11.12.18 (E)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 897 **I**

Unique Paper Code : 32355101

Name of the Paper : Calculus

Name of the Course : **Mathematics : G.E. for Honours**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Do any **five** questions from each of the **three** sections.
3. Each question is for **five** marks.

SECTION 1

1. Given $f(x)=2x-2,\ x_o=-2,\ \varepsilon=0.02$. Find $L=\lim\limits_{x\to x_o} f(x)$. Then find a number $\delta>0$ such that for all $x, 0<|x-x_o|<\delta \Rightarrow |f(x)-L|<\varepsilon$.

2. Find a linearization of $f(x)=\sqrt{x^2+9}$ at $x=-4$.

P.T.O.

3. The radius of a circle is increased from 2.00 to 2.02 m. Estimate the resulting change in area. Also express the estimate as a percentage of the circle's original area.

4. Evaluate the limit $\lim_{x\to 0}\dfrac{e^{1/x}-1}{e^{1/x}+1}$.

5. Use L'Hôpital's rule to find $\lim_{x\to 0^+}\dfrac{\ln(e^x-1)}{\ln x}$

6. Sketch the graph of a function $f(x)=x^3-3x+1$.

7. Find the volume of the solid generated by revolving the region between the y-axis and curve $x=2/y,\ 1\le y\le 4$, about the y-axis.

SECTION 2

8. Use the shell method to find the volume of the solid generated when the region R in the first quadrant enclosed between $y=x$ and $y=x^2$ is revolved about the y-axis.

9. Sketch the graph of $r=1-2\cos\theta$ and identify its symmetries.

10. Find the area of the surface generated by revolving the curve $y=\sqrt{x}$, $0\le x\le 1$, about the x-axis.

11. Suppose a person on a hang glider is spiraling upward due to rapidly rising air on a path having acceleration vector $a(t)=-3\cos t\,\boldsymbol{i}-3\sin t\,\boldsymbol{j}+2\boldsymbol{k}$. It is also known that initially (at time $t=0$), the glider departed from the point (3,0,0) with velocity $v(0)=3\boldsymbol{j}$. Find the glider's position as a function of t.

12. Find the unit tangent vector of the curve

$$r(t)=t^2\,\boldsymbol{i}+2\cos t\,\boldsymbol{j}+2\sin t\,\boldsymbol{k}.$$

13. Determine whether $\int_{-\infty}^{-1}\frac{1}{x}dx$ converges?

14. Find the arc length parameterization of the helix

$$r(t)=\cos 4t\,\boldsymbol{i}+\sin 4t\,\boldsymbol{j}+3t\,\boldsymbol{k},\quad 0\le t\le 2\pi.$$

SECTION 3

15. Show that the ellipse $x=a\cos t,\ y=b\sin t,\ a>b>0$, has its largest curvature on its major axis and its smallest curvature on its minor axis.

16. Find the binormal vector $\vec{B}$ and the torsion function τ for the space curve

$$\vec{r}(t)=(\cos t+t\sin t)\hat{i}+(\sin t-t\cos t)\hat{j}+3\hat{k}$$

P.T.O.

17. Show that the function

$$f(x, y) = \frac{xy}{|xy|}$$

has no limit as (x, y) approaches $(0, 0)$.

18. If $w = \ln(x^2 + y^2 + z^2)$, $x = ue^v \sin u$, $y = ue^v \cos u$ and $z = ue^v$ find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ using chain rule at the point (u, v) =

$(-2, 0)$.

19. Find the directions in which the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$ increase and decrease most rapidly at the point $P_0(1, 1, 1)$. Then find the derivatives of the function in those directions.

20. Find parametric equations for the line tangent to the curve of intersection of the surfaces

$$x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0 \text{ and } x^3 + y^2 + z^2 = 11$$

at the point $(1, 1, 3)$.

21. Find the absolute maxima and minima of the function

$$T(x, y) = x^2 + xy + y^2 - 6x + 2$$

on the rectangular plate $0 \le x \le 5;\ -3 \le y \le 0$.