | 4.025  | 4.437  |
| 12       | 1.356    | 1.782 | 2.179  | 2.681  | 3.055  | 3.930  | 4.318  |
| 13       | 1.350    | 1.771 | 2.160  | 2.650  | 3.012  | 3.852  | 4.221  |
| 14       | 1.345    | 1.761 | 2.145  | 2.624  | 2.977  | 3.787  | 4.140  |
| 15       | 1.341    | 1.753 | 2.131  | 2.602  | 2.947  | 3.733  | 4.073  |
| 16       | 1.337    | 1.746 | 2.120  | 2.583  | 2.921  | 3.686  | 4.015  |
| 17       | 1.333    | 1.740 | 2.110  | 2.567  | 2.898  | 3.646  | 3.965  |
| 18       | 1.330    | 1.734 | 2.101  | 2.552  | 2.878  | 3.610  | 3.922  |
| 19       | 1.328    | 1.729 | 2.093  | 2.539  | 2.861  | 3.579  | 3.883  |
| 20       | 1.325    | 1.725 | 2.086  | 2.528  | 2.845  | 3.552  | 3.850  |
| 21       | 1.323    | 1.721 | 2.080  | 2.518  | 2.831  | 3.527  | 3.819  |
| 22       | 1.321    | 1.717 | 2.074  | 2.508  | 2.819  | 3.505  | 3.792  |
| 23       | 1.319    | 1.714 | 2.069  | 2.500  | 2.807  | 3.485  | 3.767  |
| 24       | 1.318    | 1.711 | 2.064  | 2.492  | 2.797  | 3.467  | 3.745  |
| 25       | 1.316    | 1.708 | 2.060  | 2.485  | 2.787  | 3.450  | 3.725  |
| 26       | 1.315    | 1.706 | 2.056  | 2.479  | 2.779  | 3.435  | 3.707  |
| 27       | 1.314    | 1.703 | 2.052  | 2.473  | 2.771  | 3.421  | 3.690  |
| 28       | 1.313    | 1.701 | 2.048  | 2.467  | 2.763  | 3.408  | 3.674  |
| 29       | 1.311    | 1.699 | 2.045  | 2.462  | 2.756  | 3.396  | 3.659  |
| 30       | 1.310    | 1.697 | 2.042  | 2.457  | 2.750  | 3.385  | 3.646  |
| 32       | 1.309    | 1.694 | 2.037  | 2.449  | 2.738  | 3.365  | 3.622  |
| 34       | 1.307    | 1.691 | 2.032  | 2.441  | 2.728  | 3.348  | 3.601  |
| 36       | 1.306    | 1.688 | 2.028  | 2.434  | 2.719  | 3.333  | 3.582  |
| 38       | 1.304    | 1.686 | 2.024  | 2.429  | 2.712  | 3.319  | 3.566  |
| 40       | 1.303    | 1.684 | 2.021  | 2.423  | 2.704  | 3.307  | 3.551  |
| 50       | 1.299    | 1.676 | 2.009  | 2.403  | 2.678  | 3.262  | 3.496  |
| 60       | 1.296    | 1.671 | 2.000  | 2.390  | 2.660  | 3.232  | 3.460  |
| 120      | 1.289    | 1.658 | 1.980  | 2.358  | 2.617  | 3.160  | 3.373  |
| $\infty$ | 1.282    | 1.645 | 1.960  | 2.326  | 2.576  | 3.090  | 3.291  |

Table A.9 Critical Values for F Distributions

|          |                                 | $\nu_1 = \text{numerator df}$ |         |         |         |         |         |         |         |         |
|----------|---------------------------------|-------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\alpha$ | $\nu_2 = \text{denominator df}$ | 1                             | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       |
|          |                                 |                               |         |         |         |         |         |         |         |         |
| .100     | 1                               | 39.86                         | 49.50   | 53.59   | 55.83   | 57.24   | 58.20   | 58.91   | 59.44   | 59.86   |
| .050     | 1                               | 161.45                        | 199.50  | 215.71  | 224.58  | 230.16  | 233.99  | 236.77  | 238.88  | 240.54  |
| .010     | 1                               | 4052.20                       | 4999.50 | 5403.40 | 5624.60 | 5763.60 | 5859.00 | 5928.40 | 5981.10 | 6022.50 |
| .001     | 1                               | 405.284                       | 500.000 | 540.379 | 562.500 | 576.405 | 585.937 | 592.873 | 598.144 | 602.284 |
| .100     | 2                               | 8.53                          | 9.00    | 9.16    | 9.24    | 9.29    | 9.33    | 9.35    | 9.37    | 9.38    |
| .050     | 2                               | 18.51                         | 19.00   | 19.16   | 19.25   | 19.30   | 19.33   | 19.35   | 19.37   | 19.38   |
| .010     | 2                               | 98.50                         | 99.00   | 99.17   | 99.25   | 99.30   | 99.33   | 99.36   | 99.37   | 99.39   |
| .001     | 2                               | 998.50                        | 999.00  | 999.17  | 999.25  | 999.30  | 999.33  | 999.36  | 999.37  | 999.39  |
| .100     | 3                               | 5.54                          | 5.46    | 5.39    | 5.34    | 5.31    | 5.28    | 5.27    | 5.25    | 5.24    |
| .050     | 3                               | 10.13                         | 9.55    | 9.28    | 9.12    | 9.01    | 8.94    | 8.89    | 8.85    | 8.81    |
| .010     | 3                               | 34.12                         | 30.82   | 29.46   | 28.71   | 28.24   | 27.91   | 27.67   | 27.49   | 27.35   |
| .001     | 3                               | 167.03                        | 148.50  | 141.11  | 137.10  | 134.58  | 132.85  | 131.58  | 130.62  | 129.86  |
| .100     | 4                               | 4.54                          | 4.32    | 4.19    | 4.11    | 4.05    | 4.01    | 3.98    | 3.95    | 3.94    |
| .050     | 4                               | 7.71                          | 6.94    | 6.59    | 6.39    | 6.26    | 6.16    | 6.09    | 6.04    | 6.00    |
| .010     | 4                               | 21.20                         | 18.00   | 16.69   | 15.98   | 15.52   | 15.21   | 14.98   | 14.80   | 14.66   |
| .001     | 4                               | 74.14                         | 61.25   | 56.18   | 53.44   | 51.71   | 50.53   | 49.66   | 49.00   | 48.47   |
| .100     | 5                               | 4.06                          | 3.78    | 3.62    | 3.52    | 3.45    | 3.40    | 3.37    | 3.34    | 3.32    |
| .050     | 5                               | 6.61                          | 5.79    | 5.41    | 5.19    | 5.05    | 4.95    | 4.88    | 4.82    | 4.77    |
| .010     | 5                               | 16.26                         | 13.27   | 12.06   | 11.39   | 10.97   | 10.67   | 10.46   | 10.29   | 10.16   |
| .001     | 5                               | 47.18                         | 37.12   | 33.20   | 31.09   | 29.75   | 28.83   | 28.16   | 27.65   | 27.24   |
| .100     | 6                               | 3.78                          | 3.46    | 3.29    | 3.18    | 3.11    | 3.05    | 3.01    | 2.98    | 2.96    |
| .050     | 6                               | 5.99                          | 5.14    | 4.76    | 4.53    | 4.39    | 4.28    | 4.21    | 4.15    | 4.10    |
| .010     | 6                               | 13.75                         | 10.92   | 9.78    | 9.15    | 8.75    | 8.47    | 8.26    | 8.10    | 7.98    |
| .001     | 6                               | 35.51                         | 27.00   | 23.70   | 21.92   | 20.80   | 20.03   | 19.46   | 19.03   | 18.69   |
| .100     | 7                               | 3.59                          | 3.26    | 3.07    | 2.96    | 2.88    | 2.83    | 2.78    | 2.75    | 2.72    |
| .050     | 7                               | 5.59                          | 4.74    | 4.35    | 4.12    | 3.97    | 3.87    | 3.79    | 3.73    | 3.68    |
| .010     | 7                               | 12.25                         | 9.55    | 8.45    | 7.85    | 7.46    | 7.19    | 6.99    | 6.84    | 6.72    |
| .001     | 7                               | 29.25                         | 21.69   | 18.77   | 17.20   | 16.21   | 15.52   | 15.02   | 14.63   | 14.33   |
| .100     | 8                               | 3.46                          | 3.11    | 2.92    | 2.81    | 2.73    | 2.67    | 2.62    | 2.59    | 2.56    |
| .050     | 8                               | 5.32                          | 4.46    | 4.07    | 3.84    | 3.69    | 3.58    | 3.50    | 3.44    | 3.39    |
| .010     | 8                               | 11.26                         | 8.65    | 7.59    | 7.01    | 6.63    | 6.37    | 6.18    | 6.03    | 5.91    |
| .001     | 8                               | 25.41                         | 18.49   | 15.83   | 14.39   | 13.48   | 12.86   | 12.40   | 12.05   | 11.77   |
| .100     | 9                               | 3.36                          | 3.01    | 2.81    | 2.69    | 2.61    | 2.55    | 2.51    | 2.47    | 2.44    |
| .050     | 9                               | 5.12                          | 4.26    | 3.86    | 3.63    | 3.48    | 3.37    | 3.29    | 3.23    | 3.18    |
| .010     | 9                               | 10.56                         | 8.02    | 6.99    | 6.42    | 6.06    | 5.80    | 5.61    | 5.47    | 5.35    |
| .001     | 9                               | 22.86                         | 16.39   | 13.90   | 12.56   | 11.71   | 11.13   | 10.70   | 10.37   | 10.11   |
| .100     | 10                              | 3.29                          | 2.92    | 2.73    | 2.61    | 2.52    | 2.46    | 2.41    | 2.38    | 2.35    |
| .050     | 10                              | 4.96                          | 4.10    | 3.71    | 3.48    | 3.33    | 3.22    | 3.14    | 3.07    | 3.02    |
| .010     | 10                              | 10.04                         | 7.56    | 6.55    | 5.99    | 5.64    | 5.39    | 5.20    | 5.06    | 4.94    |
| .001     | 10                              | 21.04                         | 14.91   | 12.55   | 11.28   | 10.48   | 9.93    | 9.52    | 9.20    | 8.96    |
| .100     | 11                              | 3.23                          | 2.86    | 2.66    | 2.54    | 2.45    | 2.39    | 2.34    | 2.30    | 2.27    |
| .050     | 11                              | 4.84                          | 3.98    | 3.59    | 3.36    | 3.20    | 3.09    | 3.01    | 2.95    | 2.90    |
| .010     | 11                              | 9.65                          | 7.21    | 6.22    | 5.67    | 5.32    | 5.07    | 4.89    | 4.74    | 4.63    |
| .001     | 11                              | 19.69                         | 13.81   | 11.56   | 10.35   | 9.58    | 9.05    | 8.66    | 8.35    | 8.12    |
| .100     | 12                              | 3.18                          | 2.81    | 2.61    | 2.48    | 2.39    | 2.33    | 2.28    | 2.24    | 2.21    |
| .050     | 12                              | 4.75                          | 3.89    | 3.49    | 3.26    | 3.11    | 3.00    | 2.91    | 2.85    | 2.80    |
| .010     | 12                              | 9.33                          | 6.93    | 5.95    | 5.41    | 5.06    | 4.82    | 4.64    | 4.50    | 4.39    |
| .001     | 12                              | 18.64                         | 12.97   | 10.80   | 9.63    | 8.89    | 8.38    | 8.00    | 7.71    | 7.48    |

(continued)

Table A.9 Critical Values for F Distributions (cont.)

