7.12.18 (M)

This question paper contains 4 printed pages.

Your Roll No.

1 : 102 Sl. No. of Ques. Paper

: 32221101 Unique Paper Code

: Mathematical Physics - I Name of Paper

: B.Sc. (Hons.) Physics Name of Course

: I Semester

: 3 hours Duration

: 75 Maximum Marks

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all. Question No. 1 is compulsory.

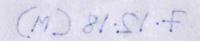
1. Do any five questions:

Do any five queeze
$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$
.

- (b) By calculating the Wronskian of the functions e^x , e^{-x} , and e-2x check whether the functions are linearly dependent or
- (c) Find the area of the triangle with vertices P(2, 3, 5), Q(4, 2-1), and R(3, 6, 4).
- (d) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 4$ at the point $(1, \sqrt{2}, -1)$.
- (e) Show that:

$$\oiint_{S} (\vec{\nabla} r^2) \cdot \vec{dS} = 6V$$

where S is the closed surface enclosing the volume V. P.T.O.



(f) Evaluate:

$$\iint\limits_{\mathbb{R}} \sqrt{x^2 + y^2} \, dx \, dy$$

(g) Verify that:

$$\int_{-\infty}^{+\infty} \delta(a-x)\delta(x-b) dx = \delta(a-b)$$

- (h) Form a differential equation whose solutions are e^{2x} and e^{3x} . $5\times 3=15$
- 2. (a) Solve the inexact equation:

$$y(1+xy) dx + x(1+xy+x^2y^2) dy = 0.$$

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x.$$

(c) Using method of undetermined coefficients, solve the differential equation:

$$\frac{d^2y}{dx^2} + 4y = 2\sin 2x.$$

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = x^2 + 5.$$

(b) Solve the differential equation using method of variation of parameter

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^{2x}.$$

4. (a) Show that

$$\left[\left(\vec{A} \times \vec{B} \right) \times \vec{C} \right] \times \vec{D} + \left[\left(\vec{B} \times \vec{A} \right) \times \vec{D} \right] \times \vec{C} +$$

$$\left[\left(\vec{C} + \vec{D} \right) \times \vec{A} \right] \times \vec{B} + \left[\left(\vec{D} \times \vec{C} \right) \times \vec{B} \right] \times \vec{A} = 0. \quad 6$$

(b) Show that:

$$\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$$

is a conservative force field and then evaluate

$$\int_{C} \vec{F} \cdot \vec{dr}$$

where C is any path from (0, 1, -1) to $(\frac{\pi}{2}, -1, 2)$.

5. (a) If \vec{a} is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that:

$$\operatorname{curl}\left(\frac{\vec{a}\times\vec{r}}{r^3}\right) = \frac{3(\vec{a}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{a}}{r^3}.$$

(b) Evaluate:

$$\iint\limits_{S} \vec{A} \cdot \hat{n} \, dS$$

where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane 2x + 3y + 6z = 12 located in the first octant.

6. (a) Evaluate:

$$\oint_C (y - \sin x) \, dx + \cos x \, dy$$

(i) directly

(ii) using Green's theorem in the plane, where C is the boundary of a triangle enclosed by the lines y = 0,

$$x = \frac{\pi}{2}$$
, and $y = \frac{2}{\pi}x$.

(b) Verify that:

$$\nabla^2 r^n = n(n+1)r^{n-2}.$$

7. (a) Verify divergence theorem for

$$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$$

taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

- (b) Express the position and velocity of a particle in cylindrical coordinates.
- 8. (a) Derive an expression for the divergence of a vector field in orthogonal curvilinear coordinate system.
 - (b) Evaluate Jacobian $J\left(\frac{x,y,z}{u_1,u_2,u_3}\right)$ for the transformation from rectangular coordinate system to spherical coordinate system.