This question paper contains 4 printed pages]

7.12.18 (M)

Maximum Marks: 75

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Roll No.							
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S. No. of Question Paper : 88

Unique Paper Code : 32351101

Name of the Paper : Calculus

Name of the Course : B.Sc. (H) Mathematics

Semester : I

Duration : 3 Hours

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

(Attempt any four questions from Section I)

1. If $y = \log (x + \sqrt{x^2 + 1})$, show that :

$$(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+n^2y_n=0.$$

2. Sketch the graph of the function

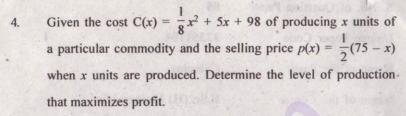
$$f(x)=\frac{3x-5}{x-2}$$

by determining all critical points, intervals of increase and decrease, points of relative maxima and minima, concavity of the graph, inflection points and horizontal and vertical asymptotes.



3. Evaluate: $\lim_{x\to 0} (e^x - 1 - x)^x$.

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Sketch the graph of $r = \sin 2\theta$ in polar coordinates. 5.

Section II

(Attempt any four questions from Section II)

Obtain the reduction formula for 6.

$$\int \sec^n x \, dx$$
.

Use it to evaluate $\int \sec^6 x \, dx$.

- Find the volume of the solid generated by revolving the region 7. enclosed by y = x, $y = 2 - x^2$ and x = 0 is revolved about the x-axis.
- Use cylindrical shells method to find the volume of the solid 8. generated when the region enclosed by $y = 2x - x^2$ and y = 0 is resolved about y-axis.
- Show that the arc length of the curve $y = \cosh x$ between 9. x = 0 and $x = \log 2$ is 3/4.
- Find the area of the surface generated by revolving the curve $y = \sqrt{9-x^2}$, $-1 \le x \le 1$, about x-axis.

Section III

(Attempt any three questions from Section III)

- Find the equation of parabola having axis y = 0 and passing through the points (3, 2) and (2, -3).
- Find the equation of ellipse with foci (1, 2) and (1, 4) and minor axis of length 2.
- Describe and sketch the graph of the conic

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Label the vertices, foci and asymptotes to the graph.

Rotate the coordinate axes to remove the xy-term in the equation

$$31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0$$

Identify the resultant conic.

Section IV

(Attempt any four questions from Section IV)

Given the vector functions 15.

$$\overrightarrow{\mathbf{F}}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and

$$\vec{\mathbf{G}}(t) = \frac{1}{t}\mathbf{i} - e^t\mathbf{j}$$

verify that

$$\lim_{t\to 1} [\overrightarrow{\mathbf{F}}(t) \times \overrightarrow{\mathbf{G}}(t)] = [\lim_{t\to 1} \overrightarrow{\mathbf{F}}(t)] \times [\lim_{t\to 1} \overrightarrow{\mathbf{G}}(t)].$$

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16. A velocity of particle moving in space is

$$\overrightarrow{\mathbf{V}}(t) = t^2 \hat{\mathbf{i}} - e^{2t} \hat{\mathbf{j}} + \sqrt{t} \hat{\mathbf{k}}$$

Find the particle's position as a function of t if the position at time t = 0 is $\mathbf{R}(0) = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

- 17. A shell is fired at ground level with a muzzle speed of 280 ft/s and at an elevation of 45° from ground level :
 - (i) Find the maximum height attained by the shell.
 - (ii) Find the time of flight and the range of the shell.
- 18. Find the tangential and normal components of the acceleration of an object that moves with position vector

$$\overrightarrow{R}(t) = \cos t \hat{i} + \sin t \hat{j} + t \hat{k}.$$

19. Find the curvature $\kappa(t)$ for the curve given by the vector equation

$$\overrightarrow{\mathbf{R}}(t) = 4 \cos t \hat{\mathbf{i}} + 4 \sin t \hat{\mathbf{j}} + t \hat{\mathbf{k}} \quad (0 \le t \le 2\pi).$$