|          |                                 | $\nu_1 = \text{numerator df}$ |         |         |         |         |         |         |         |         |         |         |
|----------|---------------------------------|-------------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $\alpha$ | $\nu_2 = \text{denominator df}$ | 10                            | 12      | 15      | 20      | 25      | 30      | 40      | 50      | 60      | 120     | 1000    |
|          |                                 |                               |         |         |         |         |         |         |         |         |         |         |
| .100     | 10                              | 60.19                         | 60.71   | 61.22   | 61.74   | 62.05   | 62.26   | 62.53   | 62.69   | 62.79   | 63.06   | 63.30   |
| .050     | 10                              | 241.88                        | 243.91  | 245.95  | 248.01  | 249.26  | 250.10  | 251.14  | 251.77  | 252.20  | 253.25  | 254.19  |
| .010     | 10                              | 6055.80                       | 6106.30 | 6157.30 | 6208.70 | 6239.80 | 6260.60 | 6286.80 | 6302.50 | 6313.00 | 6339.40 | 6362.70 |
| .001     | 10                              | 605.621                       | 610.668 | 615.764 | 620.908 | 624.017 | 626.099 | 628.712 | 630.285 | 631.337 | 633.972 | 636.301 |
| .100     | 12                              | 9.39                          | 9.41    | 9.42    | 9.44    | 9.45    | 9.46    | 9.47    | 9.47    | 9.47    | 9.48    | 9.49    |
| .050     | 12                              | 19.40                         | 19.41   | 19.43   | 19.45   | 19.46   | 19.46   | 19.47   | 19.47   | 19.48   | 19.49   | 19.49   |
| .010     | 12                              | 99.40                         | 99.42   | 99.43   | 99.45   | 99.46   | 99.47   | 99.47   | 99.48   | 99.48   | 99.49   | 99.50   |
| .001     | 12                              | 999.40                        | 999.42  | 999.43  | 999.45  | 999.46  | 999.47  | 999.47  | 999.48  | 999.48  | 999.49  | 999.50  |
| .100     | 15                              | 5.23                          | 5.22    | 5.20    | 5.18    | 5.17    | 5.17    | 5.16    | 5.15    | 5.15    | 5.14    | 5.13    |
| .050     | 15                              | 8.79                          | 8.74    | 8.70    | 8.66    | 8.63    | 8.62    | 8.59    | 8.58    | 8.57    | 8.55    | 8.53    |
| .010     | 15                              | 27.23                         | 27.05   | 26.87   | 26.69   | 26.58   | 26.50   | 26.41   | 26.35   | 26.32   | 26.22   | 26.14   |
| .001     | 15                              | 129.25                        | 128.32  | 127.37  | 126.42  | 125.84  | 125.45  | 124.96  | 124.66  | 124.47  | 123.97  | 123.53  |
| .100     | 20                              | 3.92                          | 3.90    | 3.87    | 3.84    | 3.83    | 3.82    | 3.80    | 3.80    | 3.79    | 3.78    | 3.76    |
| .050     | 20                              | 5.96                          | 5.91    | 5.86    | 5.80    | 5.77    | 5.75    | 5.72    | 5.70    | 5.69    | 5.66    | 5.63    |
| .010     | 20                              | 14.55                         | 14.37   | 14.20   | 14.02   | 13.91   | 13.84   | 13.75   | 13.69   | 13.65   | 13.56   | 13.47   |
| .001     | 20                              | 48.05                         | 47.41   | 46.76   | 46.10   | 45.70   | 45.43   | 45.09   | 44.88   | 44.75   | 44.40   | 44.09   |
| .100     | 25                              | 3.30                          | 3.27    | 3.24    | 3.21    | 3.19    | 3.17    | 3.16    | 3.15    | 3.14    | 3.12    | 3.11    |
| .050     | 25                              | 4.74                          | 4.68    | 4.62    | 4.56    | 4.52    | 4.50    | 4.46    | 4.44    | 4.43    | 4.40    | 4.37    |
| .010     | 25                              | 10.05                         | 9.89    | 9.72    | 9.55    | 9.45    | 9.38    | 9.29    | 9.24    | 9.20    | 9.11    | 9.03    |
| .001     | 25                              | 26.92                         | 26.42   | 25.91   | 25.39   | 25.08   | 24.87   | 24.60   | 24.44   | 24.33   | 24.06   | 23.82   |
| .100     | 30                              | 2.94                          | 2.90    | 2.87    | 2.84    | 2.81    | 2.80    | 2.78    | 2.77    | 2.76    | 2.74    | 2.72    |
| .050     | 30                              | 4.06                          | 4.00    | 3.94    | 3.87    | 3.83    | 3.81    | 3.77    | 3.75    | 3.74    | 3.70    | 3.67    |
| .010     | 30                              | 7.87                          | 7.72    | 7.56    | 7.40    | 7.30    | 7.23    | 7.14    | 7.09    | 7.06    | 6.97    | 6.89    |
| .001     | 30                              | 18.41                         | 17.99   | 17.56   | 17.12   | 16.85   | 16.67   | 16.44   | 16.31   | 16.21   | 15.98   | 15.77   |
| .100     | 40                              | 2.70                          | 2.67    | 2.63    | 2.59    | 2.57    | 2.56    | 2.54    | 2.52    | 2.51    | 2.49    | 2.47    |
| .050     | 40                              | 3.64                          | 3.57    | 3.51    | 3.44    | 3.40    | 3.38    | 3.34    | 3.32    | 3.30    | 3.27    | 3.23    |
| .010     | 40                              | 6.62                          | 6.47    | 6.31    | 6.16    | 6.06    | 5.99    | 5.91    | 5.86    | 5.82    | 5.74    | 5.66    |
| .001     | 40                              | 14.08                         | 13.71   | 13.32   | 12.93   | 12.69   | 12.53   | 12.33   | 12.20   | 12.12   | 11.91   | 11.72   |
| .100     | 50                              | 2.54                          | 2.50    | 2.46    | 2.42    | 2.40    | 2.38    | 2.36    | 2.35    | 2.34    | 2.32    | 2.30    |
| .050     | 50                              | 3.35                          | 3.28    | 3.22    | 3.15    | 3.11    | 3.08    | 3.04    | 3.02    | 3.01    | 2.97    | 2.93    |
| .010     | 50                              | 5.81                          | 5.67    | 5.52    | 5.36    | 5.26    | 5.20    | 5.12    | 5.07    | 5.03    | 4.95    | 4.87    |
| .001     | 50                              | 11.54                         | 11.19   | 10.84   | 10.48   | 10.26   | 10.11   | 9.92    | 9.80    | 9.73    | 9.53    | 9.36    |
| .100     | 60                              | 2.42                          | 2.38    | 2.34    | 2.30    | 2.27    | 2.25    | 2.23    | 2.22    | 2.21    | 2.18    | 2.16    |
| .050     | 60                              | 3.14                          | 3.07    | 3.01    | 2.94    | 2.89    | 2.86    | 2.83    | 2.80    | 2.79    | 2.75    | 2.71    |
| .010     | 60                              | 5.26                          | 5.11    | 4.96    | 4.81    | 4.71    | 4.65    | 4.57    | 4.52    | 4.48    | 4.40    | 4.32    |
| .001     | 60                              | 9.89                          | 9.57    | 9.24    | 8.90    | 8.69    | 8.55    | 8.37    | 8.26    | 8.19    | 8.00    | 7.84    |
| .100     | 100                             | 2.32                          | 2.28    | 2.24    | 2.20    | 2.17    | 2.16    | 2.13    | 2.12    | 2.11    | 2.08    | 2.06    |
| .050     | 100                             | 2.98                          | 2.91    | 2.85    | 2.77    | 2.73    | 2.70    | 2.66    | 2.64    | 2.62    | 2.58    | 2.54    |
| .010     | 100                             | 4.85                          | 4.71    | 4.56    | 4.41    | 4.31    | 4.25    | 4.17    | 4.12    | 4.08    | 4.00    | 3.92    |
| .001     | 100                             | 8.75                          | 8.45    | 8.13    | 7.80    | 7.60    | 7.47    | 7.30    | 7.19    | 7.12    | 6.94    | 6.78    |
| .100     | 120                             | 2.25                          | 2.21    | 2.17    | 2.12    | 2.10    | 2.08    | 2.05    | 2.04    | 2.03    | 2.00    | 1.98    |
| .050     | 120                             | 2.85                          | 2.79    | 2.72    | 2.65    | 2.60    | 2.57    | 2.53    | 2.51    | 2.49    | 2.45    | 2.41    |
| .010     | 120                             | 4.54                          | 4.40    | 4.25    | 4.10    | 4.01    | 3.94    | 3.86    | 3.81    | 3.78    | 3.69    | 3.61    |
| .001     | 120                             | 7.92                          | 7.63    | 7.32    | 7.01    | 6.81    | 6.68    | 6.52    | 6.42    | 6.35    | 6.18    | 6.02    |
| .100     | 200                             | 2.19                          | 2.15    | 2.10    | 2.06    | 2.03    | 2.01    | 1.99    | 1.97    | 1.96    | 1.93    | 1.91    |
| .050     | 200                             | 2.75                          | 2.69    | 2.62    | 2.54    | 2.50    | 2.47    | 2.43    | 2.40    | 2.38    | 2.34    | 2.30    |
| .010     | 200                             | 4.30                          | 4.16    | 4.01    | 3.86    | 3.76    | 3.70    | 3.62    | 3.57    | 3.54    | 3.45    | 3.37    |
| .001     | 200                             | 7.29                          | 7.00    | 6.71    | 6.40    | 6.22    | 6.09    | 5.93    | 5.83    | 5.76    | 5.59    | 5.44    |

(continued)

Table A.9 Critical Values for F Distributions (cont.)

|          |      | $\nu_1 = \text{numerator df}$ |       |       |      |      |      |      |      |      |
|----------|------|-------------------------------|-------|-------|------|------|------|------|------|------|
| $\alpha$ |      | 1                             | 2     | 3     | 4    | 5    | 6    | 7    | 8    | 9    |
| 13       | .100 | 3.14                          | 2.76  | 2.56  | 2.43 | 2.35 | 2.28 | 2.23 | 2.20 | 2.16 |
|          | .050 | 4.57                          | 3.81  | 3.41  | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 |
|          | .010 | 9.07                          | 6.70  | 5.74  | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 |
|          | .001 | 17.82                         | 12.31 | 10.21 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 |
| 14       | .100 | 3.10                          | 2.73  | 2.52  | 2.39 | 2.31 | 2.24 | 2.19 | 2.15 | 2.12 |
|          | .050 | 4.60                          | 3.74  | 3.34  | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 |
|          | .010 | 8.86                          | 6.51  | 5.56  | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 |
|          | .001 | 17.14                         | 11.78 | 9.73  | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 |
| 15       | .100 | 3.07                          | 2.70  | 2.49  | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 |
|          | .050 | 4.54                          | 3.68  | 3.29  | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
|          | .010 | 8.68                          | 6.36  | 5.42  | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
|          | .001 | 16.59                         | 11.34 | 9.34  | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 |
| 16       | .100 | 3.05                          | 2.67  | 2.46  | 2.33 | 2.24 | 2.18 | 2.13 | 2.09 | 2.06 |
|          | .050 | 4.49                          | 3.63  | 3.24  | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 |
|          | .010 | 8.53                          | 6.23  | 5.29  | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 |
|          | .001 | 16.12                         | 10.97 | 9.01  | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 |
| 17       | .100 | 3.03                          | 2.64  | 2.44  | 2.31 | 2.22 | 2.15 | 2.10 | 2.06 | 2.03 |
|          | .050 | 4.45                          | 3.59  | 3.20  | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 |
|          | .010 | 8.40                          | 6.11  | 5.19  | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 |
|          | .001 | 15.72                         | 10.66 | 8.73  | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 |
| 18       | .100 | 3.01                          | 2.62  | 2.42  | 2.29 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 |
|          | .050 | 4.41                          | 3.55  | 3.16  | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 |
|          | .010 | 8.29                          | 6.01  | 5.09  | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 |
|          | .001 | 15.38                         | 10.39 | 8.49  | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 |
| 19       | .100 | 2.99                          | 2.61  | 2.40  | 2.27 | 2.18 | 2.11 | 2.06 | 2.02 | 1.98 |
|          | .050 | 4.38                          | 3.52  | 3.13  | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 |
|          | .010 | 8.18                          | 5.93  | 5.01  | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 |
|          | .001 | 15.08                         | 10.16 | 8.28  | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 |
| 20       | .100 | 2.97                          | 2.59  | 2.38  | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 |
|          | .050 | 4.35                          | 3.49  | 3.10  | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
|          | .010 | 8.10                          | 5.85  | 4.94  | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
|          | .001 | 14.82                         | 9.95  | 8.10  | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 |
| 21       | .100 | 2.96                          | 2.57  | 2.36  | 2.23 | 2.14 | 2.08 | 2.02 | 1.98 | 1.95 |
|          | .050 | 4.32                          | 3.47  | 3.07  | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 |
|          | .010 | 8.02                          | 5.78  | 4.87  | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 |
|          | .001 | 14.59                         | 9.77  | 7.94  | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 |
| 22       | .100 | 2.95                          | 2.56  | 2.35  | 2.22 | 2.13 | 2.06 | 2.01 | 1.97 | 1.93 |
|          | .050 | 4.30                          | 3.44  | 3.05  | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 |
|          | .010 | 7.95                          | 5.72  | 4.82  | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 |
|          | .001 | 14.38                         | 9.61  | 7.80  | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 |
| 23       | .100 | 2.94                          | 2.55  | 2.34  | 2.21 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 |
|          | .050 | 4.28                          | 3.42  | 3.03  | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 |
|          | .010 | 7.88                          | 5.66  | 4.76  | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 |
|          | .001 | 14.20                         | 9.47  | 7.67  | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 |
| 24       | .100 | 2.93                          | 2.54  | 2.33  | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 |
|          | .050 | 4.26                          | 3.40  | 3.01  | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
|          | .010 | 7.82                          | 5.61  | 4.72  | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |
|          | .001 | 14.03                         | 9.34  | 7.55  | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 |

