4/12/18 (M)

[This question paper contains 4 printed pages]

**Your Roll No.** : ........................

**Sl. No. of Q. Paper** : **618** **I**

Unique Paper Code : 32227502

Name of the Course : **B.Sc.(Hons.) Physics : DSE - I**

Name of the Paper : Advanced Mathematical Physics

Semester : V

***Time : 3 Hours*** ***Maximum Marks : 75***

**Instructions for Candidates :**

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **five** questions in all taking at least **two** questions from each section.

(c) **All** questions carry equal marks.

(d) Attend **all** parts of each question together.

## Section - A

**1.** (a) Determine whether the identity element exist or not for the binary operation * defined as: $a * b = a^b$ 4

P.T.O.

(b) Let $T : R^3 \to R^3$ be defined by

$T(x, y, z) = (x+y-2z,\ x+2y+z,\ 2x+2y-3z)$.

Show that T is a non-singular transformation. 5

(c) Linear transformation T on $R^2$ is defined as :

$T(x, y) = (3x - 4y,\ x + 5y)$ 6

Find the matrix representation of T relative to the u-basis :

$\{u_1 = (1, 3) \text{ and } u_2 = (2, 5)\}$.

**2.** (a) Determine whether (1, 2, 5) ; (2, 5, 1) ; (1, 5, 2) are linearly dependent or not. 5

(b) Consider the following subspace of $R^4$ :

$W = \{(a, b, c, d) : a + b = 0,\ c = 2d\}$

Find the dimension and basis of W. 2,3

(c) Assume that $A$, $I - A$, $I - A^{-1}$ are all non-singular matrices, show that : 5

$$(I - A)^{-1} + (I - A^{-1})^{-1} = I.$$

**3.** (a) Given a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, prove that its eigenvalue equation is given by

$$\lambda^2 - \lambda \mathrm{Tr}(A) + \det(A) = 0.$$ 5

(b) If $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, prove that :

$$e^{\theta B} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$ 10







**4.** (a) Find the condition for the following matrix to be orthogonal : 7

$$\begin{bmatrix} a+b & b-a \\ a-b & a+b \end{bmatrix}.$$

(b) Evaluate $C^{20}$, where $C = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$. 8

## Section - B

**5.** (a) Show that every second order tensor can be expressed as a sum of symmetric and skew-symmetric tensor. 3

(b) Prove that $(\vec{A} \times \vec{B}) \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{A}(\vec{B} \cdot \vec{C})$, using tensors. 7

(c) Given a vector $\vec{A} = (x,\ x + y,\ x + y + z)$.

Find the matrix elements of the second order skew-symmetric tensor associated with it. 5

**6.** (a) Prove that $\epsilon_{ijk}$ is an isotropic tensor of order three. 3

(b) Prove that :

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = (\vec{\nabla} \times \vec{A}) \cdot \vec{B} - (\vec{\nabla} \times \vec{B}) \cdot \vec{A}$$ 7

(c) Using tensors, show that scalar product of two vectors is invariant. 5

**7.** (a) Stress tensor $(p_{ij})$ satisfies the equations $p_{ij}\epsilon_{ijk}=0$ and $p_{ij}=f_i n_j$, where $f_k$ is the restoring force per unit area across the plane along $x_k$-axis and $\hat{n}$ is the unit vector normal to that surface. Prove that stress tensor is a symmetric tensor of order two. 7

(b) Stress tensor and strain tensor are related as

$$p_{ij}=\omega_{ijks}e_{ks},$$

where, elastic tensor $\omega_{ijks}$ is symmetric in i, j and k, s and its general form is $\omega_{ijks}=\lambda\delta_{ij}\delta_{ks}+\mu\delta_{ik}\delta_{js}+\gamma\,\delta_{is}\delta_{jk}$.

Prove that :

(i) $\omega_{ijks}=\lambda\delta_{ij}\delta_{ks}+\mu\left(\delta_{ik}\delta_{js}+\gamma_{is}\delta_{jk}\right)$

(ii) $P_{ii}=(3\lambda+2\mu)e_{ii}$ 4,4

**8.** (a) A covariant tensor has components xy, $2y-z^2$, xz in cartesian co-ordinates. Find its covariant components in cylindrical co-ordinates. 10

(b) Prove that $g^{jk}$ is a symmetric contravariant tensor of order two. 5