4/12/18 (M)

[This question paper contains 8 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **611** **I**

Unique Paper Code : 32357505

Name of the Course : **B.Sc.(Hons.) Mathematics : DSE-I**

Name of the Paper : Discrete Mathematics

Semester : V

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Do any **two** parts from each question.

Section - I

1. (a) Define covering relation in an ordered set. Prove that if X is any set, then in the ordered set $\wp(X)$ equipped with the set inclusion relation given by $A \le B$ if and only if $A \subseteq B$ for all $A, B \in \wp(X)$, a subset B of X covers a subset A of X if and only if $B = A \cup \{b\}$, for some $b \in X \sim A$. 6

(b) Let $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ if and only if m divides n and let $\wp(\mathbb{N})$ be the power set of $\mathbb{N}$ equipped with the partial order given by $A \leq B$ if and only if $A \subseteq B$ for all $A, B \in \wp(\mathbb{N})$. In which of the following cases is the map $\varphi : P \to Q$ order-preserving ?

(i) $P = Q = \mathbb{N}_0$ and $\varphi(x) = nx \ \forall x \in P$, where $n \in \mathbb{N}_0$ is fixed. 3

(ii) $P = Q = \wp(\mathbb{N})$ and φ defined by 3

$$\varphi(A) = \begin{cases} \{1\} \text{ if } 1 \in A \\ \{2\} \text{ if } 2 \in A \text{ but } 1 \notin A \\ \varnothing \text{ otherwise} \end{cases}$$

(c) Let P = {a, b, c, d, e, f, u, v}. Draw a diagram of the ordered set $(P, \leq)$ where

$v < a < c < d < e < u$, $a < f < u$,

$v < b < c$, $b < f$

Also, find out $a \vee b$, $a \wedge b$, $e \vee f$ and $e \wedge f$. 6

2. (a) Let V be a vector space and let M = Sub V, the set of all subspaces of V. Prove that $(M, \subseteq)$ is a lattice as an ordered set but is not a sublattice of the lattice $(L, \subseteq)$, where $L = \wp(V)$, the power set of V. 6.5

(b) Prove that in a lattice L, the following inequalities are satisfied :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c) \ \forall a, b, c \in L$ 3

(ii) $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a) \quad \forall a, b, c \in L$ 3.5

(c) Let $(L, \leq)$ be a lattice as an ordered set. Define two binary operations + and. on L by $x+y = x \vee y = \sup\{x, y\}$ and $x \cdot y = x \wedge y = \inf\{x, y\}$. Prove that (L, +, .) is an algebraic lattice. 6.5

Section - II

3. (a) Define a distributive lattice. Prove that a homomorphic image of a distributive lattice is distributive . 6

(b) Use the Quine-McCluskey method to find a minimal form of :

$xyz' + xy'z + xy'z' + x'yz + x'y'z$ 6

(c) (i) Find the conjunctive normal form of : 3

$$(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'.$$

(ii) Find the disjunctive normal form of : 3

$$x_1'x_2 + x_3(x_1' + x_2).$$

4. (a) (i) Prove that $(x \wedge y)' = x' \vee y'$ and $(x \vee y)' = x' \wedge y'$ for all *x*, *y* in a Boolean algebra B. 3.5

(ii) Show that the lattice ({1, 2, 4, 5, 10, 20}, gcd, lcm) does not form a Boolean algebra for the set of positive divisors of 20. 3

(b) Using the Karnaugh Diagrams, find a minimum form for p and q where :

$$p = (x_1 + x_2)(x_1 + x_3) + x_1x_2x_3$$ 3.5

$$q = x_1x_2'x_3 + x_1x_2'x_3' + x_1'x_2x_3 + x_1'x_2'x_3 + x_1'x_2'x_3'$$ 3

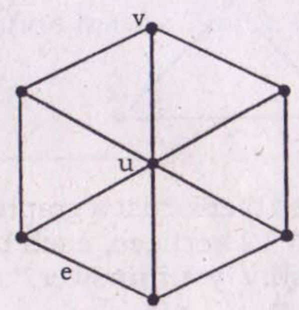
(c) Draw the contact diagram and give the symbolic representation (using seven gates) of the circuit given by

$$p = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1x_3 + x_1'x_2)(x_2' + x_3)$$

6.5

Section - III

5. (a) (i) Draw pictures of the subgraphs G \{e}, G \{v} and G \{u} of the following graph G. 3



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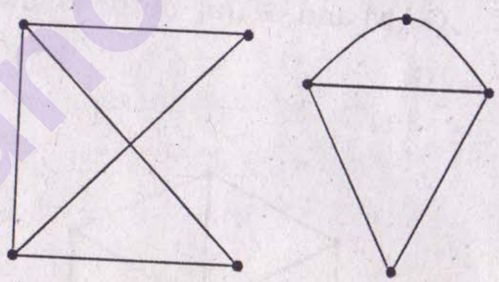
(ii) Answer the Königsberg bridge problem and explain your answer with graph. 3

(b) (i) Draw K_4 and $K_{3,4}$. 3

(ii) For the below pair of graphs, either label the graphs so as to exihibit an isomorphism or explain why graphs are not isomorphic. 3

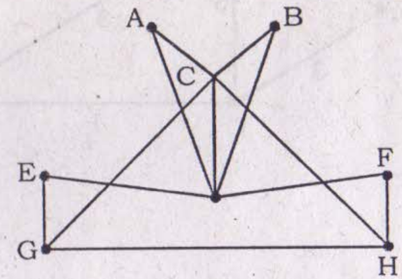


(c) (i) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 4. Justify your answer. 2

(ii) A complete graph with more than two vertices is not bipartite. Justify this statement. 2

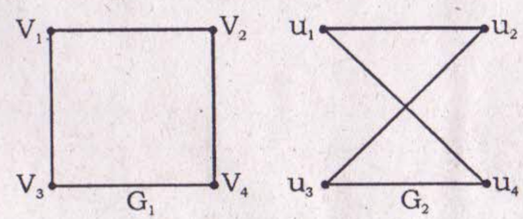
(iii) Draw a graph whose degree sequence is 1,1,1,1,1,1. 2

6. (a) Consider the Graph G given below. Is it Hamiltonian ? Is it Eulerian ? Explain your answers. 6.5



(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.

6.5



(c) Apply the improved version of Dijkstra's Algorithm to find a shortest path from A to Z. Write steps. 6.5