(continued)

Table A.9 Critical Values for F Distributions (cont.)

| $\nu_1 = \text{numerator df}$ |      |      |      |      |      |      |      |      |      |      |  |  |  |
|-------------------------------|------|------|------|------|------|------|------|------|------|------|--|--|--|
| 10                            | 12   | 15   | 20   | 25   | 30   | 40   | 50   | 60   | 120  | 1000 |  |  |  |
| 2.14                          | 2.10 | 2.05 | 2.01 | 1.98 | 1.96 | 1.93 | 1.92 | 1.90 | 1.88 | 1.85 |  |  |  |
| 2.67                          | 2.60 | 2.53 | 2.46 | 2.41 | 2.38 | 2.34 | 2.31 | 2.30 | 2.25 | 2.21 |  |  |  |
| 4.10                          | 3.96 | 3.82 | 3.66 | 3.57 | 3.51 | 3.43 | 3.38 | 3.34 | 3.25 | 3.18 |  |  |  |
| 6.80                          | 6.52 | 6.23 | 5.93 | 5.75 | 5.63 | 5.47 | 5.37 | 5.30 | 5.14 | 4.99 |  |  |  |
| 2.10                          | 2.05 | 2.01 | 1.96 | 1.93 | 1.91 | 1.89 | 1.87 | 1.86 | 1.83 | 1.80 |  |  |  |
| 2.60                          | 2.53 | 2.46 | 2.39 | 2.34 | 2.31 | 2.27 | 2.24 | 2.22 | 2.18 | 2.14 |  |  |  |
| 3.94                          | 3.80 | 3.66 | 3.51 | 3.41 | 3.35 | 3.27 | 3.22 | 3.18 | 3.09 | 3.02 |  |  |  |
| 6.40                          | 6.13 | 5.85 | 5.56 | 5.38 | 5.25 | 5.10 | 5.00 | 4.94 | 4.77 | 4.62 |  |  |  |
| 2.06                          | 2.02 | 1.97 | 1.92 | 1.89 | 1.87 | 1.85 | 1.83 | 1.82 | 1.79 | 1.76 |  |  |  |
| 2.54                          | 2.48 | 2.40 | 2.33 | 2.23 | 2.25 | 2.20 | 2.18 | 2.16 | 2.11 | 2.07 |  |  |  |
| 3.80                          | 3.67 | 3.52 | 3.37 | 3.28 | 3.21 | 3.13 | 3.08 | 3.05 | 2.96 | 2.88 |  |  |  |
| 6.08                          | 5.81 | 5.54 | 5.25 | 5.07 | 4.95 | 4.80 | 4.70 | 4.64 | 4.47 | 4.33 |  |  |  |
| 2.03                          | 1.99 | 1.94 | 1.89 | 1.86 | 1.84 | 1.81 | 1.79 | 1.78 | 1.75 | 1.72 |  |  |  |
| 2.49                          | 2.42 | 2.35 | 2.28 | 2.23 | 2.19 | 2.15 | 2.12 | 2.11 | 2.06 | 2.02 |  |  |  |
| 3.69                          | 3.55 | 3.41 | 3.26 | 3.16 | 3.10 | 3.02 | 2.97 | 2.93 | 2.84 | 2.76 |  |  |  |
| 5.81                          | 5.55 | 5.27 | 4.99 | 4.82 | 4.70 | 4.54 | 4.45 | 4.39 | 4.23 | 4.08 |  |  |  |
| 2.00                          | 1.96 | 1.91 | 1.86 | 1.83 | 1.81 | 1.78 | 1.76 | 1.75 | 1.72 | 1.69 |  |  |  |
| 2.45                          | 2.38 | 2.31 | 2.23 | 2.18 | 2.15 | 2.10 | 2.08 | 2.06 | 2.01 | 1.97 |  |  |  |
| 3.59                          | 3.46 | 3.31 | 3.16 | 3.07 | 3.00 | 2.92 | 2.87 | 2.83 | 2.75 | 2.66 |  |  |  |
| 5.58                          | 5.32 | 5.05 | 4.78 | 4.60 | 4.48 | 4.33 | 4.24 | 4.18 | 4.02 | 3.87 |  |  |  |
| 1.98                          | 1.93 | 1.89 | 1.84 | 1.80 | 1.78 | 1.75 | 1.74 | 1.72 | 1.69 | 1.66 |  |  |  |
| 2.41                          | 2.34 | 2.27 | 2.19 | 2.14 | 2.11 | 2.06 | 2.04 | 2.02 | 1.97 | 1.92 |  |  |  |
| 3.51                          | 3.37 | 3.23 | 3.08 | 2.98 | 2.92 | 2.84 | 2.78 | 2.75 | 2.66 | 2.58 |  |  |  |
| 5.39                          | 5.13 | 4.87 | 4.59 | 4.42 | 4.30 | 4.15 | 4.06 | 4.00 | 3.84 | 3.69 |  |  |  |
| 1.96                          | 1.91 | 1.86 | 1.81 | 1.78 | 1.76 | 1.73 | 1.71 | 1.70 | 1.67 | 1.64 |  |  |  |
| 2.38                          | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 2.00 | 1.98 | 1.93 | 1.88 |  |  |  |
| 3.43                          | 3.30 | 3.15 | 3.00 | 2.91 | 2.84 | 2.76 | 2.71 | 2.67 | 2.58 | 2.50 |  |  |  |
| 5.22                          | 4.97 | 4.70 | 4.43 | 4.26 | 4.14 | 3.99 | 3.90 | 3.84 | 3.68 | 3.53 |  |  |  |
| 1.94                          | 1.89 | 1.84 | 1.79 | 1.76 | 1.74 | 1.71 | 1.69 | 1.68 | 1.64 | 1.61 |  |  |  |
| 2.35                          | 2.28 | 2.20 | 2.12 | 2.07 | 2.04 | 1.99 | 1.97 | 1.95 | 1.90 | 1.85 |  |  |  |
| 3.37                          | 3.23 | 3.09 | 2.94 | 2.84 | 2.78 | 2.69 | 2.64 | 2.61 | 2.52 | 2.43 |  |  |  |
| 5.08                          | 4.82 | 4.56 | 4.29 | 4.12 | 4.00 | 3.86 | 3.77 | 3.70 | 3.54 | 3.40 |  |  |  |
| 1.92                          | 1.87 | 1.83 | 1.78 | 1.74 | 1.72 | 1.69 | 1.67 | 1.66 | 1.62 | 1.59 |  |  |  |
| 2.32                          | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.94 | 1.92 | 1.87 | 1.82 |  |  |  |
| 3.31                          | 3.17 | 3.03 | 2.88 | 2.79 | 2.72 | 2.64 | 2.58 | 2.55 | 2.46 | 2.37 |  |  |  |
| 4.95                          | 4.70 | 4.44 | 4.17 | 4.00 | 3.88 | 3.74 | 3.64 | 3.58 | 3.42 | 3.28 |  |  |  |
| 1.90                          | 1.86 | 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.65 | 1.64 | 1.60 | 1.57 |  |  |  |
| 2.30                          | 2.23 | 2.15 | 2.07 | 2.02 | 1.98 | 1.94 | 1.91 | 1.89 | 1.84 | 1.79 |  |  |  |
| 3.26                          | 3.12 | 2.98 | 2.83 | 2.73 | 2.67 | 2.58 | 2.53 | 2.50 | 2.40 | 2.32 |  |  |  |
| 4.83                          | 4.58 | 4.33 | 4.06 | 3.89 | 3.78 | 3.63 | 3.54 | 3.48 | 3.32 | 3.17 |  |  |  |
| 1.89                          | 1.84 | 1.80 | 1.74 | 1.71 | 1.69 | 1.66 | 1.64 | 1.62 | 1.59 | 1.55 |  |  |  |
| 2.27                          | 2.20 | 2.13 | 2.05 | 2.00 | 1.96 | 1.91 | 1.88 | 1.86 | 1.81 | 1.76 |  |  |  |
| 3.21                          | 3.07 | 2.93 | 2.78 | 2.69 | 2.62 | 2.54 | 2.48 | 2.45 | 2.35 | 2.27 |  |  |  |
| 4.73                          | 4.48 | 4.23 | 3.96 | 3.79 | 3.68 | 3.53 | 3.44 | 3.38 | 3.22 | 3.08 |  |  |  |
| 1.88                          | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.64 | 1.62 | 1.61 | 1.57 | 1.54 |  |  |  |
| 2.25                          | 2.18 | 2.11 | 2.03 | 1.97 | 1.94 | 1.89 | 1.86 | 1.84 | 1.79 | 1.74 |  |  |  |
| 3.17                          | 3.03 | 2.89 | 2.74 | 2.64 | 2.58 | 2.49 | 2.44 | 2.40 | 2.31 | 2.22 |  |  |  |
| 4.64                          | 4.39 | 4.14 | 3.87 | 3.71 | 3.59 | 3.45 | 3.36 | 3.29 | 3.14 | 2.99 |  |  |  |

Table A.9 Critical Values for F Distributions (cont.)

