

256-6/12/18 (M)

[This question paper contains 4 printed pages]

**Your Roll No.** : .....

**Sl. No. of Q. Paper** : 138 I

**Unique Paper Code** : 42351101

**Name of the Course** : B.Sc.(Mathematical Sciences)/B.Sc. (Prog.)

**Name of the Paper** : Calculus and Matrices

**Semester** : I

**Time : 3 Hours** **Maximum Marks : 75**

**Instructions for Candidates :**

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt any **Two** questions from each section.

**Section - I**

1. (a) Prove that the set  $\{X_1, X_2\}$  of vectors in  $R^n$  is linearly independent iff  $X_1$  and  $X_2$  are collinear. 6  
(b) Define a subspace of a vector space. Examine whether the subset 6  
 $W = \{(a, b, 2); a, b \in R\}$  of  $R^3$  is a subspace or not.
2. (a) Define Linear Transformation. Find and sketch the image of unit square with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 1)$  and  $(1, 1)$  under the dilation of factor 3. 6

P.T.O.



- (b) Define eigen value of a matrix. Find eigen values and corresponding eigen vectors of matrix 6

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

3. (a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & 0 \end{bmatrix} \text{ using Elementary}$$

Transformations.

- (b) Solve, if consistent, the system of equations :

$$x + y + 3z = 1$$

$$2x + 3y - z = 3$$

$$5x + 7y + z = 7$$

6, 6

### Section - II

4. (a) Discuss the convergence of the following sequences : 6

(i)  $\left( (-1)^n \cdot \frac{1}{n} \right)$

(ii)  $(x^n)$

where  $-1 < x < 1$

- (b) Sketch the graph of the function

$$f(x) = \frac{1}{2}x^2 - 3x + \frac{11}{2}$$

Mention the transformations used at each step. 6

- (c) Radium is known to decay at the rate proportional to the amount present. If half life of radium is 1600 years, what percentage of radium will remain in a given sample after 800 years ? 6

5. (a) If  $y = \sin(m \sin^{-1} x)$ , prove that

$$(1-x^2)y_{n+2} = (2n+1)xy_{n+1} + (n^2-m^2)y_n$$

- (b) Find the Taylor's series generated by  $f(x) = \frac{1}{x}$

at  $x = 2$ . When does this series converges to  $\frac{1}{x}$ .

6

- (c) Verify that  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz$ , where  $z$  is given

$$\text{by } z = (x^2 + xy + y^2)^{-1}.$$

6

6. (a) Find the  $n$ th order derivative of the function given by  $y = \sin(ax + b)$ , where  $a, b$  are fixed constants. 6



- (b) Define heat equation and hence verify that  $\varphi(x, t) = e^{-c^2 x^2 t} \sin \pi x$  is a solution of heat equation. 6

- (c) Find the limit of the following sequences : 6

(i)  $\frac{1^2 + 2 + 3^2 + \dots + n^2}{6n^3 - n^2 + 3n + 4}$  (ii)  $\left(5^{\frac{1}{n}}\right)$  6

### Section - III

7. (a) Show that modulus of sum of two complex numbers is always less than or equal to the sum of their moduli. 3.5
- (b) Form an equation in lowest degree with real coefficients which has  $2-3i$  and  $3+2i$  as two of its roots. 4
8. (a) Solve the equation  $z^7 + z = 0$ . 4
- (b) Simplify  $\frac{(\cos \theta + i \sin \theta)^4}{(\sin \theta + i \cos \theta)^5}$ . 3.5
9. (a) Find the equation of circle whose radius is 3 and whose centre has affix  $1-i$ . 3.5
- (b) Find the equation of the right bisectors of the line joining the points  $z_1$  and  $z_2$ . 4