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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **89**

Unique Paper Code : **32351102** **I**

Name of the Paper : **Algebra**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **I**

Duration : **3 Hours** **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Find polar representation of the complex number : 6

$$z = \sin a + i(1 + \cos a),\ a \in [0, 2\pi).$$

(*b*) Find $|z|$ and arg z, arg $(-z)$ for : 6

(*i*) $z = (1 - i)(6 + 6i)$

(*ii*) $z = (7 - 7\sqrt{3}i)(-1 - i)$.

(*c*) Solve the equation : 6

$$z^4 = 5(z - 1)(z^2 - z + 1).$$

2. (*a*) For $a, b \in \mathbf{Z}$, define $a \sim b$ iff $a^2 - b^2$ is divisible by 3 : 6

(*i*) Prove that $\sim$ is an equivalence relation on Z.

(*ii*) Find the equivalence classes of 0 and 1.

(*b*) Define : 6

$$f : \mathbf{Z} \to \mathbf{Z} \text{ by } f(x) = x^2 - 5x + 5$$

(*i*) Is f one-to-one ?

(*ii*) Is f onto ?

Justify each answer.

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(*c*) Show that the open intervals (0, 1) and (4, 6) have the same cardinality. 6

3. (*a*) Suppose a, b and c are three non-zero integers with a and c relatively prime. Show that : 6

$$\gcd(a, bc) = \gcd(a, b).$$

(*b*) (*i*) Solve the following congruence if possible. If no solution exists, explain why not :

$$4x \equiv 2 (\text{mod } 6).$$

(*ii*) Find three positive and three negative integers in $\overline{5}$ w.r.t. congruence mod 7. 6

(*c*) Use mathematical induction to establish the following inequality : 6

$$n\,! > n^3, \text{ for all } n \geq 6.$$

4. (*a*) Find the general solution to the following linear system : 6½

$$3x_2 - 6x_3 + 6x_4 + 4x_5 = -5$$
$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$
$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15.$$

(*b*) Let $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$.

Is u in the subspace of $\mathbf{R}^3$ spanned by the columns of A. Why or why not ? 6½

(*c*) Let :

$$v_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}.$$

(*i*) For what values of h is v_3 in span $\{v_1, v_2\}$?

(*ii*) For what values of h is $\{v_1, v_2, v_3\}$ linearly dependent ? Justify each answer. 6½

5. (*a*) Let $A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 5 & 1 \\ 3 & 6 & 0 \end{bmatrix}$, and define by $T : R^3 \to R^3$ by $T(x) = Ax$. Find all x in R^3 such that $T(x) = 0$. Does $b = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ belong to range of T ? 6½

(*b*) A linear transformation $T : R^2 \to R^2$ first reflects points through the x_1-axis and then reflects points through the x_2-axis. Show that T can also be described as a linear transformation that rotates points about the origin. What is the angle of that rotation ? 6½

(*c*) Let

$$A = \begin{bmatrix} 2 & -3 & -4 \\ -8 & 8 & 6 \\ 6 & -7 & -7 \end{bmatrix} \text{ and } u = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}.$$

Is u in Nul A ? Is u in Col A ? Justify each answer. 6½

6. (*a*) Given $b_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, b_2 = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$ and $B = \{b_1, b_2\}$ is basis of subspace H of R^2.

(*i*) Determine if $x = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ belongs to H.

(*ii*) Find $[x]_B$, the B-coordinate vector of x. 6½

(*b*) Determine the basis of the null space of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix}.$$ 6½

(*c*) Is $\lambda = -2$ an eigenvalue of $\begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

If so, find one corresponding eigenvector. 6½