|          |      | $\nu_1$ = numerator df |      |      |      |      |      |      |      |      |
|----------|------|------------------------|------|------|------|------|------|------|------|------|
| $\alpha$ |      | 1                      | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    |
| 25       | .100 | 2.92                   | 2.53 | 2.32 | 2.18 | 2.09 | 2.02 | 1.97 | 1.93 | 1.89 |
|          | .050 | 4.24                   | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 |
|          | .010 | 7.77                   | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 |
|          | .001 | 13.88                  | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 |
| 26       | .100 | 2.91                   | 2.52 | 2.31 | 2.17 | 2.08 | 2.01 | 1.96 | 1.92 | 1.88 |
|          | .050 | 4.23                   | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 |
|          | .010 | 7.72                   | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 |
|          | .001 | 13.74                  | 9.12 | 7.36 | 6.41 | 5.80 | 5.38 | 5.07 | 4.83 | 4.64 |
| 27       | .100 | 2.90                   | 2.51 | 2.30 | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 |
|          | .050 | 4.21                   | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 |
|          | .010 | 7.68                   | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 |
|          | .001 | 13.61                  | 9.02 | 7.27 | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 |
| 28       | .100 | 2.89                   | 2.50 | 2.29 | 2.16 | 2.06 | 2.00 | 1.94 | 1.90 | 1.87 |
|          | .050 | 4.20                   | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 |
|          | .010 | 7.64                   | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 |
|          | .001 | 13.50                  | 8.93 | 7.19 | 6.25 | 5.66 | 5.24 | 4.93 | 4.69 | 4.50 |
| 29       | .100 | 2.89                   | 2.50 | 2.28 | 2.15 | 2.06 | 1.99 | 1.93 | 1.89 | 1.86 |
|          | .050 | 4.18                   | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 |
|          | .010 | 7.60                   | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 |
|          | .001 | 13.39                  | 8.85 | 7.12 | 6.19 | 5.59 | 5.18 | 4.87 | 4.64 | 4.45 |
| 30       | .100 | 2.88                   | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 |
|          | .050 | 4.17                   | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
|          | .010 | 7.56                   | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
|          | .001 | 13.29                  | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 |
| 40       | .100 | 2.84                   | 2.44 | 2.23 | 2.09 | 2.00 | 1.93 | 1.87 | 1.83 | 1.79 |
|          | .050 | 4.08                   | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 |
|          | .010 | 7.31                   | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 |
|          | .001 | 12.61                  | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 |
| 50       | .100 | 2.81                   | 2.41 | 2.20 | 2.06 | 1.97 | 1.90 | 1.84 | 1.80 | 1.76 |
|          | .050 | 4.03                   | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 |
|          | .010 | 7.17                   | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 |
|          | .001 | 12.22                  | 7.96 | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 |
| 60       | .100 | 2.79                   | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 |
|          | .050 | 4.00                   | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
|          | .010 | 7.08                   | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
|          | .001 | 11.97                  | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 |
| 100      | .100 | 2.76                   | 2.36 | 2.14 | 2.00 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 |
|          | .050 | 3.94                   | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 |
|          | .010 | 6.90                   | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 |
|          | .001 | 11.50                  | 7.41 | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 |
| 200      | .100 | 2.73                   | 2.33 | 2.11 | 1.97 | 1.88 | 1.80 | 1.75 | 1.70 | 1.66 |
|          | .050 | 3.89                   | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 |
|          | .010 | 6.76                   | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 |
|          | .001 | 11.15                  | 7.15 | 5.63 | 4.81 | 4.29 | 3.92 | 3.65 | 3.43 | 3.26 |
| 1000     | .100 | 2.71                   | 2.31 | 2.09 | 1.95 | 1.85 | 1.78 | 1.72 | 1.68 | 1.64 |
|          | .050 | 3.85                   | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 |
|          | .010 | 6.66                   | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 |
|          | .001 | 10.89                  | 6.96 | 5.46 | 4.65 | 4.14 | 3.78 | 3.51 | 3.30 | 3.13 |

(continued)

Table A.9 Critical Values for F Distributions (cont.)

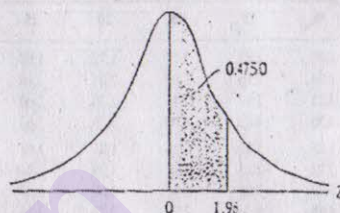
|      |      | $\nu_1$ = numerator df |      |      |      |      |      |      |      |      |      |
|------|------|------------------------|------|------|------|------|------|------|------|------|------|
|      |      | 10                     | 12   | 15   | 20   | 25   | 30   | 40   | 50   | 60   | 100  |
| 25   | .100 | 1.87                   | 1.82 | 1.77 | 1.72 | 1.68 | 1.66 | 1.63 | 1.61 | 1.59 | 1.56 |
|      | .050 | 2.24                   | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.84 | 1.82 | 1.77 |
|      | .010 | 3.13                   | 2.99 | 2.85 | 2.70 | 2.60 | 2.54 | 2.45 | 2.40 | 2.36 | 2.27 |
|      | .001 | 4.56                   | 4.31 | 4.06 | 3.79 | 3.63 | 3.52 | 3.37 | 3.28 | 3.22 | 3.06 |
| 26   | .100 | 1.86                   | 1.81 | 1.76 | 1.71 | 1.67 | 1.65 | 1.61 | 1.59 | 1.58 | 1.54 |
|      | .050 | 2.22                   | 2.15 | 2.07 | 1.99 | 1.94 | 1.90 | 1.85 | 1.82 | 1.80 | 1.75 |
|      | .010 | 3.09                   | 2.96 | 2.81 | 2.66 | 2.57 | 2.50 | 2.42 | 2.36 | 2.33 | 2.23 |
|      | .001 | 4.48                   | 4.24 | 3.99 | 3.72 | 3.56 | 3.44 | 3.30 | 3.21 | 3.15 | 2.99 |
| 27   | .100 | 1.85                   | 1.80 | 1.75 | 1.70 | 1.66 | 1.64 | 1.60 | 1.58 | 1.57 | 1.53 |
|      | .050 | 2.20                   | 2.13 | 2.06 | 1.97 | 1.92 | 1.88 | 1.84 | 1.81 | 1.79 | 1.73 |
|      | .010 | 3.06                   | 2.93 | 2.78 | 2.63 | 2.54 | 2.47 | 2.38 | 2.33 | 2.29 | 2.20 |
|      | .001 | 4.41                   | 4.17 | 3.92 | 3.66 | 3.49 | 3.38 | 3.23 | 3.14 | 3.08 | 2.92 |
| 28   | .100 | 1.84                   | 1.79 | 1.74 | 1.69 | 1.65 | 1.63 | 1.59 | 1.57 | 1.56 | 1.52 |
|      | .050 | 2.19                   | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.79 | 1.77 | 1.71 |
|      | .010 | 3.03                   | 2.90 | 2.75 | 2.60 | 2.51 | 2.44 | 2.35 | 2.30 | 2.26 | 2.17 |
|      | .001 | 4.35                   | 4.11 | 3.86 | 3.60 | 3.43 | 3.32 | 3.18 | 3.09 | 3.02 | 2.86 |
| 29   | .100 | 1.83                   | 1.78 | 1.73 | 1.68 | 1.64 | 1.62 | 1.58 | 1.56 | 1.55 | 1.51 |
|      | .050 | 2.18                   | 2.10 | 2.03 | 1.94 | 1.89 | 1.85 | 1.81 | 1.77 | 1.75 | 1.69 |
|      | .010 | 3.00                   | 2.87 | 2.73 | 2.57 | 2.48 | 2.41 | 2.33 | 2.27 | 2.23 | 2.14 |
|      | .001 | 4.29                   | 4.05 | 3.80 | 3.54 | 3.38 | 3.27 | 3.12 | 3.03 | 2.97 | 2.81 |
| 30   | .100 | 1.82                   | 1.77 | 1.72 | 1.67 | 1.63 | 1.61 | 1.57 | 1.55 | 1.54 | 1.50 |
|      | .050 | 2.16                   | 2.09 | 2.01 | 1.93 | 1.88 | 1.84 | 1.79 | 1.76 | 1.74 | 1.68 |
|      | .010 | 2.98                   | 2.84 | 2.70 | 2.55 | 2.45 | 2.39 | 2.30 | 2.25 | 2.21 | 2.11 |
|      | .001 | 4.24                   | 4.00 | 3.75 | 3.49 | 3.33 | 3.22 | 3.07 | 2.98 | 2.92 | 2.76 |
| 40   | .100 | 1.76                   | 1.71 | 1.66 | 1.61 | 1.57 | 1.54 | 1.51 | 1.48 | 1.47 | 1.42 |
|      | .050 | 2.08                   | 2.00 | 1.92 | 1.84 | 1.78 | 1.74 | 1.69 | 1.66 | 1.64 | 1.58 |
|      | .010 | 2.80                   | 2.66 | 2.52 | 2.37 | 2.27 | 2.20 | 2.11 | 2.06 | 2.02 | 1.92 |
|      | .001 | 3.87                   | 3.64 | 3.40 | 3.14 | 2.98 | 2.87 | 2.73 | 2.64 | 2.57 | 2.41 |
| 50   | .100 | 1.73                   | 1.68 | 1.63 | 1.57 | 1.53 | 1.50 | 1.46 | 1.44 | 1.42 | 1.38 |
|      | .050 | 2.03                   | 1.95 | 1.87 | 1.78 | 1.73 | 1.69 | 1.63 | 1.60 | 1.58 | 1.51 |
|      | .010 | 2.70                   | 2.56 | 2.42 | 2.27 | 2.17 | 2.10 | 2.01 | 1.95 | 1.91 | 1.80 |
|      | .001 | 3.67                   | 3.44 | 3.20 | 2.95 | 2.79 | 2.68 | 2.53 | 2.44 | 2.38 | 2.21 |
| 60   | .100 | 1.71                   | 1.66 | 1.60 | 1.54 | 1.50 | 1.48 | 1.44 | 1.41 | 1.40 | 1.35 |
|      | .050 | 1.99                   | 1.92 | 1.84 | 1.75 | 1.69 | 1.65 | 1.59 | 1.56 | 1.53 | 1.47 |
|      | .010 | 2.63                   | 2.50 | 2.35 | 2.20 | 2.10 | 2.03 | 1.94 | 1.88 | 1.84 | 1.73 |
|      | .001 | 3.54                   | 3.32 | 3.08 | 2.83 | 2.67 | 2.55 | 2.41 | 2.32 | 2.25 | 2.08 |
| 100  | .100 | 1.66                   | 1.61 | 1.56 | 1.49 | 1.45 | 1.42 | 1.38 | 1.35 | 1.34 | 1.28 |
|      | .050 | 1.93                   | 1.85 | 1.77 | 1.68 | 1.62 | 1.57 | 1.52 | 1.48 | 1.45 | 1.38 |
|      | .010 | 2.50                   | 2.37 | 2.22 | 2.07 | 1.97 | 1.89 | 1.80 | 1.74 | 1.69 | 1.57 |
|      | .001 | 3.30                   | 3.07 | 2.84 | 2.59 | 2.43 | 2.32 | 2.17 | 2.08 | 2.01 | 1.83 |
| 200  | .100 | 1.63                   | 1.58 | 1.52 | 1.46 | 1.41 | 1.38 | 1.34 | 1.31 | 1.29 | 1.23 |
|      | .050 | 1.88                   | 1.80 | 1.72 | 1.62 | 1.56 | 1.52 | 1.46 | 1.41 | 1.39 | 1.30 |
|      | .010 | 2.41                   | 2.27 | 2.13 | 1.97 | 1.87 | 1.79 | 1.69 | 1.63 | 1.58 | 1.45 |
|      | .001 | 3.12                   | 2.90 | 2.67 | 2.42 | 2.26 | 2.15 | 2.00 | 1.90 | 1.83 | 1.64 |
| 1000 | .100 | 1.61                   | 1.55 | 1.49 | 1.43 | 1.38 | 1.35 | 1.30 | 1.27 | 1.25 | 1.18 |
|      | .050 | 1.84                   | 1.76 | 1.68 | 1.58 | 1.52 | 1.47 | 1.41 | 1.36 | 1.33 | 1.24 |
|      | .010 | 2.34                   | 2.20 | 2.06 | 1.90 | 1.79 | 1.72 | 1.61 | 1.54 | 1.50 | 1.35 |
|      | .001 | 2.99                   | 2.77 | 2.54 | 2.30 | 2.14 | 2.02 | 1.87 | 1.77 | 1.69 | 1.49 |

TABLE D.1 AREAS UNDER THE STANDARDIZED NORMAL DISTRIBUTION

Example

$$\Pr(0 \leq Z \leq 1.96) = 0.4750$$

$$\Pr(Z \geq 1.96) = 0.5 - 0.4750 = 0.025$$



| Z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| 0.1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| 0.2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| 0.3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| 0.4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| 0.5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| 0.6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| 0.7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| 0.8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| 0.9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4908 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4985 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |

Note: This table gives the area in the right-hand tail of the distribution (i.e.,  $Z \geq 0$ ). But since the normal distribution is symmetrical about  $Z = 0$ , the area in the left-hand tail is the same as the area in the corresponding right-hand tail. For example,  $\Pr(-1.96 \leq Z \leq 0) = 0.4750$ . Therefore,  $\Pr(-1.96 \leq Z \leq 1.96) = 2(0.4750) = 0.95$ .

