The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Friday, June 21, 2019 — 9:15 a.m. to 12:15 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. You may remove this sheet from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

   Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

1 On the set of axes on the next page, triangle *ABC* is graphed. Triangles *A'B'C'* and *A''B''C''*, the images of triangle *ABC*, are graphed after a sequence of rigid motions.

Identify which sequence of rigid motions maps △*ABC* onto △*A'B'C'* and then maps △*A'B'C'* onto △*A''B''C''*.

(1) a rotation followed by another rotation

(2) a translation followed by a reflection

(3) a reflection followed by a translation

(4) a reflection followed by a rotation

The set of axes for question 1 is on the next page.
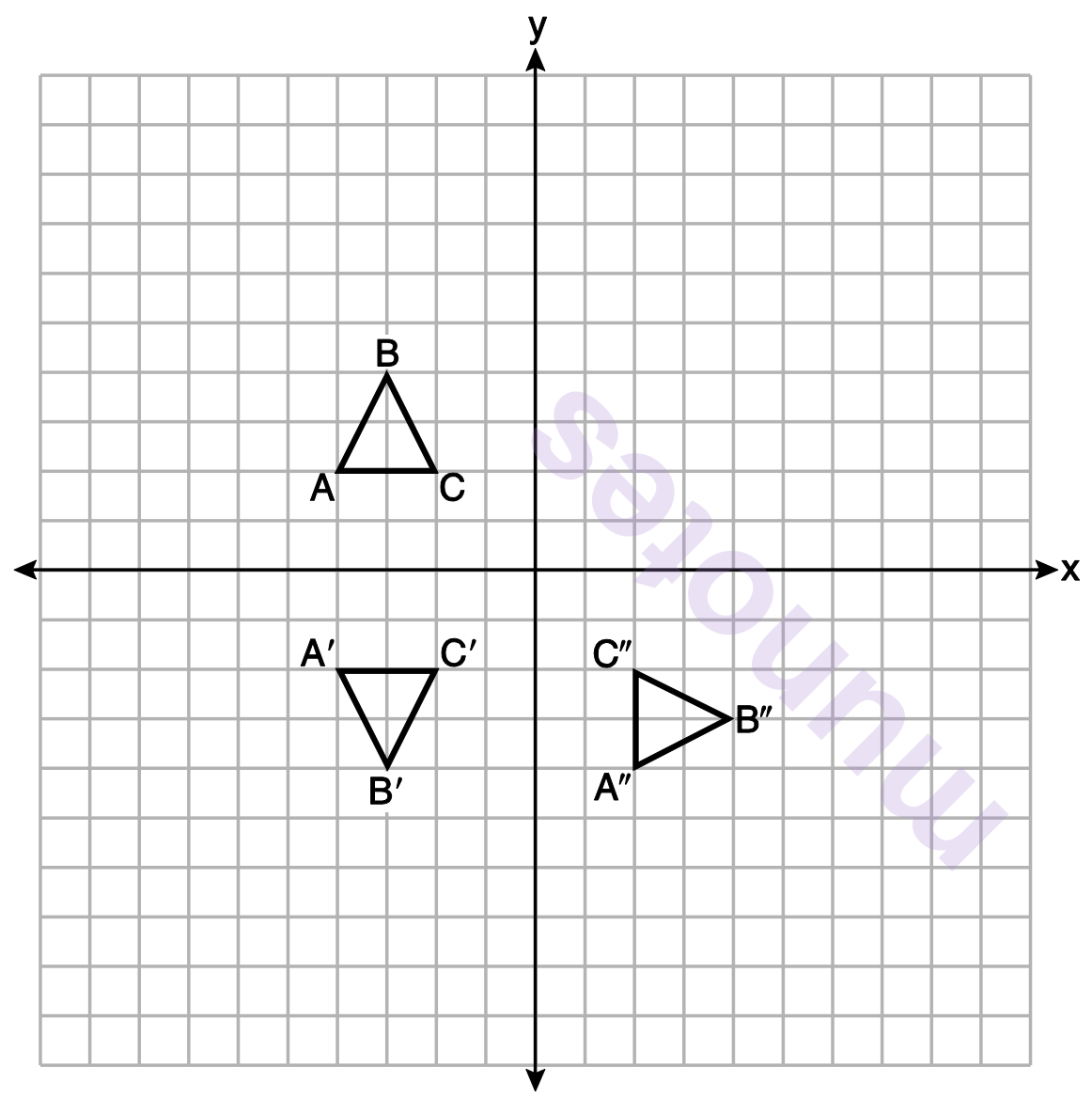
2 The table below shows the population and land area, in square miles, of four counties in New York State at the turn of the century.

