This question paper contains 4 printed pages] 5/12/18 (E)

Roll No. [| | | | | | | | | | |]

S. No. of Question Paper : 1414

Unique Paper Code : 62357502 I

Name of the Paper : Differential Equations

Name of the Course : B.A. (Prog.) Mathematics : DSE-2

Semester : V

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* the questions by selecting

any *two* parts from each question.

1. (*a*) Solve the initial value problem : 6

$$\left(3x^2y^4 + 2xy\right)dx + \left(2x^3y^3 - x^2\right)dy = 0; \ y(1) = 2,$$

(*b*) Solve : 6

$$x = y + a \ln p.$$

(*c*) Solve : 6

$$(y + x + 5)dy - (y - x + 1)dx = 0.$$

P.T.O.

2. (*a*) Solve : 6.5

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = x.$$

(*b*) Solve : 6.5

$$(x+3)^2\frac{d^2y}{dx^2} - 4(x+3)\frac{dy}{dx} + 6y = x.$$

(*c*) Consider the following differential equation : 6.5

$$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 0$$

(*i*) Show that x, x^2 and x^4 are solution of above differential equation.

(*ii*) Show that the solutions x, x^2 and x^4 are linearly independent.

(*iii*) Write the general solution of the above differential equation.

3. (*a*) Using the method of variation of parameters to find the general solution of : 6.5

$$\frac{d^2y}{dx^2} + y = \tan x.$$

(*b*) Use the method of undetermined coefficients to find the general solution of the differential equation : 6.5

$$4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = e^x + 2\cos 2x.$$

(*c*) Given that $y = x$ is a solution of the differential equation : 6.5

$$(x^2+1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$

Find a linearly independent solution by reducing the order and write the general solution.

4. (*a*) Solve : 6

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}.$$

(*b*) Solve : 6

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0.$$

(*c*) Solve : 6

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0,$$

$$\frac{dy}{dt} + 5x + 3y = 0.$$

5. (*a*) Find the general solution of the differential equation : 6.5

$$x^2 p + y^2 q = (x + y)z.$$

(*b*) Find the complete integral of the differential equation : 6.5

$$(p^2 + q^2)x^2 - qz = 0.$$

(*c*) (*i*) Classify the partial differential equation as elliptic, parabolic or hyperbolic : 2.5

$$4u_{xx} - 4u_{xy} + 5u_{yy} = 0.$$

(*ii*) Eliminate the parameters *a* and *b* from the following equation to find the corresponding partial differential equation : 4

$$z = x + ax^2 y^2 + b.$$

6. (*a*) Find the complete integral of the equation : 6

$$(p + q)(z - xp - yq) = 1.$$

(*b*) Eliminate the arbitrary function f from the equation $z = x + y + f(xy)$ to find the corresponding partial differential equation. 6

(*c*) Find the general solution of the partial differential equation : 6

$$y^2 p - xyq = x(z - 2y).$$