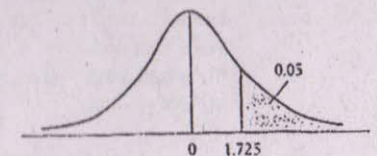
TABLE D.2 PERCENTAGE POINTS OF THE  $t$  DISTRIBUTION

Example

$$\Pr(t > 2.086) = 0.025$$

$$\Pr(t > 1.725) = 0.05 \quad \text{for } df = 20$$

$$\Pr(|t| > 1.725) = 0.10$$



| df  | Fr<br>0.50 | 0.25<br>0.20 | 0.10<br>0.10 | 0.05<br>0.05 | 0.01<br>0.02 | 0.005<br>0.010 | 0.001<br>0.002 |
|-----|------------|--------------|--------------|--------------|--------------|----------------|----------------|
| 1   | 1.000      | 3.078        | 6.314        | 12.706       | 31.821       | 63.657         | 318.31         |
| 2   | 0.816      | 1.886        | 2.920        | 4.303        | 6.965        | 9.925          | 22.327         |
| 3   | 0.765      | 1.638        | 2.353        | 3.182        | 4.541        | 5.841          | 10.214         |
| 4   | 0.741      | 1.533        | 2.132        | 2.776        | 3.747        | 4.604          | 7.173          |
| 5   | 0.727      | 1.476        | 2.015        | 2.571        | 3.365        | 4.032          | 5.893          |
| 6   | 0.718      | 1.440        | 1.943        | 2.447        | 3.143        | 3.707          | 5.208          |
| 7   | 0.711      | 1.415        | 1.895        | 2.365        | 2.996        | 3.499          | 4.785          |
| 8   | 0.706      | 1.397        | 1.860        | 2.306        | 2.896        | 3.355          | 4.501          |
| 9   | 0.703      | 1.383        | 1.833        | 2.262        | 2.821        | 3.250          | 4.297          |
| 10  | 0.700      | 1.372        | 1.812        | 2.228        | 2.764        | 3.169          | 4.144          |
| 11  | 0.697      | 1.363        | 1.796        | 2.201        | 2.718        | 3.106          | 4.025          |
| 12  | 0.695      | 1.356        | 1.782        | 2.179        | 2.681        | 3.055          | 3.930          |
| 13  | 0.694      | 1.350        | 1.771        | 2.160        | 2.650        | 3.012          | 3.852          |
| 14  | 0.692      | 1.345*       | 1.761        | 2.145        | 2.624        | 2.977          | 3.787          |
| 15  | 0.691      | 1.341        | 1.753        | 2.131        | 2.602        | 2.947          | 3.733          |
| 16  | 0.690      | 1.337        | 1.746        | 2.120        | 2.583        | 2.921          | 3.686          |
| 17  | 0.689      | 1.333        | 1.740        | 2.110        | 2.567        | 2.898          | 3.646          |
| 18  | 0.688      | 1.330        | 1.734        | 2.101        | 2.552        | 2.878          | 3.610          |
| 19  | 0.688      | 1.328        | 1.729        | 2.093        | 2.539        | 2.861          | 3.579          |
| 20  | 0.687      | 1.325        | 1.725        | 2.086        | 2.528        | 2.845          | 3.552          |
| 21  | 0.686      | 1.323        | 1.721        | 2.080        | 2.518        | 2.831          | 3.527          |
| 22  | 0.686      | 1.321        | 1.717        | 2.074        | 2.508        | 2.819          | 3.505          |
| 23  | 0.685      | 1.319        | 1.714        | 2.069        | 2.500        | 2.807          | 3.485          |
| 24  | 0.685      | 1.318        | 1.711        | 2.064        | 2.492        | 2.797          | 3.467          |
| 25  | 0.684      | 1.316        | 1.708        | 2.060        | 2.485        | 2.787          | 3.450          |
| 26  | 0.684      | 1.315        | 1.706        | 2.056        | 2.479        | 2.779          | 3.435          |
| 27  | 0.684      | 1.314        | 1.703        | 2.052        | 2.473        | 2.771          | 3.421          |
| 28  | 0.683      | 1.313        | 1.701        | 2.048        | 2.467        | 2.763          | 3.408          |
| 29  | 0.683      | 1.311        | 1.699        | 2.045        | 2.462        | 2.756          | 3.396          |
| 30  | 0.683      | 1.310        | 1.697        | 2.042        | 2.457        | 2.750          | 3.385          |
| 40  | 0.681      | 1.303        | 1.684        | 2.021        | 2.423        | 2.704          | 3.307          |
| 60  | 0.679      | 1.296        | 1.671        | 2.000        | 2.390        | 2.660          | 3.232          |
| 120 | 0.677      | 1.289        | 1.658        | 1.980        | 2.358        | 2.617          | 3.160          |
| ∞   | 0.674      | 1.282        | 1.645        | 1.960        | 2.326        | 2.576          | 3.090          |

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

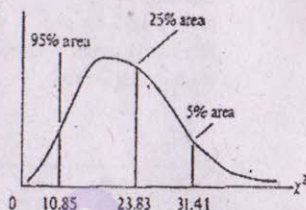
TABLE D.4 UPPER PERCENTAGE POINTS OF THE  $\chi^2$  DISTRIBUTION

Example

$$\Pr(\chi^2 > 10.85) = 0.95$$

$$\Pr(\chi^2 > 23.83) = 0.25 \quad \text{for } df = 20$$

$$\Pr(\chi^2 > 31.41) = 0.05$$



| Degrees of freedom | Pr | .995                     | .990                    | .975                    | .950                    | .900     |
|--------------------|----|--------------------------|-------------------------|-------------------------|-------------------------|----------|
| 1                  |    | $392704 \times 10^{-10}$ | $157088 \times 10^{-9}$ | $982069 \times 10^{-9}$ | $393214 \times 10^{-8}$ | .0157938 |
| 2                  |    | .0100251                 | .0201007                | .0506356                | .102587                 | .210729  |
| 3                  |    | .0717212                 | .114832                 | .215795                 | .351846                 | .584357  |
| 4                  |    | .206990                  | .297110                 | .484419                 | .710721                 | 1.063623 |
| 5                  |    | .411740                  | .554300                 | .831211                 | 1.145476                | 1.61031  |
| 6                  |    | .575272                  | .72085                  | 1.237347                | 1.63539                 | 2.20413  |
| 7                  |    | .689265                  | 1.239043                | 1.68987                 | 2.16735                 | 2.83311  |
| 8                  |    | 1.344419                 | 1.649482                | 2.17973                 | 2.73264                 | 3.48954  |
| 9                  |    | 1.734926                 | 2.087912                | 2.70039                 | 3.32511                 | 4.16816  |
| 10                 |    | 2.15585                  | 2.55821                 | 3.24697                 | 3.94030                 | 4.86513  |
| 11                 |    | 2.60321                  | 3.05347                 | 3.81575                 | 4.57481                 | 5.57779  |
| 12                 |    | 3.07382                  | 3.57056                 | 4.40379                 | 5.22603                 | 6.30380  |
| 13                 |    | 3.56503                  | 4.10691                 | 5.00874                 | 5.89186                 | 7.04150  |
| 14                 |    | 4.07468                  | 4.66043                 | 5.62672                 | 6.57063                 | 7.78953  |
| 15                 |    | 4.60094                  | 5.22935                 | 6.26214                 | 7.26094                 | 8.54675  |
| 16                 |    | 5.14224                  | 5.81221                 | 6.90766                 | 7.96164                 | 9.31223  |
| 17                 |    | 5.69724                  | 6.40776                 | 7.56418                 | 8.67176                 | 10.0852  |
| 18                 |    | 6.26481                  | 7.01491                 | 8.23075                 | 9.39046                 | 10.8649  |
| 19                 |    | 6.84398                  | 7.63273                 | 8.90655                 | 10.1170                 | 11.6503  |
| 20                 |    | 7.43386                  | 8.26040                 | 9.59033                 | 10.8508                 | 12.4426  |
| 21                 |    | 8.03368                  | 8.89720                 | 10.28293                | 11.5913                 | 13.2396  |
| 22                 |    | 8.64272                  | 9.54249                 | 10.9823                 | 12.3380                 | 14.0415  |
| 23                 |    | 9.26042                  | 10.19567                | 11.6885                 | 13.0905                 | 14.8479  |
| 24                 |    | 9.88623                  | 10.8564                 | 12.4011                 | 13.8484                 | 15.6587  |
| 25                 |    | 10.5197                  | 11.5240                 | 13.1197                 | 14.6114                 | 16.4734  |
| 26                 |    | 11.1603                  | 12.1981                 | 13.8439                 | 15.3791                 | 17.2919  |
| 27                 |    | 11.8076                  | 12.8786                 | 14.5733                 | 16.1513                 | 18.1138  |
| 28                 |    | 12.4613                  | 13.5648                 | 15.3079                 | 16.9279                 | 18.9392  |
| 29                 |    | 13.1211                  | 14.2585                 | 16.0471                 | 17.7083                 | 19.7677  |
| 30                 |    | 13.7867                  | 14.9535                 | 16.7908                 | 18.4926                 | 20.5992  |
| 40                 |    | 20.7065                  | 22.1643                 | 24.4331                 | 26.5093                 | 28.0505  |
| 50                 |    | 27.9907                  | 29.7067                 | 32.3574                 | 34.7642                 | 37.6885  |
| 60                 |    | 35.5346                  | 37.4848                 | 40.4817                 | 43.1879                 | 46.4589  |
| 70                 |    | 43.2752                  | 45.4418                 | 48.7576                 | 51.7393                 | 55.3299  |
| 80                 |    | 51.1720                  | 53.5400                 | 57.1532                 | 60.3915                 | 64.2778  |
| 90                 |    | 59.1953                  | 61.7541                 | 65.6466                 | 69.1260                 | 73.2912  |
| 100*               |    | 67.3276                  | 70.0648                 | 74.2219                 | 77.9295                 | 82.3581  |

\*For  $df$  greater than 100 the expression  $\sqrt{2k-1} - \sqrt{(2k-1)} = Z$  follows the standardized normal distribution, where  $k$  represents the degrees of freedom.