County	2000 Census Population	2000 Land Area (mi²)
Broome	200,536	706.82
Dutchess	280,150	801.59
Niagara	219,846	522.95
Saratoga	200,635	811.84

Which county had the greatest population density?

(1) Broome

(2) Dutchess

(3) Niagara

(4) Saratoga

3 If a rectangle is continuously rotated around one of its sides, what is the three-dimensional figure formed?

(1) rectangular prism

(2) cylinder

(3) sphere

(4) cone

4 Which transformation carries the parallelogram below onto itself?
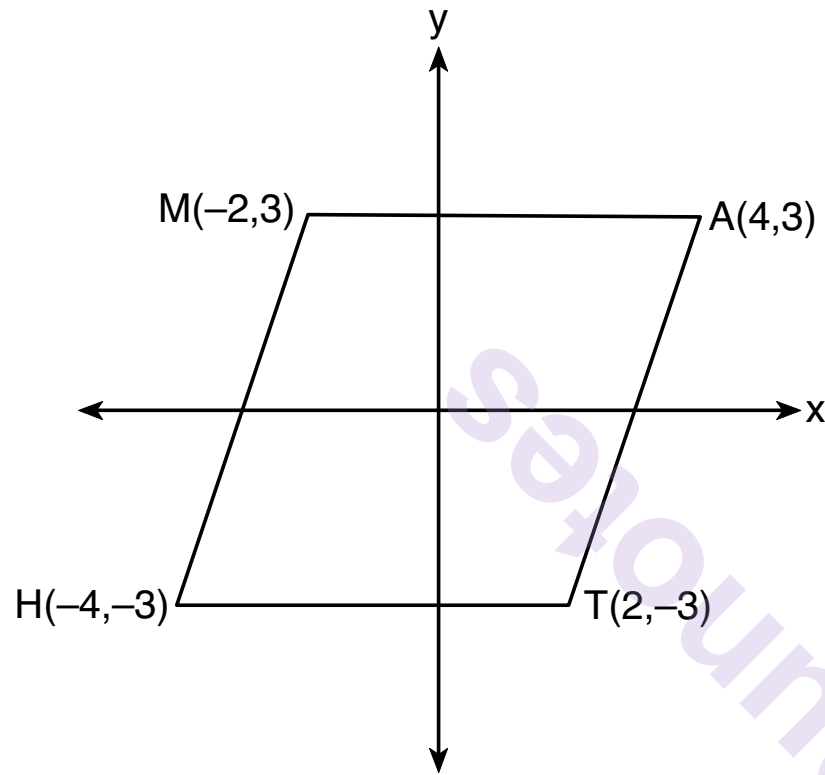
(1) a reflection over $y = x$

(2) a reflection over $y = -x$

(3) a rotation of 90° counterclockwise about the origin

(4) a rotation of 180° counterclockwise about the origin

5 After a dilation centered at the origin, the image of $\overline{CD}$ is $\overline{C'D'}$. If the coordinates of the endpoints of these segments are $C(6,-4)$, $D(2,-8)$, $C'(9,-6)$, and $D'(3,-12)$, the scale factor of the dilation is

(1) $\dfrac{3}{2}$ (3) 3

(2) $\dfrac{2}{3}$ (4) $\dfrac{1}{3}$

6 A tent is in the shape of a right pyramid with a square floor. The square floor has side lengths of 8 feet. If the height of the tent at its center is 6 feet, what is the volume of the tent, in cubic feet?

