6|12|18 (E)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1026 **I**

Unique Paper Code : 32355301

Name of the Paper : Differential Equations

Name of the Course : **Generic Elective for Hons. : Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.

1. (a) Using exactness, solve the following differential equation

$$\cos(x+y)dx + \left(3y^2 + 2y + \cos(x+y)\right)dy = 0 . \qquad (6)$$

(b) Solve the initial value problem

$$y' \tan x = 2y - 8, \quad y\left(\frac{\pi}{2}\right) = 0 . \qquad (6)$$

(c) Find the orthogonal trajectories of the given family of curves

$$y^2 = 2x^2 + c.$$ (6)

2. (a) Solve the differential equation :

$$y' = Ay - By^2.$$ (6)

(b) Solve the initial value problem :

$$y'' + 0.4y + 9.04y = 0, \quad y(0) = 0, \quad y'(0) = 3.$$ (6)

(c) Show that the functions e^{-2x}, e^{-x}, e^{x} and e^{2x} form a basis of a differential equation on any interval. (6)

3. (a) Find a homogeneous linear ordinary differential equation for which two functions x^{-3} and $x^{-3}\ln x$ are solutions. Show also their linear independence by considering their Wronskian. (6)

(b) Use the method of Variation of Parameters to find a general solution of the following non-homogeneous ordinary differential equation :

$$y'' - 2y' + y = e^{x}\sin x.$$ (6)

(c) Solve the following differential equation :

$$\left(xD^2 + 4D\right)y = 0.$$

Also find the solution satisfying $y(1) = 12, \quad y'(1) = -6$. (6)

4. (a) Use the method of undetermined coefficients to find the particular solution of the differential equation :

$$y'' + 4y' + 5y = 25x^2 + 13\sin 2x.$$

Also find its general solution. (6.5)

(b) Find the solution of the linear system

$$\frac{dx}{dt} = 5x + 3y,$$
$$\frac{dy}{dt} = 4x + y,$$

that satisfies the initial conditions $x(0) = 0, y(0) = 8$. (6.5)

(c) Reduce the equation to canonical form and obtain the general solution :

$$u_x - yu_y = u + 1.$$ (6.5)

5. (a) Find a power series solution, in powers of x of the differential equation :

$$y'' - y' = 0.$$ (6.5)

(b) Find the solution of quasi-linear partial differential equation :

$$x(y^2+u)u_x - y(x^2+u)u_y = (x^2-y^2)u$$

with Cauchy data $u=1$ on $x+y=0$. (6.5)

(c) Find the general solution of the linear partial differential equation :

$$x^2u_x + y^2u_y + z(x+y)u_z = 0.$$ (6.5)

6. (a) Find the solution of the following partial differential equation by the method of separation of variables :

$$u_x - u_y = u, \quad u(x,0) = 4e^{-3x}.$$ (6.5)

(b) Reduce the equation

$$yu_{xx} + 3yu_{xy} + 3u_x = 0, y \neq 0$$

to canonical form and hence find its general solution. (6.5)

(c) Form partial differential equations by

(i) eliminating the arbitrary constants a and b from the relation

$$(x-a)^2 + (y-b)^2 + z^2 = r^2.$$

(ii) eliminating the arbitrary function f from the relation

$$z = xy + f(x^2+y^2).$$ (6.5)