This question paper contains 4 printed pages] 7.12.18 (M)

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **88**

Unique Paper Code : **32351101** I

Name of the Paper : **Calculus**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

(Attempt any *four* questions from Section I)

1. If $y = \log (x+\sqrt{x^2+1})$, show that :

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

2. Sketch the graph of the function

$$f(x) = \frac{3x-5}{x-2}$$

by determining all critical points, intervals of increase and decrease, points of relative maxima and minima, concavity of the graph, inflection points and horizontal and vertical asymptotes.

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3. Evaluate : $\lim_{x\to 0}(e^x - 1 - x)^x$.

4. Given the cost $C(x) = \frac{1}{8}x^2 + 5x + 98$ of producing x units of a particular commodity and the selling price $p(x) = \frac{1}{2}(75 - x)$ when x units are produced. Determine the level of production that maximizes profit.

5. Sketch the graph of $r = \sin 2\theta$ in polar coordinates.

Section II

(Attempt any *four* questions from Section II)

6. Obtain the reduction formula for

$$\int \sec^n x \, dx.$$

Use it to evaluate $\int \sec^6 x \, dx$.

7. Find the volume of the solid generated by revolving the region enclosed by $y = x$, $y = 2 - x^2$ and $x = 0$ is revolved about the x-axis.

8. Use cylindrical shells method to find the volume of the solid generated when the region enclosed by $y = 2x - x^2$ and $y = 0$ is resolved about y-axis.

9. Show that the arc length of the curve $y = \cosh x$ between $x = 0$ and $x = \log 2$ is 3/4.

10. Find the area of the surface generated by revolving the curve $y = \sqrt{9 - x^2}$, $-1 \le x \le 1$, about x-axis.

Section III

(Attempt any *three* questions from Section III)

11. Find the equation of parabola having axis $y = 0$ and passing through the points (3, 2) and (2, –3).

12. Find the equation of ellipse with foci (1, 2) and (1, 4) and minor axis of length 2.

13. Describe and sketch the graph of the conic

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Label the vertices, foci and asymptotes to the graph.

14. Rotate the coordinate axes to remove the xy-term in the equation

$$31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0.$$

Identify the resultant conic.

Section IV

(Attempt any *four* questions from Section IV)

15. Given the vector functions

$$\vec{F}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and

$$\vec{G}(t) = \frac{1}{t}\mathbf{i} - e^t\mathbf{j}$$

verify that

$$\lim_{t\to 1}[\vec{F}(t) \times \vec{G}(t)] = [\lim_{t\to 1}\vec{F}(t)] \times [\lim_{t\to 1}\vec{G}(t)].$$

16. A velocity of particle moving in space is

$$\vec{V}(t) = t^2\hat{\mathbf{i}} - e^{2t}\hat{\mathbf{j}} + \sqrt{t}\hat{\mathbf{k}}\cdot$$

Find the particle's position as a function of t if the position at time $t = 0$ is $\vec{R}(0) = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}.$

17. A shell is fired at ground level with a muzzle speed of 280 ft/s and at an elevation of 45° from ground level :

(i) Find the maximum height attained by the shell.

(ii) Find the time of flight and the range of the shell.

18. Find the tangential and normal components of the acceleration of an object that moves with position vector

$$\vec{R}(t) = \cos t\hat{\mathbf{i}} + \sin t\hat{\mathbf{j}} + t\hat{\mathbf{k}}.$$

19. Find the curvature $\kappa(t)$ for the curve given by the vector equation

$$\vec{R}(t) = 4\cos t\hat{\mathbf{i}} + 4\sin t\hat{\mathbf{j}} + t\hat{\mathbf{k}}\ (0 \le t \le 2\pi).$$