(1) 48   (3) 192

(2) 128   (4) 384

7 The line $-3x + 4y = 8$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?
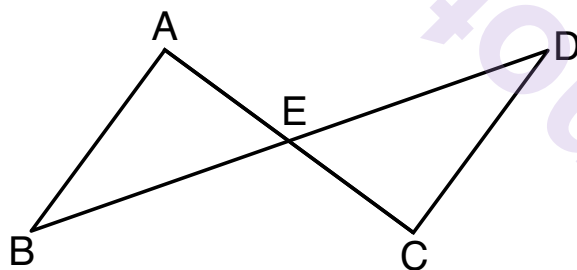
(1) $y = \frac{4}{3}x + 8$ (3) $y = -\frac{3}{4}x - 8$

(2) $y = \frac{3}{4}x + 8$ (4) $y = -\frac{4}{3}x - 8$

8 In the diagram below, $\overline{AC}$ and $\overline{BD}$ intersect at E.

Which information is always sufficient to prove $\triangle ABE \cong \triangle CDE$?

(1) $\overline{AB} \parallel \overline{CD}$

(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BE} \cong \overline{DE}$

(3) E is the midpoint of $\overline{AC}$.

(4) $\overline{BD}$ and $\overline{AC}$ bisect each other.

9 The expression sin 57° is equal to

(1) tan 33°   (3) tan 57°

(2) cos 33°   (4) cos 57°

10 What is the volume of a hemisphere that has a diameter of 12.6 cm, to the *nearest tenth of a cubic centimeter*?

(1) 523.7   (3) 4189.6

(2) 1047.4   (4) 8379.2

11 In the diagram below of $\triangle ABC$, D is a point on $\overline{BA}$, E is a point on $\overline{BC}$, and $\overline{DE}$ is drawn.
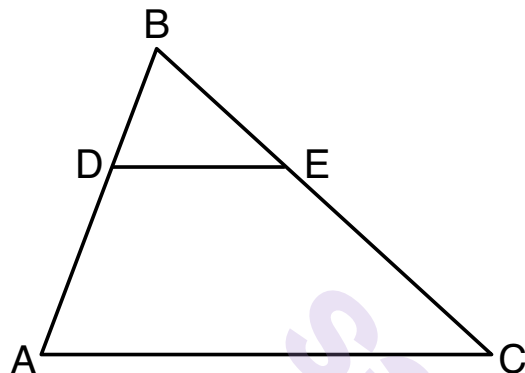
If $BD = 5$, $DA = 12$, and $BE = 7$, what is the length of $\overline{BC}$ so that $\overline{AC} \parallel \overline{DE}$?
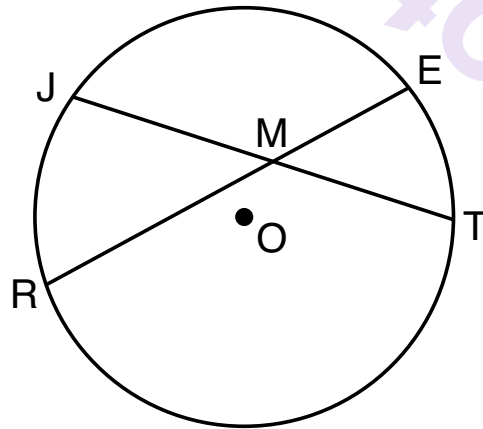
(1) 23.8

