17.12.18 (E)

This question paper contains 4 printed pages]

**Roll No.** | | | | | | | | | | | |

S. No. of Question Paper : **1279**

Unique Paper Code : **62351101** I

Name of the Paper : **Calculus**

Name of the Course : **B.A. (Prog.) Mathematics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*



Attempt any *two* parts from each question.

1. (*a*) Examine the continuity of the function

$$f(x) = \begin{cases} 2x, & \text{if } 0 \le x \le 1, \\ 2 - x, & \text{if } 1 < x \le 2, \\ x^2 - 2x, & \text{if } x > 2 \end{cases}$$

at $x = 1$ and $x = 2$. 6

(*b*) Show that the

$$\lim_{x \to 1} \frac{|x - 1|}{x - 1}$$

doesn't exist. 6

P.T.O.

(c) Discuss the differentiability of

$$f(x) = x\,\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}},\ x \neq 0$$

$f(0) = 0$ at the origin. 6

2. (a) If

$$y = \sin mx + \cos nx,$$

then show that

$$y_n = m^n\left[1 + (-1)^n \sin 2mx\right].$$ 6

(b) If

$$z = x\cos\left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right),$$

then show that :

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0,\ (x \neq 0).$$ 6

(c) If

$$z = \tan^{-1}\frac{x^3 + y^3}{x - y},$$

prove that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sin 2z.$$ 6





3. (a) Show that the length of the portion of the tangent to the curve :

$$x = a\cos^3\theta,\ y = a\sin^3\theta$$

Intercepted between the co-ordinate axes is constant. 6½

(b) Find the radius of curvature at any point on the curve :

$$y = c\cosh x/c.$$ 6½

(c) Prove that the radius of curvature at the point $(-2a, 2a)$ on the curve :

$x^2y = a(x^2 + y^2)$ is $-2a$. 6½

4. (a) Find the asymptotes of the curve :

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0.$$ 6½

(b) Find the equation of the tangent at $(-1, -2)$ to the curve :

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0,$$

and show that this point is a cusp. 6½

(c) Trace the curve :

$$y = x^3 - 12x - 16.$$ 6½

5. (*a*) State Lagrange's Mean value theorem in the interval $[a, a + h]$. Prove that for any quadratic function $px^2 + qx + r$, the value of $\theta$ in Lagrange's theorem is always 1/2 whatever $p, q, r, a, h$ may be. 6

(*b*) Show that :

$$\frac{\tan x}{x} > \frac{x}{\sin x} \text{ for } 0 < x < \frac{\pi}{2}.$$ 6

(*c*) Prove that :

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\dots\dots .$$ 6

6. (*a*) State and prove Rolle's Mean Value theorem. 6½

(*b*) Evaluate :

$$\lim_{x \to 0} \frac{xe^x - \log(1 + x)}{x^2}.$$ 6½

(*c*) Investigate the maximum and minimum values of the function *f*, defined by

$$f(x) = (x - 3)^2 (x + 3), \text{ for all } x \in \mathbf{R}.$$ 6½