15.12.18(M)

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 92

Unique Paper Code : 32351303 I

Name of the Paper : C-7 Multivariate Calculus

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : III

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Section I

Attempt any *six* questions from this section.

1. Let f be the function defined by $f(x, y) = \dfrac{x^2 + 2y^2}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$.

   (*a*) Find $\lim\limits_{(x, y) \to (2, 1)} f(x, y)$.

   (*b*) Prove that f has no limit at (0, 0).

P.T.O.

2. The temperature at the point (x, y) on a given metal plate in the xy-plane is determined according to the formula $T(x, y) = x^3 + 2xy^2 + y$ degrees. Compute the rate at which the temperature changes with distance if we start at (2, 1) and move :

(*a*) parallel to the vector **j**.

(*b*) parallel to the vector **i**.

3. The Company sells two brands X and Y of a commercial soap, in thousand-pound units. If x units of brand X and y units of brand Y are sold, the unit price for brand X is $p(x) = 4{,}000 - 500x$ and for brand Y is $q(y) = 3{,}000 - 450y$.

(*a*) Find the total revenue R in terms of p and q.

(*b*) Suppose the brand X sells for $ 500 per unit and brand Y sells for $ 750 per unit. Estimate the change in total revenue if the unit prices are increased by $ 20 for brand X and $ 18 for brand Y.

4. If

$$w = f\left(\frac{r-s}{s}\right),$$

show that

$$r\frac{\partial w}{\partial r} + s\frac{\partial w}{\partial s} = 0.$$

5. Find the directional derivative of $f(x, y) = e^{x^2y^2}$ at P(1, –1) in the direction toward Q(2, 3).

6. Find the absolute extrema of $f(x, y) = 2\sin x + 5\cos y$ in the rectangular region with vertices (0, 0), (2, 0), (2, 5) and (0, 5).

7. Let $\mathbf{R} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and $r = \|\mathbf{R}\|$, evaluate $\operatorname{div}\left(\frac{1}{r^3}\mathbf{R}\right)$.

Section II

Attempt any *five* questions from this section.

8. By using iterated integral, compute

$$\iint_R x\sqrt{1-x^2}\,e^{3y}\,dA,$$

where R is the rectangle $0 \le x \le 1$, $0 \le y \le 2$.

16. Use Green's theorem, to find the work done by the force field

$$\mathbf{F}(x, y) = (3y - 4x)\mathbf{i} + (4x - y)\mathbf{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

17. Use Stokes' theorem, to evaluate the line integral

$$\oint_C (3y\,dx + 2z\,dy - 5x\,dz)$$

where C is the intersection of the xy-plane and the hemisphere

$$z = \sqrt{1 - x^2 - y^2},$$

traversed counterclockwise as viewed from above.

18. Evaluate

$$\iint_S (\mathbf{F} \cdot \mathbf{N})\,dS,$$

where $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + x^3y^3\mathbf{k}$ and S is the surface of the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes, with outward unit normal vector N.