(3) 15.6

(2) 16.8

(4) 8.6

12 A quadrilateral must be a parallelogram if

(1) one pair of sides is parallel and one pair of angles is congruent

(2) one pair of sides is congruent and one pair of angles is congruent

(3) one pair of sides is both parallel and congruent

(4) the diagonals are congruent

13 In the diagram below of circle O, chords $\overline{JT}$ and $\overline{ER}$ intersect at M.

If $EM = 8$ and $RM = 15$, the lengths of $\overline{JM}$ and $\overline{TM}$ could be
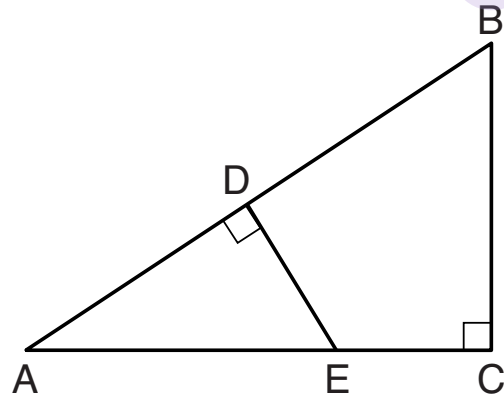
(1) 12 and 9.5   (3) 16 and 7.5

(2) 14 and 8.5   (4) 18 and 6.5

14 Triangles *JOE* and *SAM* are drawn such that $\angle E \cong \angle M$ and $\overline{EJ} \cong \overline{MS}$. Which mapping would *not* always lead to $\triangle JOE \cong \triangle SAM$?

(1) $\angle J$ maps onto $\angle S$ (3) $\overline{EO}$ maps onto $\overline{MA}$

(2) $\angle O$ maps onto $\angle A$ (4) $\overline{JO}$ maps onto $\overline{SA}$

15 In $\triangle ABC$ shown below, $\angle ACB$ is a right angle, E is a point on $\overline{AC}$, and $\overline{ED}$ is drawn perpendicular to hypotenuse $\overline{AB}$.

If $AB = 9$, $BC = 6$, and $DE = 4$, what is the length of $\overline{AE}$?

(1) 5   (3) 7

(2) 6   (4) 8

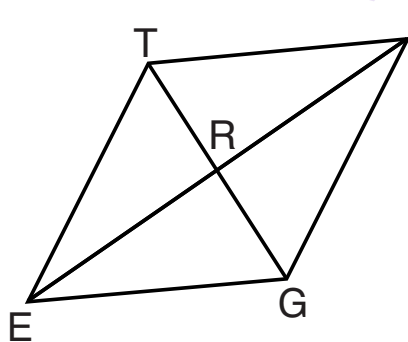**16** Which equation represents a line parallel to the line whose equation is $-2x + 3y = -4$ and passes through the point (1,3)?

(1) $y - 3 = -\dfrac{3}{2}(x - 1)$

(3) $y + 3 = -\dfrac{3}{2}(x + 1)$

(2) $y - 3 = \dfrac{2}{3}(x - 1)$

(4) $y + 3 = \dfrac{2}{3}(x + 1)$

17 In rhombus *TIGE*, diagonals $\overline{TG}$ and $\overline{IE}$ intersect at *R*. The perimeter of *TIGE* is 68, and *TG* = 16.

What is the length of diagonal $\overline{IE}$?

(1) 15

(3) 34

(2) 30

(4) 52

18 In circle O two secants, $\overline{ABP}$ and $\overline{CDP}$, are drawn to external point P. If $m\widehat{AC} = 72°$, and $m\widehat{BD} = 34°$, what is the measure of $\angle P$?

(1) 19°

(3) 53°

(2) 38°

(4) 106°

19 What are the coordinates of point C on the directed segment from $A(-8,4)$ to $B(10,-2)$ that partitions the segment such that AC:CB is 2:1?
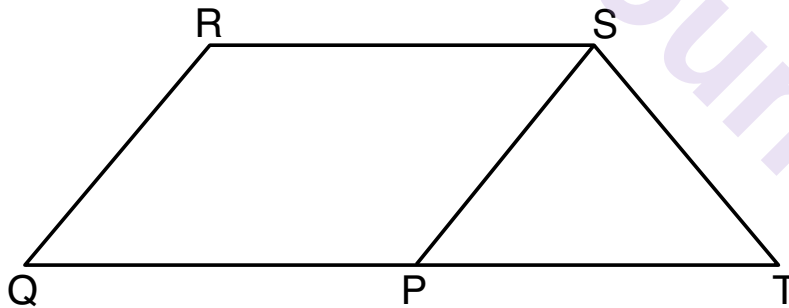
(1) (1,1)

(3) (2,−2)

(2) (−2,2)

(4) (4,0)

20 The equation of a circle is $x^2 + 8x + y^2 - 12y = 144$. What are the coordinates of the center and the length of the radius of the circle?

