7.12.18 (M)

This question paper contains 4 printed pages.

***Your Roll No.* ....**

I

Sl. No. of Ques. Paper	**: 102**
Unique Paper Code	**: 32221101**
Name of Paper	**: Mathematical Physics - I**
Name of Course	**: B.Sc. (Hons.) Physics**
Semester	**: I**
Duration	**: 3 hours**
Maximum Marks	**: 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt five questions in all.
Question No. 1 is compulsory.

1. Do any *five* questions :

(a) Solve : $\frac{dy}{dx} = \left(1+x^2\right)\left(1+y^2\right)$.

(b) By calculating the Wronskian of the functions e^x, e^{-x}, and e^{-2x} check whether the functions are linearly dependent or independent.

(c) Find the area of the triangle with vertices P(2, 3, 5), Q(4, 2–1), and R(3, 6, 4).

(d) Find the unit vector normal to the surface $x^2 + y^2 + z^2 = 4$ at the point $\left(1, \sqrt{2}, -1\right)$.

(e) Show that :

$$\oiint_S \left(\vec{\nabla} r^2\right) \cdot \overrightarrow{dS} = 6V$$

where S is the closed surface enclosing the volume V.

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(f) Evaluate :

$$\iint_R \sqrt{x^2+y^2}\, dx\, dy$$

(g) Verify that :

$$\int_{-\infty}^{+\infty} \delta(a-x)\delta(x-b)\, dx = \delta(a-b)$$

(h) Form a differential equation whose solutions are e^{2x} and e^{3x}. 5×3=15

2. (a) Solve the inexact equation :

$$y(1+xy)\, dx + x(1+xy+x^2y^2)\, dy = 0.$$ 5

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x.$$ 4

(c) Using method of undetermined coefficients, solve the differential equation :

$$\frac{d^2y}{dx^2} + 4y = 2\sin 2x.$$ 6

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = x^2 + 5.$$ 9

(b) Solve the differential equation using method of variation of parameter

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^{2x}.$$ 6

4. (a) Show that

$$\left[\left(\vec{A}\times\vec{B}\right)\times\vec{C}\right]\times\vec{D} + \left[\left(\vec{B}\times\vec{A}\right)\times\vec{D}\right]\times\vec{C} + \left[\left(\vec{C}+\vec{D}\right)\times\vec{A}\right]\times\vec{B} + \left[\left(\vec{D}\times\vec{C}\right)\times\vec{B}\right]\times\vec{A} = 0.$$ 6

(b) Show that :

$$\vec{F} = \left(y^2\cos x + z^3\right)\hat{i} + (2y\sin x - 4)\hat{j} + \left(3xz^2+2\right)\hat{k}$$

is a conservative force field and then evaluate

$$\int_C \vec{F}\cdot\overrightarrow{dr}$$

where C is any path from (0, 1, –1) to $\left(\frac{\pi}{2}, -1, 2\right)$. 9

5. (a) If $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that :

$$\text{curl}\left(\frac{\vec{a}\times\vec{r}}{r^3}\right) = \frac{3(\vec{a}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{a}}{r^3}.$$ 7

(b) Evaluate :

$$\iint_S \vec{A}\cdot\hat{n}\, dS$$

where $\vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ located in the first octant. 8

6. (a) Evaluate :

$$\oint_C (y - \sin x)\, dx + \cos x\, dy$$

(i) directly

(ii) using Green's theorem in the plane, where C is the boundary of a triangle enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$, and $y = \frac{2}{\pi}x$. 10

(b) Verify that :

$$\nabla^2 r^n = n(n+1)r^{n-2}.$$ 5

7. (a) Verify divergence theorem for

$$\vec{F} = \left(x^2 - yz\right)\hat{i} + \left(y^2 - xz\right)\hat{j} + \left(z^2 - xy\right)\hat{k}$$

taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b, 0 \le z \le c$. 10

(b) Express the position and velocity of a particle in cylindrical coordinates. 5

8. (a) Derive an expression for the divergence of a vector field in orthogonal curvilinear coordinate system. 10

(b) Evaluate Jacobian $J\left(\frac{x, y, z}{u_1, u_2, u_3}\right)$ for the transformation from rectangular coordinate system to spherical coordinate system. 5