Lib- 10/12/18 (M)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **168**

Unique Paper Code : **42357501** **IC**

Name of the Paper : **Differential Equations**

Name of the Course : **B.Sc. (Math Sci.)/B.Sc. (Prog.) : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Solve : 6½

$$(2x + \tan y)\, dx + (x - x^2 \tan y) dy = 0.$$

(*b*) Solve : 6½

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3},\ y(1) = 2.$$

(*c*) Solve : 6½

$$p^2 - 9p + 18 = 0.$$

2. (*a*) Solve the initial value problem : 6½

$$\frac{d^2 y}{dx^2} + 4y = 8 \sin 2x, \quad y(0) = 6,\ y'(0) = 8.$$

(*b*) Find the general solution of the differential equation : 6½

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = 2x \ln x.$$

(*c*) For the differential equation : 6½

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0,$$

show that e^x and e^{4x} are solutions on the interval $-\infty < x < \infty$. Are these linearly independent ? Justify.

Also find the solution that satisfies the conditions $y(0) = 1$, $y'(0) = 4$.

3. (*a*) Using the method of variation of parameters, solve the differential equation : 6

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x.$$

(*b*) Given that $y = e^{2x}$ is a solution of : 6

$$(2x + 1)\frac{d^2y}{dx^2} - 4(x + 1)\frac{dy}{dx} + 4y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution.

(*c*) Find the general solution of : 6

$$x^2\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 10y = 3x^4 + 6x^3,$$

given that $y = x^2$ and $y = x^5$ are linearly independent solutions of the corresponding homogeneous equation.

4. (*a*) Solve : 6

$$\frac{a\,dx}{(b-c)yz} = \frac{b\,dy}{(c-a)zx} = \frac{c\,dz}{(a-b)xy}.$$

(*b*) Solve : 6

$$3\frac{dx}{dt} + 2\frac{dy}{dt} - x + y = t - 1,$$

$$\frac{dx}{dt} + \frac{dy}{dt} - x = t + 2.$$

(*c*) Check condition of integrability and solve : 6

$$zydx = xzdy + y^2dz.$$

5. (*a*) Eliminate the arbitrary function f from the equation : 6

$$f(x^2 + y^2 + z^2,\ z^2 - 2xy) = 0$$

to form the corresponding partial differential equation.

(*b*) Find the general integral of the partial differential equation : 6

$$x(x^2 + 3y^2)\,p - y\,(3x^2 + y^2)\,q = 2z(y^2 - x^2).$$

(*c*) Show that the equation : 6

$$xp = yq, \ z(xp + yq) = 2xy$$

are compatible and find their solution.

6. (*a*) Find the complete integral of the equation : 6½

$$p^2x + q^2y = z.$$

(*b*) Find the complete integral of the equation : 6½

$$pqz = p^2(xq + p^2) + q^2(yp + q^2).$$

(*c*) Reduce the following differential equation to canonical form : 6½

$$\frac{d^2z}{dx^2} - x^2\frac{\partial^2 z}{\partial y^2} = 0.$$