(1) center $(4,-6)$ and radius 12

(2) center $(-4,6)$ and radius 12

(3) center $(4,-6)$ and radius 14

(4) center $(-4,6)$ and radius 14

21 In parallelogram $PQRS$, $\overline{QP}$ is extended to point T and $\overline{ST}$ is drawn.

If $\overline{ST} \cong \overline{SP}$ and m$\angle R = 130°$, what is m$\angle PST$?
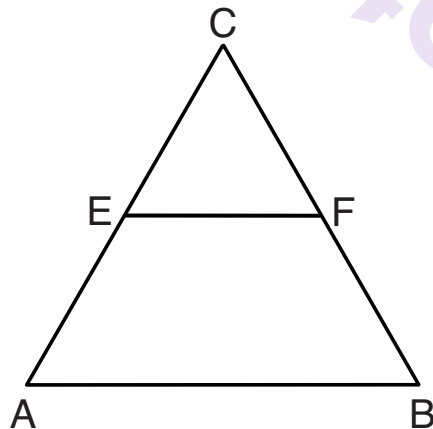
(1) 130°   (3) 65°

(2) 80°   (4) 50°

22 A 12-foot ladder leans against a building and reaches a window 10 feet above ground. What is the measure of the angle, to the *nearest degree*, that the ladder forms with the ground?

(1) 34   (3) 50

(2) 40   (4) 56

23 In the diagram of equilateral triangle *ABC* shown below, *E* and *F* are the midpoints of $\overline{AC}$ and $\overline{BC}$, respectively.

If $EF = 2x + 8$ and $AB = 7x - 2$, what is the perimeter of trapezoid *ABFE*?

(1) 36   (3) 100

(2) 60   (4) 120

24 Which information is *not* sufficient to prove that a parallelogram is a square?

(1) The diagonals are both congruent and perpendicular.

(2) The diagonals are congruent and one pair of adjacent sides are congruent.

(3) The diagonals are perpendicular and one pair of adjacent sides are congruent.

(4) The diagonals are perpendicular and one pair of adjacent sides are perpendicular.

GO RIGHT ON TO THE NEXT PAGE ➡

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Triangle $A'B'C'$ is the image of triangle ABC after a dilation with a scale factor of $\frac{1}{2}$ and centered at point A. Is triangle ABC congruent to triangle $A'B'C'$? Explain your answer.