|     | .750     | .500    | .250    | .100    | .050    | .025    | .010    | .005    |
|-----|----------|---------|---------|---------|---------|---------|---------|---------|
| 1   | .1015308 | .454937 | 1.32330 | 2.70554 | 3.84146 | 5.02389 | 6.63490 | 7.87944 |
| 2   | .575364  | 1.38629 | 2.77259 | 4.60517 | 5.99147 | 7.37776 | 9.21034 | 10.5966 |
| 3   | 1.212534 | 2.36597 | 4.10835 | 6.25139 | 7.81473 | 9.34840 | 11.3449 | 12.8381 |
| 4   | 1.92255  | 3.35670 | 5.38527 | 7.77944 | 9.48773 | 11.1433 | 13.2767 | 14.8602 |
| 5   | 2.67480  | 4.35146 | 6.62568 | 9.23635 | 11.0705 | 12.8325 | 15.0863 | 16.7496 |
| 6   | 3.45460  | 5.34812 | 7.84080 | 10.6446 | 12.5916 | 14.4494 | 16.8119 | 18.5476 |
| 7   | 4.25485  | 6.34581 | 9.03715 | 12.0170 | 14.0671 | 16.0128 | 18.4753 | 20.2777 |
| 8   | 5.07064  | 7.34412 | 10.2168 | 13.3616 | 15.5073 | 17.5346 | 20.0902 | 21.9550 |
| 9   | 5.89883  | 8.34283 | 11.3887 | 14.6837 | 16.9190 | 19.0228 | 21.6660 | 23.5893 |
| 10  | 6.73720  | 9.34182 | 12.5489 | 15.9871 | 18.3070 | 20.4831 | 23.2093 | 25.1882 |
| 11  | 7.58412  | 10.3410 | 13.7007 | 17.2750 | 19.6751 | 21.9200 | 24.7250 | 26.7569 |
| 12  | 8.43842  | 11.3403 | 14.8454 | 18.5494 | 21.0261 | 23.3367 | 26.2170 | 28.2995 |
| 13  | 9.29906  | 12.3398 | 15.9809 | 19.8119 | 22.3621 | 24.7356 | 27.6883 | 29.8194 |
| 14  | 10.1653  | 13.3393 | 17.1170 | 21.0642 | 23.6848 | 26.1190 | 29.1413 | 31.3193 |
| 15  | 11.0365  | 14.3389 | 18.2451 | 22.3072 | 24.9958 | 27.4884 | 30.5779 | 32.8013 |
| 16  | 11.9122  | 15.3385 | 19.3688 | 23.5418 | 26.2962 | 28.8454 | 31.9999 | 34.2672 |
| 17  | 12.7919  | 16.3381 | 20.4887 | 24.7690 | 27.5871 | 30.1910 | 33.4087 | 35.7185 |
| 18  | 13.6753  | 17.3379 | 21.6049 | 25.9894 | 28.8693 | 31.5264 | 34.8053 | 37.1594 |
| 19  | 14.5620  | 18.3376 | 22.7178 | 27.2006 | 30.1435 | 32.8523 | 36.1908 | 38.5822 |
| 20  | 15.4518  | 19.3374 | 23.8277 | 28.4120 | 31.4104 | 34.1596 | 37.5682 | 39.9953 |
| 21  | 16.3444  | 20.3372 | 24.9348 | 29.6151 | 32.6705 | 35.4789 | 38.9321 | 41.4010 |
| 22  | 17.2396  | 21.3370 | 26.0393 | 30.8133 | 33.9244 | 36.7807 | 40.2894 | 42.7955 |
| 23  | 18.1373  | 22.3369 | 27.1413 | 32.0069 | 35.1725 | 38.0757 | 41.6384 | 44.1813 |
| 24  | 19.0372  | 23.3367 | 28.2412 | 33.1953 | 36.4151 | 39.3641 | 42.9798 | 45.5585 |
| 25  | 19.9393  | 24.3366 | 29.3389 | 34.3816 | 37.6525 | 40.6465 | 44.3141 | 46.9278 |
| 26  | 20.8434  | 25.3364 | 30.4345 | 35.5631 | 38.8852 | 41.9232 | 45.6417 | 48.2899 |
| 27  | 21.7494  | 26.3363 | 31.5294 | 36.7412 | 40.1133 | 43.1944 | 46.9530 | 49.6449 |
| 28  | 22.6572  | 27.3363 | 32.6205 | 37.9159 | 41.3372 | 44.4607 | 48.2782 | 50.9933 |
| 29  | 23.5666  | 28.3362 | 33.7109 | 39.0875 | 42.5569 | 45.7222 | 49.5979 | 52.3356 |
| 30  | 24.4776  | 29.3360 | 34.7998 | 40.2580 | 43.7729 | 46.9792 | 50.8922 | 53.6720 |
| 40  | 33.6603  | 39.3354 | 45.6180 | 51.8050 | 55.7585 | 59.3417 | 63.6907 | 66.7659 |
| 50  | 42.9421  | 49.3349 | 56.3306 | 63.1671 | 67.5048 | 71.4202 | 76.1539 | 79.4900 |
| 60  | 52.2938  | 59.3347 | 66.9014 | 74.3970 | 79.0819 | 83.2976 | 88.3794 | 91.9517 |
| 70  | 61.6983  | 69.3344 | 77.5766 | 85.5271 | 90.5312 | 95.0231 | 100.425 | 104.215 |
| 80  | 71.1445  | 79.3343 | 88.1303 | 96.5782 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90  | 80.6247  | 89.3342 | 98.6499 | 107.565 | 113.145 | 118.136 | 124.116 | 128.259 |
| 100 | 90.1332  | 99.3341 | 109.141 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

Source: Abridged from E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 6, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.

TABLE D.5A  
DURBIN-WATSON  $d$  STATISTIC: SIGNIFICANCE POINTS OF  $d_L$  AND  $d_U$  AT 0.05 LEVEL OF SIGNIFICANCE

|     | $K=1$ | $K=2$ | $K=3$ | $K=4$ | $K=5$ | $K=6$ | $K=7$ | $K=8$ | $K=9$ | $K=10$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| $n$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$  |
| 6   | 0.910 | 1.400 | —     | —     | —     | —     | —     | —     | —     | —      |
| 7   | 0.700 | 1.350 | 0.467 | 1.699 | —     | —     | —     | —     | —     | —      |
| 8   | 0.703 | 1.332 | 0.559 | 1.777 | 0.368 | 2.287 | —     | —     | —     | —      |
| 9   | 0.624 | 1.330 | 0.623 | 1.699 | 0.455 | 2.125 | 0.296 | 2.508 | —     | —      |
| 10  | 0.679 | 1.323 | 0.697 | 1.641 | 0.535 | 2.016 | 0.376 | 2.414 | 0.243 | 2.822  |
| 11  | 0.627 | 1.324 | 0.658 | 1.604 | 0.593 | 1.929 | 0.444 | 2.263 | 0.316 | 2.645  |
| 12  | 0.571 | 1.331 | 0.612 | 1.573 | 0.655 | 1.854 | 0.512 | 2.177 | 0.378 | 2.476  |
| 13  | 1.010 | 1.340 | 0.661 | 1.562 | 0.715 | 1.816 | 0.574 | 2.084 | 0.445 | 2.390  |
| 14  | 1.045 | 1.350 | 0.705 | 1.551 | 0.767 | 1.779 | 0.632 | 2.030 | 0.505 | 2.298  |
| 15  | 1.077 | 1.361 | 0.746 | 1.543 | 0.814 | 1.750 | 0.685 | 1.977 | 0.562 | 2.220  |
| 16  | 1.105 | 1.371 | 0.782 | 1.535 | 0.857 | 1.728 | 0.734 | 1.935 | 0.615 | 2.157  |
| 17  | 1.133 | 1.381 | 0.815 | 1.528 | 0.897 | 1.710 | 0.779 | 1.900 | 0.664 | 2.104  |
| 18  | 1.159 | 1.391 | 0.845 | 1.533 | 0.933 | 1.695 | 0.820 | 1.872 | 0.710 | 2.060  |
| 19  | 1.180 | 1.401 | 0.874 | 1.538 | 0.967 | 1.683 | 0.859 | 1.848 | 0.752 | 2.023  |
| 20  | 1.201 | 1.411 | 0.903 | 1.546 | 1.000 | 1.674 | 0.894 | 1.828 | 0.796 | 1.991  |
| 21  | 1.221 | 1.420 | 0.930 | 1.553 | 1.030 | 1.669 | 0.927 | 1.812 | 0.839 | 1.964  |
| 22  | 1.239 | 1.429 | 0.957 | 1.561 | 1.059 | 1.664 | 0.958 | 1.797 | 0.883 | 1.940  |
| 23  | 1.257 | 1.437 | 0.983 | 1.569 | 1.086 | 1.660 | 0.986 | 1.783 | 0.926 | 1.918  |
| 24  | 1.273 | 1.445 | 1.008 | 1.576 | 1.112 | 1.656 | 1.012 | 1.770 | 0.968 | 1.897  |
| 25  | 1.289 | 1.454 | 1.032 | 1.583 | 1.137 | 1.652 | 1.037 | 1.757 | 1.009 | 1.878  |
| 26  | 1.302 | 1.461 | 1.055 | 1.590 | 1.161 | 1.648 | 1.061 | 1.744 | 1.049 | 1.860  |
| 27  | 1.316 | 1.469 | 1.077 | 1.597 | 1.184 | 1.644 | 1.084 | 1.731 | 1.088 | 1.843  |
| 28  | 1.328 | 1.476 | 1.099 | 1.604 | 1.206 | 1.640 | 1.106 | 1.718 | 1.126 | 1.827  |
| 29  | 1.341 | 1.483 | 1.120 | 1.611 | 1.227 | 1.636 | 1.127 | 1.705 | 1.163 | 1.812  |
| 30  | 1.352 | 1.489 | 1.140 | 1.618 | 1.247 | 1.632 | 1.147 | 1.692 | 1.199 | 1.797  |
| 31  | 1.363 | 1.495 | 1.159 | 1.625 | 1.266 | 1.628 | 1.166 | 1.679 | 1.234 | 1.783  |
| 32  | 1.373 | 1.502 | 1.177 | 1.632 | 1.284 | 1.624 | 1.184 | 1.666 | 1.268 | 1.769  |
| 33  | 1.383 | 1.508 | 1.195 | 1.639 | 1.301 | 1.620 | 1.201 | 1.653 | 1.301 | 1.756  |
| 34  | 1.393 | 1.514 | 1.212 | 1.646 | 1.318 | 1.616 | 1.218 | 1.640 | 1.333 | 1.743  |
| 35  | 1.402 | 1.519 | 1.229 | 1.653 | 1.334 | 1.612 | 1.234 | 1.627 | 1.364 | 1.730  |
| 36  | 1.411 | 1.525 | 1.245 | 1.659 | 1.349 | 1.608 | 1.249 | 1.614 | 1.394 | 1.717  |
| 37  | 1.419 | 1.530 | 1.260 | 1.665 | 1.363 | 1.604 | 1.263 | 1.601 | 1.423 | 1.704  |
| 38  | 1.427 | 1.536 | 1.274 | 1.671 | 1.377 | 1.600 | 1.277 | 1.588 | 1.451 | 1.691  |
| 39  | 1.435 | 1.542 | 1.287 | 1.677 | 1.390 | 1.596 | 1.290 | 1.575 | 1.478 | 1.678  |
| 40  | 1.442 | 1.547 | 1.299 | 1.683 | 1.403 | 1.592 | 1.303 | 1.562 | 1.504 | 1.665  |
| 45  | 1.475 | 1.569 | 1.335 | 1.715 | 1.435 | 1.625 | 1.335 | 1.595 | 1.536 | 1.697  |
| 50  | 1.503 | 1.585 | 1.369 | 1.747 | 1.467 | 1.657 | 1.367 | 1.627 | 1.568 | 1.729  |
| 55  | 1.526 | 1.601 | 1.399 | 1.777 | 1.496 | 1.687 | 1.396 | 1.657 | 1.597 | 1.759  |
| 60  | 1.549 | 1.615 | 1.426 | 1.806 | 1.523 | 1.716 | 1.423 | 1.686 | 1.625 | 1.788  |
| 65  | 1.567 | 1.629 | 1.450 | 1.834 | 1.547 | 1.743 | 1.447 | 1.713 | 1.650 | 1.815  |
| 70  | 1.583 | 1.641 | 1.472 | 1.859 | 1.568 | 1.768 | 1.468 | 1.738 | 1.672 | 1.839  |
| 75  | 1.598 | 1.652 | 1.492 | 1.881 | 1.587 | 1.790 | 1.487 | 1.759 | 1.691 | 1.859  |
| 80  | 1.611 | 1.662 | 1.508 | 1.900 | 1.603 | 1.809 | 1.503 | 1.777 | 1.708 | 1.876  |
| 85  | 1.624 | 1.671 | 1.521 | 1.916 | 1.617 | 1.824 | 1.517 | 1.792 | 1.722 | 1.890  |
| 90  | 1.636 | 1.679 | 1.532 | 1.930 | 1.629 | 1.837 | 1.528 | 1.804 | 1.734 | 1.902  |
| 95  | 1.645 | 1.687 | 1.541 | 1.942 | 1.639 | 1.848 | 1.537 | 1.814 | 1.744 | 1.912  |
| 100 | 1.654 | 1.694 | 1.549 | 1.953 | 1.647 | 1.857 | 1.545 | 1.823 | 1.753 | 1.921  |
| 150 | 1.724 | 1.748 | 1.598 | 1.993 | 1.693 | 1.774 | 1.579 | 1.786 | 1.805 | 1.961  |
| 200 | 1.755 | 1.778 | 1.629 | 1.733 | 1.759 | 1.725 | 1.810 | 1.718 | 1.820 | 1.971  |

