

✓ 10.12.18(M)
This question paper contains 4 printed pages.

Your Roll No.

S. No. of Paper : 753 I
Unique Paper Code : 32357502
Name of the Paper : Mathematical Modelling and Graph Theory
Name of the Course : B.Sc. (H) Mathematics : DSE-2
Semester : V
Duration : 3 hours
Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions, selecting three parts from each question.

1. (a) Solve the initial value problem using the Laplace transform: 6

$$x'' - 6x' + 8x = 2; x(0) = 0, x'(0) = 0.$$

- (b) (i) Find the inverse Laplace transform of: 2

$$F(s) = \frac{5s - 6}{s^2 - 3s}.$$

- (ii) Show that: 2

$$L = \{t \sinh kt\} = \frac{2ks}{(s^2 - k^2)^2}$$

- (iii) Find the inverse Laplace transform of: 2

$$F(s) = \frac{1}{s^4 - 16}.$$

- (c) Find two linearly independent Frobenius series solutions of: 6

P. T. O.

$$2x^2y'' + xy' - (3 - 2x^2)y = 0.$$

- (d) Use power series to solve the initial value problem: 6

$$(x^2 - 6x + 10)y'' - 4(x - 3)y' + 6y = 0; y(3) = 2, y'(3) = 0.$$

2. (a) Explain Middle-Square Method and use it to generate 10 random numbers taking $x_0 = 1009$. Comment about the results. Was there cycling? Illustrate. 6

- (b) Using Monte Carlo Simulation, write an algorithm to calculate the area trapped between the two curves $y = x^2$ and $y = 6 - x$ and the x - and y -axes. 6

- (c) Using the simplex method: 6

$$\begin{aligned} &\text{Maximize } 3x + y \\ &\text{subject to } 2x + y \leq 6, \\ &\quad x + 3y \leq 9, \\ &\quad x, y \geq 0. \end{aligned}$$

- (d) Using algebraic analysis: 6

$$\begin{aligned} &\text{Maximize } 10x + 35y \\ &\text{subject to } 4x + 3y \leq 24, \\ &\quad 4x + y \leq 20, \\ &\quad x, y \geq 0. \end{aligned}$$

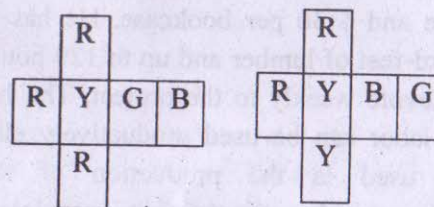
3. (a) (i) Determine the number of edges of $K_{9,10}$, Q_5 and C_{10} . 3

- (ii) Define complete bipartite graph. How many vertices and edges does a complete graph $K_{m,n}$ have? 3

- (b) (i) Prove that in any balanced signed graph every cycle has an even number of edges. 4

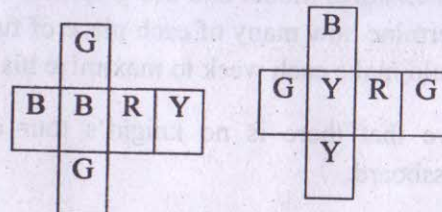
- (ii) Draw a simple connected graph with degree sequence $(3, 3, 3, 3, 3, 5, 5, 5)$. 2

- (c) Determine whether the given four cubes having four colors, can be stacked in a manner so that each side of the stack formed will have all the four colors exactly once. 6



Cube 1

Cube 2



Cube 3

Cube 4

- (d) Define a r -regular graph. Prove that, a r -regular graph with n vertices has $nr/2$ edges. 6

4. (a) Use the factorization:

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$$s^4 + 4a^4 = (s^2 - 2as + 2a^2)(s^2 + 2as + 2a^2)$$

and apply inverse Laplace transform to show that:

$$L^{-1} \left\{ \frac{s^3}{s^4 + 4a^4} \right\} = \cosh at \cos at.$$

- (b) Find the general solutions in power of x of the following differential equation:

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$$y'' + xy' + y = 0.$$

- (c) A carpenter realizes a net unit profit of \$ 25 per table and \$ 30 per bookcase. He has up to 690 board-feet of lumber and up to 120 hours of labor to devote weekly to the project. The lumber and the labor can be used productively elsewhere if not used in the production of tables and bookcases. He estimates that it requires 20 board-feet of lumber and 5 hours of labor to complete a table and 30 board-feet of lumber and 4 hours of labor to complete a bookcase. Form a mathematical model and use graphical analysis to determine how many of each piece of furniture he should make each week to maximize his profit. 7
- (d) Prove that there is no knight's tour on a 7×7 chessboard. 7