Work space for question 25 is continued on the next page.
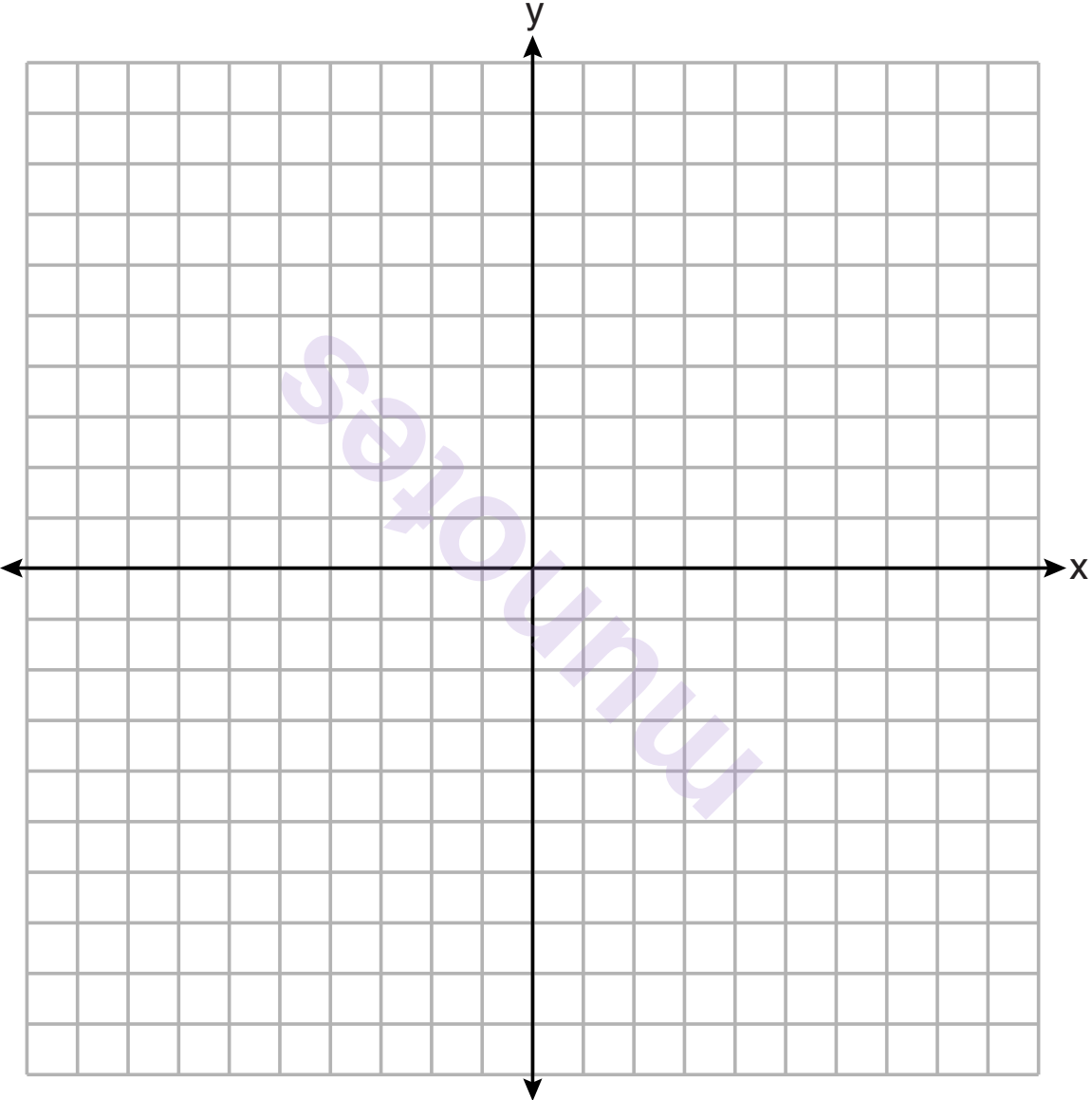
Question 25 continued

26 Determine and state the area of triangle PQR, whose vertices have coordinates $P(-2,-5)$, $Q(3,5)$, and $R(6,1)$.

[The use of the set of axes on the next page is optional.]

The set of axes for question 26 is on the next page.

Question 26 continued

27 A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a 68° angle with the ground. Find the length of the support wire to the *nearest foot*.
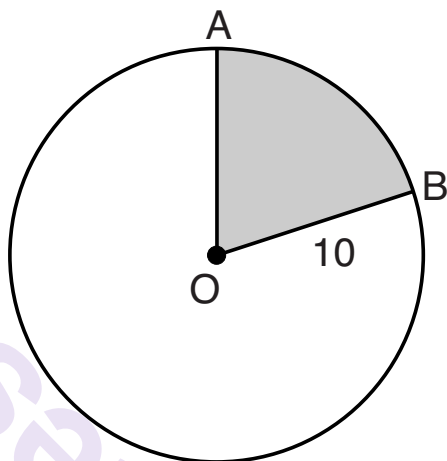
Work space for question 27 is continued on the next page.