|     | $K=11$ | $K=12$ | $K=13$ | $K=14$ | $K=15$ | $K=16$ | $K=17$ | $K=18$ | $K=19$ | $K=20$ |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $n$ | $d_L$  | $d_U$  | $d_L$  | $d_U$  | $d_L$  | $d_U$  | $d_L$  | $d_U$  | $d_L$  | $d_U$  |
| 16  | 0.996  | 3.503  | —      | —      | —      | —      | —      | —      | —      | —      |
| 17  | 0.136  | 3.376  | 0.087  | 3.657  | —      | —      | —      | —      | —      | —      |
| 18  | 0.177  | 3.258  | 0.123  | 3.441  | 0.018  | 3.603  | —      | —      | —      | —      |
| 19  | 0.230  | 3.159  | 0.160  | 3.333  | 0.111  | 3.495  | 0.070  | 3.642  | —      | —      |
| 20  | 0.283  | 3.061  | 0.200  | 3.234  | 0.145  | 3.395  | 0.100  | 3.542  | 0.063  | 3.678  |
| 21  | 0.307  | 2.976  | 0.240  | 3.141  | 0.182  | 3.300  | 0.132  | 3.448  | 0.091  | 3.583  |
| 22  | 0.340  | 2.897  | 0.281  | 3.057  | 0.220  | 3.211  | 0.166  | 3.358  | 0.120  | 3.493  |
| 23  | 0.351  | 2.826  | 0.321  | 2.979  | 0.257  | 3.128  | 0.202  | 3.272  | 0.155  | 3.403  |
| 24  | 0.431  | 2.761  | 0.362  | 2.906  | 0.297  | 3.053  | 0.239  | 3.193  | 0.186  | 3.327  |
| 25  | 0.470  | 2.702  | 0.400  | 2.844  | 0.335  | 2.983  | 0.275  | 3.118  | 0.221  | 3.251  |
| 26  | 0.508  | 2.649  | 0.438  | 2.784  | 0.373  | 2.919  | 0.312  | 3.051  | 0.259  | 3.179  |
| 27  | 0.544  | 2.600  | 0.475  | 2.730  | 0.409  | 2.856  | 0.348  | 2.987  | 0.291  | 3.112  |
| 28  | 0.573  | 2.555  | 0.510  | 2.680  | 0.445  | 2.805  | 0.383  | 2.928  | 0.325  | 3.050  |
| 29  | 0.612  | 2.515  | 0.544  | 2.634  | 0.478  | 2.755  | 0.418  | 2.874  | 0.359  | 2.992  |
| 30  | 0.643  | 2.477  | 0.577  | 2.592  | 0.512  | 2.708  | 0.451  | 2.823  | 0.392  | 2.937  |
| 31  | 0.674  | 2.443  | 0.608  | 2.553  | 0.545  | 2.665  | 0.484  | 2.776  | 0.425  | 2.887  |
| 32  | 0.703  | 2.411  | 0.638  | 2.517  | 0.576  | 2.625  | 0.515  | 2.733  | 0.457  | 2.840  |
| 33  | 0.731  | 2.382  | 0.668  | 2.484  | 0.606  | 2.588  | 0.546  | 2.692  | 0.489  | 2.794  |
| 34  | 0.758  | 2.355  | 0.695  | 2.454  | 0.634  | 2.554  | 0.575  | 2.654  | 0.518  | 2.754  |
| 35  | 0.783  | 2.330  | 0.722  | 2.425  | 0.662  | 2.521  | 0.604  | 2.619  | 0.547  | 2.718  |
| 36  | 0.808  | 2.306  | 0.748  | 2.398  | 0.688  | 2.490  | 0.631  | 2.586  | 0.575  | 2.683  |
| 37  | 0.831  | 2.285  | 0.772  | 2.374  | 0.714  | 2.461  | 0.657  | 2.555  | 0.602  | 2.648  |
| 38  | 0.854  | 2.265  | 0.796  | 2.351  | 0.739  | 2.436  | 0.683  | 2.525  | 0.628  | 2.614  |
| 39  | 0.875  | 2.246  | 0.819  | 2.329  | 0.763  | 2.413  | 0.707  | 2.499  | 0.653  | 2.585  |
| 40  | 0.896  | 2.228  | 0.840  | 2.308  | 0.785  | 2.391  | 0.731  | 2.473  | 0.679  | 2.557  |
| 45  | 0.958  | 2.195  | 0.908  | 2.225  | 0.887  | 2.290  | 0.838  | 2.387  | 0.788  | 2.439  |
| 50  | 1.004  | 2.193  | 1.019  | 2.163  | 0.973  | 2.225  | 0.927  | 2.287  | 0.882  | 2.350  |
| 55  | 1.123  | 2.062  | 1.087  | 2.116  | 1.045  | 2.170  | 1.003  | 2.225  | 0.951  | 2.291  |
| 60  | 1.184  | 2.031  | 1.145  | 2.079  | 1.106  | 2.127  | 1.064  | 2.177  | 1.023  | 2.247  |
| 65  | 1.231  | 2.005  | 1.195  | 2.043  | 1.160  | 2.093  | 1.124  | 2.158  | 1.086  | 2.183  |
| 70  | 1.272  | 1.995  | 1.238  | 2.026  | 1.206  | 2.060  | 1.172  | 2.105  | 1.139  | 2.148  |
| 75  | 1.308  | 1.970  | 1.277  | 2.006  | 1.247  | 2.043  | 1.215  | 2.080  | 1.184  | 2.118  |
| 80  | 1.340  | 1.957  | 1.311  | 1.991  | 1.283  | 2.024  | 1.253  | 2.059  | 1.224  | 2.093  |
| 85  | 1.369  | 1.945  | 1.342  | 1.977  | 1.315  | 2.009  | 1.287  | 2.043  | 1.258  | 2.075  |
| 90  | 1.395  | 1.937  | 1.369  | 1.960  | 1.344  | 1.995  | 1.318  | 2.025  | 1.286  | 2.056  |
| 95  | 1.418  | 1.929  | 1.394  | 1.955  | 1.370  | 1.984  | 1.343  | 2.012  | 1.321  | 2.040  |
| 100 | 1.439  | 1.923  | 1.416  | 1.948  | 1.393  | 1.974  | 1.371  | 2.000  | 1.347  | 2.028  |
| 150 | 1.578  | 1.892  | 1.564  | 1.908  | 1.590  | 1.924  | 1.525  | 1.940  | 1.519  | 1.915  |
| 200 | 1.654  | 1.885  | 1.643  | 1.895  | 1.632  | 1.908  | 1.621  | 1.919  | 1.610  | 1.931  |

Source: This table is an extension of the original Durbin-Watson table and is reproduced from R. E. Savin and K. J. White, "The Durbin-Watson Test for Serial Correlation with Extreme Small Samples or Many Regressors," *Econometrica*, vol. 45, November 1977, pp. 1289-90 and as corrected by R. W. Farebrother, *Econometrica*, vol. 48, September 1980, p. 1554. Reprinted by permission of the Econometric Society.

## EXAMPLE

If  $n = 40$  and  $K = 4$ ,  $d_L = 1.285$  and  $d_U = 1.721$ . If a computed  $d$ -value is less than 1.285, there is evidence of positive first-order serial correlation; if it is greater than

1.721, there is no evidence of positive first-order serial correlation; but if  $d$  lies between the lower and the upper limit, there is inconclusive evidence regarding the presence or absence of positive first-order serial correlation.

