12.12.18 (E)

This question paper contains 5 printed pages.

Your Roll No. .....

Sl. No. of Ques. Paper	: 1319	I
Unique Paper Code	: 62354343	
Name of Paper	: **Analytic Geometry and Applied Algebra**	
Name of Course	: **B.A. (Prog.) Mathematics**	
Semester	: **III**	
Duration	: **3 hours**	
Maximum Marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has six questions in all.

Attempt any two parts from each question.

All questions are compulsory.

1. (a) Describe and draw the graph of the equation :

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$ 6½

(b) Writing the basic steps, describe and draw the graph of the equation :

$$(x+2)^2 = -(y+2).$$ 6½

(c) Identify and sketch the curve :

$$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$$ 6½

2. (a) Find the equation of the ellipse with foci $(\pm 1, 0)$ and $b = \sqrt{2}$. Also state the reflection property of the ellipse. 6

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(b) Find the equation of the parabola whose vertex is at (5, –3); axis is parallel to y-axis and passes through (9, 5). 6

(c) Find the equation of the hyperbola with the vertices $(0, \pm 2)$ and asymptotes $y = \pm\frac{2}{3}x$. Also sketch its graph. 6

3. (a) Let any $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 60°$. Find the $x'y'$-coordinates of the point whose xy-coordinates are (–2, 6). Also find the equation of the curve $\sqrt{3}xy + y^2 = 6$ in $x'y'$-coordinates. 6

(b) Rotate the coordinate axes to remove the xy-term of the conic :

$$6x^2 + 24xy - y^2 - 12x + 26y + 11 = 0.$$

Then name the conic. 6

(c) (i) Find the angle that the vector $-\sqrt{3}\mathbf{i} + \mathbf{j}$ makes with positive x-axis. 3

(ii) Find the orthogonal projection of vector $v = 6\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ on the vector $\mathbf{b} = \mathbf{i} - 2\mathbf{y} - 2\mathbf{k}$. 3

4. (a) Find the equation of two spheres that are centered at the origin and are tangent to the sphere of radius 1 centered at (3, –2, 4). 6½

(b) (i) Find the direction cosines of the vector $\mathbf{v} = 2\mathbf{i} + 3\mathbf{y} - 6\mathbf{k}$, if it makes angles α, β and γ with x-axis, y-axis and z-axis, respectively. Then show that :

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$ 3

(ii) For any two vectors $\mathbf{u}$ and $\mathbf{v}$, prove that :

$$\mathbf{u}.\mathbf{v} = \frac{1}{4}\|\mathbf{u}+\mathbf{v}\|^2 - \frac{1}{4}\|\mathbf{u}-\mathbf{v}\|^2.$$ 3½

(c) Let $\mathbf{u} = \mathbf{i} - 3\mathbf{i} + 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j}$, and $\mathbf{w} = 2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. Find the length of $3\mathbf{u} - 5\mathbf{v} + 2\mathbf{w}$. Also find the volume of the parallelopiped with adjacent edges $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$. 6½

5. (a) (i) Find the parametric equation of line passing through the point (1, 2, –3) and parallel to the vector $\mathbf{u} = 4\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$. 3

(ii) Find the equation of plane through the point (–1, 2, –5) and perpendicular to the planes $2x - y - z = 1$ and $x + y - 2z = 8$.

(b) Show that the lines :

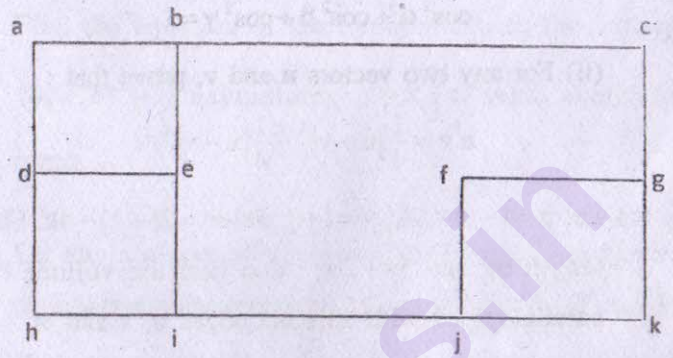
$$L_1 : x = -2 + t, \quad y = 3 + 2t, \quad z = 4 - t$$

$$L_2 : x = 3 - t, \quad y = 4 - 2t, \quad z = t$$

are parallel. Also find the equation of the plane they determine. 6

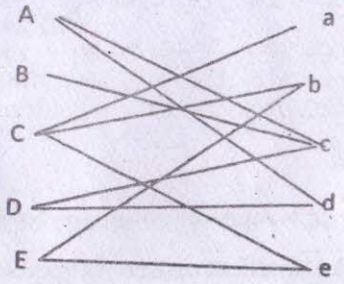
(c) Let the graph represent a section of a city's street map. What is the smallest number of policemen that should be positioned at corners (vertices) so that they can keep

every block (edge) under surveillance? Give a detailed logical analysis. 6

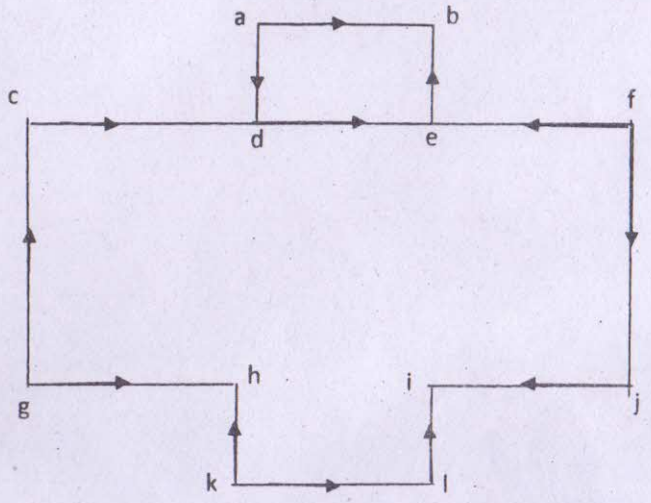


6. (a) Three pitchers of sizes 10 litres, 4 litres and 7 litres are given. If initially 10 litres pitcher is full and the other two are empty, find a minimal sequence of pouring so as to have exactly 3 litres of water in two pitchers. 6½

(b) (i) Find a matching or explain why none exists for the following graph:



(ii) Find a vertex basis for the following graph : 3½



(c) Find a maximum independent set in the following graph. Justify your answer. 6½