TABLE D.5B

DURBIN-WATSON  $d$  STATISTIC: SIGNIFICANCE POINTS OF  $d_L$  AND  $d_U$  AT 0.01 LEVEL OF SIGNIFICANCE

|     | $K=1$ |       | $K=2$ |       | $K=3$ |       | $K=4$ |       | $K=5$ |       | $K=6$ |       | $K=7$ |       | $K=8$ |       | $K=9$ |       | $K=10$ |       |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|-------|
| $n$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$ | $d_0$ | $d_1$  | $d_0$ |
| 6   | 0.390 | 1.142 | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —      | —     |
| 7   | 0.435 | 1.036 | 0.294 | 1.676 | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —      | —     |
| 8   | 0.497 | 1.003 | 0.345 | 1.469 | 0.229 | 2.102 | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —      | —     |
| 9   | 0.554 | 0.958 | 0.408 | 1.389 | 0.279 | 1.875 | 0.183 | 2.433 | —     | —     | —     | —     | —     | —     | —     | —     | —     | —     | —      | —     |
| 10  | 0.604 | 0.901 | 0.466 | 1.303 | 0.340 | 1.733 | 0.230 | 2.193 | 0.150 | 2.600 | —     | —     | —     | —     | —     | —     | —     | —     | —      | —     |
| 11  | 0.650 | 0.840 | 0.519 | 1.297 | 0.386 | 1.640 | 0.286 | 2.030 | 0.193 | 2.453 | 0.124 | 2.892 | —     | —     | —     | —     | —     | —     | —      | —     |
| 12  | 0.697 | 0.823 | 0.569 | 1.274 | 0.449 | 1.575 | 0.339 | 1.913 | 0.244 | 2.290 | 0.164 | 2.665 | 0.105 | 3.053 | —     | —     | —     | —     | —      | —     |
| 13  | 0.736 | 0.808 | 0.616 | 1.261 | 0.499 | 1.526 | 0.391 | 1.826 | 0.294 | 2.150 | 0.211 | 2.490 | 0.140 | 2.638 | 0.090 | 3.182 | —     | —     | —      | —     |
| 14  | 0.775 | 0.864 | 0.660 | 1.254 | 0.547 | 1.490 | 0.441 | 1.757 | 0.343 | 2.049 | 0.257 | 2.354 | 0.163 | 2.667 | 0.122 | 2.961 | 0.075 | 3.287 | —      | —     |
| 15  | 0.811 | 0.870 | 0.700 | 1.252 | 0.591 | 1.464 | 0.488 | 1.704 | 0.391 | 1.967 | 0.303 | 2.244 | 0.226 | 2.530 | 0.161 | 2.817 | 0.107 | 3.171 | 0.068  | 3.374 |
| 16  | 0.844 | 0.866 | 0.737 | 1.252 | 0.633 | 1.446 | 0.532 | 1.663 | 0.437 | 1.900 | 0.349 | 2.153 | 0.269 | 2.416 | 0.200 | 2.681 | 0.142 | 2.944 | 0.094  | 3.371 |
| 17  | 0.874 | 1.102 | 0.772 | 1.255 | 0.672 | 1.432 | 0.574 | 1.630 | 0.480 | 1.847 | 0.393 | 2.076 | 0.313 | 2.319 | 0.241 | 2.568 | 0.179 | 2.811 | 0.127  | 3.353 |
| 18  | 0.902 | 1.116 | 0.805 | 1.259 | 0.708 | 1.422 | 0.613 | 1.604 | 0.522 | 1.803 | 0.435 | 2.015 | 0.355 | 2.238 | 0.282 | 2.457 | 0.215 | 2.697 | 0.169  | 3.335 |
| 19  | 0.928 | 1.132 | 0.835 | 1.265 | 0.742 | 1.415 | 0.650 | 1.584 | 0.561 | 1.757 | 0.476 | 1.963 | 0.396 | 2.169 | 0.322 | 2.381 | 0.255 | 2.597 | 0.199  | 3.318 |
| 20  | 0.952 | 1.147 | 0.863 | 1.271 | 0.773 | 1.411 | 0.685 | 1.567 | 0.598 | 1.737 | 0.515 | 1.918 | 0.426 | 2.110 | 0.362 | 2.306 | 0.294 | 2.577 | 0.232  | 3.301 |
| 21  | 0.975 | 1.161 | 0.890 | 1.277 | 0.803 | 1.408 | 0.718 | 1.554 | 0.633 | 1.712 | 0.552 | 1.881 | 0.474 | 2.058 | 0.400 | 2.244 | 0.331 | 2.547 | 0.265  | 3.284 |
| 22  | 0.997 | 1.174 | 0.914 | 1.284 | 0.831 | 1.407 | 0.748 | 1.543 | 0.657 | 1.691 | 0.587 | 1.849 | 0.510 | 2.015 | 0.437 | 2.186 | 0.366 | 2.527 | 0.294  | 3.268 |
| 23  | 1.018 | 1.187 | 0.938 | 1.291 | 0.858 | 1.407 | 0.777 | 1.534 | 0.689 | 1.673 | 0.620 | 1.821 | 0.545 | 1.977 | 0.473 | 2.140 | 0.404 | 2.502 | 0.324  | 3.251 |
| 24  | 1.037 | 1.199 | 0.960 | 1.299 | 0.882 | 1.407 | 0.805 | 1.528 | 0.728 | 1.655 | 0.652 | 1.797 | 0.578 | 1.944 | 0.507 | 2.097 | 0.439 | 2.478 | 0.354  | 3.234 |
| 25  | 1.055 | 1.211 | 0.981 | 1.305 | 0.906 | 1.409 | 0.831 | 1.523 | 0.758 | 1.645 | 0.682 | 1.776 | 0.610 | 1.915 | 0.541 | 2.069 | 0.475 | 2.453 | 0.409  | 3.217 |
| 26  | 1.072 | 1.222 | 1.001 | 1.312 | 0.928 | 1.411 | 0.855 | 1.518 | 0.783 | 1.635 | 0.711 | 1.759 | 0.640 | 1.889 | 0.572 | 2.025 | 0.505 | 2.428 | 0.449  | 3.200 |
| 27  | 1.088 | 1.233 | 1.019 | 1.319 | 0.949 | 1.415 | 0.875 | 1.515 | 0.808 | 1.629 | 0.738 | 1.743 | 0.669 | 1.867 | 0.602 | 1.997 | 0.536 | 2.403 | 0.479  | 3.183 |
| 28  | 1.104 | 1.244 | 1.037 | 1.325 | 0.969 | 1.415 | 0.900 | 1.513 | 0.832 | 1.618 | 0.764 | 1.729 | 0.696 | 1.847 | 0.630 | 1.970 | 0.566 | 2.378 | 0.509  | 3.166 |
| 29  | 1.119 | 1.254 | 1.054 | 1.332 | 0.988 | 1.418 | 0.921 | 1.512 | 0.855 | 1.611 | 0.789 | 1.718 | 0.723 | 1.830 | 0.659 | 1.947 | 0.595 | 2.353 | 0.539  | 3.149 |
| 30  | 1.133 | 1.263 | 1.070 | 1.339 | 1.006 | 1.421 | 0.941 | 1.511 | 0.877 | 1.609 | 0.812 | 1.707 | 0.748 | 1.814 | 0.684 | 1.925 | 0.622 | 2.328 | 0.569  | 3.132 |
| 31  | 1.147 | 1.273 | 1.085 | 1.345 | 1.023 | 1.425 | 0.960 | 1.510 | 0.897 | 1.601 | 0.834 | 1.696 | 0.772 | 1.800 | 0.710 | 1.905 | 0.649 | 2.303 | 0.599  | 3.115 |
| 32  | 1.160 | 1.282 | 1.100 | 1.352 | 1.040 | 1.428 | 0.979 | 1.510 | 0.917 | 1.597 | 0.856 | 1.690 | 0.794 | 1.786 | 0.734 | 1.889 | 0.674 | 2.278 | 0.629  | 3.098 |
| 33  | 1.172 | 1.291 | 1.114 | 1.358 | 1.055 | 1.432 | 0.996 | 1.510 | 0.936 | 1.594 | 0.876 | 1.685 | 0.816 | 1.776 | 0.757 | 1.874 | 0.693 | 2.253 | 0.659  | 3.081 |
| 34  | 1.184 | 1.299 | 1.128 | 1.364 | 1.070 | 1.435 | 1.012 | 1.511 | 0.954 | 1.591 | 0.896 | 1.677 | 0.837 | 1.766 | 0.779 | 1.863 | 0.722 | 2.228 | 0.689  | 3.064 |
| 35  | 1.195 | 1.307 | 1.140 | 1.370 | 1.085 | 1.438 | 1.028 | 1.512 | 0.971 | 1.589 | 0.914 | 1.671 | 0.857 | 1.757 | 0.800 | 1.854 | 0.744 | 2.203 | 0.719  | 3.047 |
| 36  | 1.206 | 1.315 | 1.153 | 1.376 | 1.099 | 1.442 | 1.043 | 1.513 | 0.989 | 1.588 | 0.932 | 1.666 | 0.877 | 1.749 | 0.821 | 1.845 | 0.766 | 2.178 | 0.749  | 3.030 |
| 37  | 1.217 | 1.323 | 1.165 | 1.382 | 1.112 | 1.446 | 1.058 | 1.514 | 1.004 | 1.586 | 0.950 | 1.662 | 0.895 | 1.742 | 0.841 | 1.835 | 0.787 | 2.153 | 0.779  | 3.013 |
| 38  | 1.227 | 1.330 | 1.176 | 1.388 | 1.124 | 1.449 | 1.072 | 1.515 | 1.019 | 1.585 | 0.966 | 1.658 | 0.913 | 1.735 | 0.860 | 1.826 | 0.807 | 2.128 | 0.809  | 3.000 |
| 39  | 1.237 | 1.337 | 1.187 | 1.393 | 1.137 | 1.453 | 1.089 | 1.517 | 1.034 | 1.584 | 0.982 | 1.655 | 0.930 | 1.729 | 0.878 | 1.817 | 0.828 | 2.103 | 0.839  | 2.987 |
| 40  | 1.246 | 1.344 | 1.198 | 1.398 | 1.148 | 1.457 | 1.099 | 1.518 | 1.048 | 1.584 | 0.997 | 1.652 | 0.946 | 1.724 | 0.895 | 1.808 | 0.844 | 2.078 | 0.869  | 2.974 |
| 41  | 1.256 | 1.351 | 1.209 | 1.403 | 1.159 | 1.461 | 1.111 | 1.519 | 1.061 | 1.584 | 1.005 | 1.643 | 0.919 | 1.704 | 0.914 | 1.798 | 0.862 | 2.053 | 0.899  | 2.961 |
| 42  | 1.265 | 1.358 | 1.219 | 1.408 | 1.170 | 1.465 | 1.122 | 1.520 | 1.072 | 1.584 | 1.016 | 1.639 | 0.930 | 1.699 | 0.925 | 1.789 | 0.873 | 2.028 | 0.929  | 2.948 |
| 43  | 1.274 | 1.365 | 1.229 | 1.413 | 1.181 | 1.469 | 1.133 | 1.521 | 1.083 | 1.584 | 1.027 | 1.635 | 0.941 | 1.694 | 0.936 | 1.784 | 0.884 | 2.003 | 0.959  | 2.935 |
| 44  | 1.283 | 1.372 | 1.239 | 1.418 | 1.191 | 1.473 | 1.144 | 1.522 | 1.094 | 1.584 | 1.038 | 1.631 | 0.952 | 1.689 | 0.947 | 1.779 | 0.895 | 1.978 | 0.989  | 2.922 |
| 45  | 1.292 | 1.379 | 1.249 | 1.423 | 1.201 | 1.477 | 1.155 | 1.523 | 1.105 | 1.584 | 1.049 | 1.627 | 0.963 | 1.684 | 0.958 | 1.774 | 0.906 | 1.953 | 1.019  | 2.909 |
| 46  | 1.301 | 1.386 | 1.259 | 1.428 | 1.211 | 1.481 | 1.166 | 1.524 | 1.116 | 1.584 | 1.060 | 1.623 | 0.974 | 1.679 | 0.969 | 1.769 | 0.917 | 1.928 | 1.049  | 2.896 |
| 47  | 1.310 | 1.393 | 1.269 | 1.433 | 1.221 | 1.485 | 1.177 | 1.525 | 1.127 | 1.584 | 1.071 | 1.619 | 0.985 | 1.674 | 0.980 | 1.764 | 0.928 | 1.903 | 1.079  | 2.883 |
| 48  | 1.319 | 1.400 | 1.279 | 1.438 | 1.231 | 1.489 | 1.188 | 1.526 | 1.138 | 1.584 | 1.082 | 1.615 | 0.996 | 1.669 | 0.991 | 1.759 | 0.939 | 1.878 | 1.109  | 2.870 |
| 49  | 1.328 | 1.407 | 1.289 | 1.443 | 1.241 | 1.493 | 1.199 | 1.527 | 1.149 | 1.584 | 1.093 | 1.611 | 1.007 | 1.664 | 1.002 | 1.754 | 0.950 | 1.853 | 1.139  | 2.857 |
| 50  | 1.337 | 1.414 | 1.299 | 1.448 | 1.251 | 1.497 | 1.210 | 1.528 | 1.160 | 1.584 | 1.104 | 1.607 | 1.018 | 1.659 | 1.013 | 1.749 | 0.961 | 1.828 | 1.169  | 2.844 |
| 51  | 1.346 | 1.421 | 1.309 | 1.453 | 1.261 | 1.501 | 1.221 | 1.529 | 1.171 | 1.584 | 1.115 | 1.603 | 1.029 | 1.654 | 1.024 | 1.744 | 0.972 | 1.803 | 1.199  | 2.831 |
| 52  | 1.355 | 1.428 | 1.319 | 1.458 | 1.271 | 1.505 | 1.232 | 1.530 | 1.182 | 1.584 | 1.126 | 1.599 | 1.040 | 1.649 | 1.035 | 1.739 | 0.983 | 1.778 | 1.229  | 2.818 |
| 53  | 1.364 | 1.435 | 1.329 | 1.463 | 1.281 | 1.509 | 1.243 | 1.531 | 1.193 | 1.584 | 1.137 | 1.595 | 1.051 | 1.644 | 1.046 | 1.734 | 0.994 | 1.753 | 1.259  | 2.805 |
| 54  | 1.373 | 1.442 | 1.339 | 1.468 | 1.291 | 1.513 | 1.254 | 1.532 | 1.204 | 1.584 | 1.148 | 1.591 | 1.062 | 1.639 | 1.057 | 1.729 | 1.005 | 1.728 | 1.289  | 2.792 |
| 55  | 1.382 | 1.449 | 1.349 | 1.473 | 1.301 | 1.517 | 1.265 | 1.533 | 1.215 | 1.584 | 1.159 | 1.587 | 1.073 | 1.634 | 1.068 | 1.724 | 1.016 | 1.703 | 1.319  | 2.779 |
| 56  | 1.391 | 1.456 | 1.359 | 1.478 | 1.311 | 1.521 | 1.276 | 1.534 | 1.226 | 1.584 | 1.170 | 1.583 | 1.084 | 1.629 | 1.079 | 1.719 | 1.027 | 1.678 | 1.349  | 2.766 |
| 57  | 1.400 | 1.463 | 1.369 | 1.483 | 1.321 | 1.525 | 1.287 | 1.535 | 1.237 | 1.584 | 1.181 | 1.579 | 1.095 | 1.624 | 1.090 | 1.714 | 1.038 | 1.653 | 1.379  | 2.753 |
| 58  | 1.409 | 1.470 | 1.379 | 1.488 | 1.331 | 1.529 | 1.298 | 1.536 | 1.248 | 1.584 | 1.192 | 1.575 | 1.106 | 1.619 | 1.101 | 1.709 | 1.049 | 1.628 | 1.409  | 2.740 |
| 59  | 1.418 | 1.477 | 1.389 | 1.493 | 1.341 | 1.533 | 1.309 | 1.537 | 1.259 | 1.584 | 1.203 | 1.571 | 1.117 | 1.614 | 1.112 | 1.704 | 1.060 | 1.603 | 1.439  | 2.727 |
| 60  | 1.427 | 1.484 | 1.399 | 1.498 | 1.351 | 1.537 | 1.320 | 1.538 | 1.270 | 1.584 | 1.214 | 1.567 | 1.128 | 1.609 | 1.123 | 1.699 | 1.071 | 1.578 | 1.469  | 2.714 |
| 61  | 1.436 | 1     |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |        |       |