Question 27 continued

28 In the diagram below, circle O has a radius of 10.
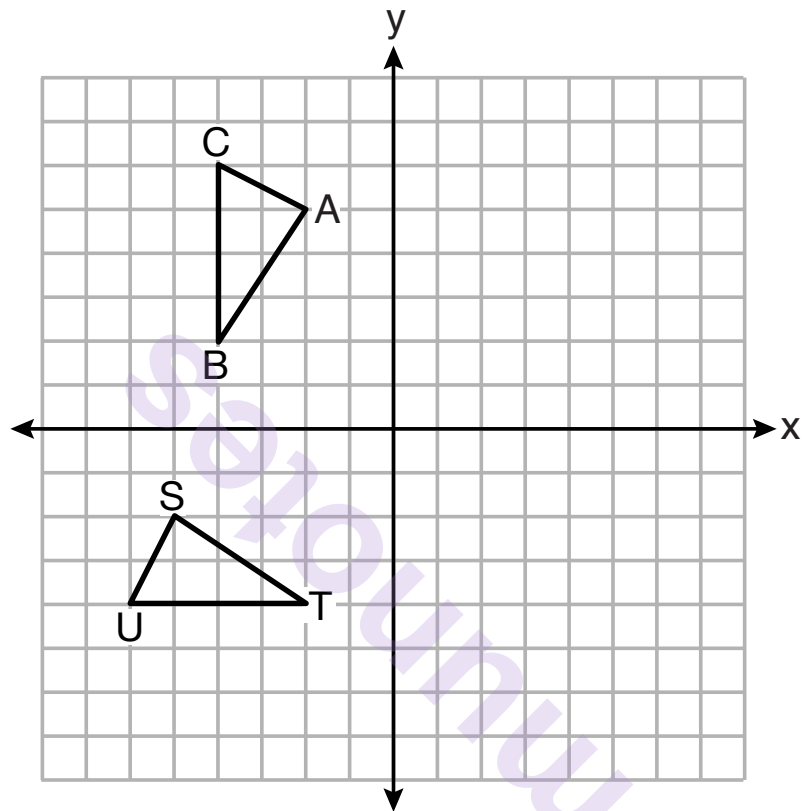

If $m\widehat{AB} = 72°$, find the area of shaded sector AOB, in terms of π.

Work space for question 28 is continued on the next page.

Question 28 continued

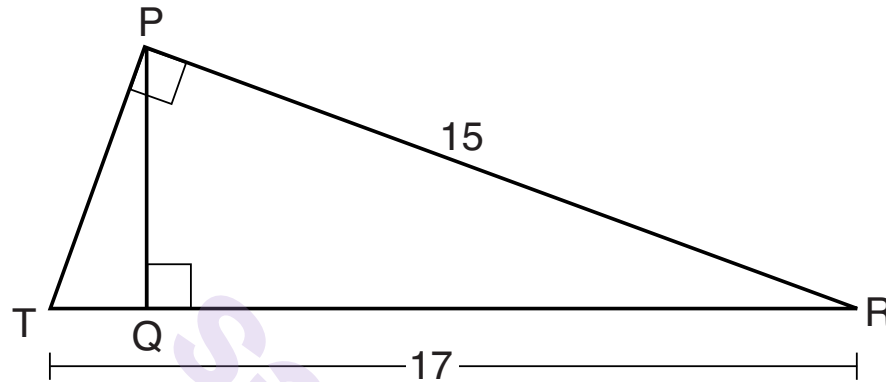**29** On the set of axes below, $\triangle ABC \cong \triangle STU$.

Question 29 continued

Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.

30 In right triangle *PRT*, m∠*P* = 90°, altitude $\overline{PQ}$ is drawn to hypotenuse $\overline{RT}$, *RT* = 17, and *PR* = 15.

Determine and state, to the *nearest tenth*, the length of $\overline{RQ}$.
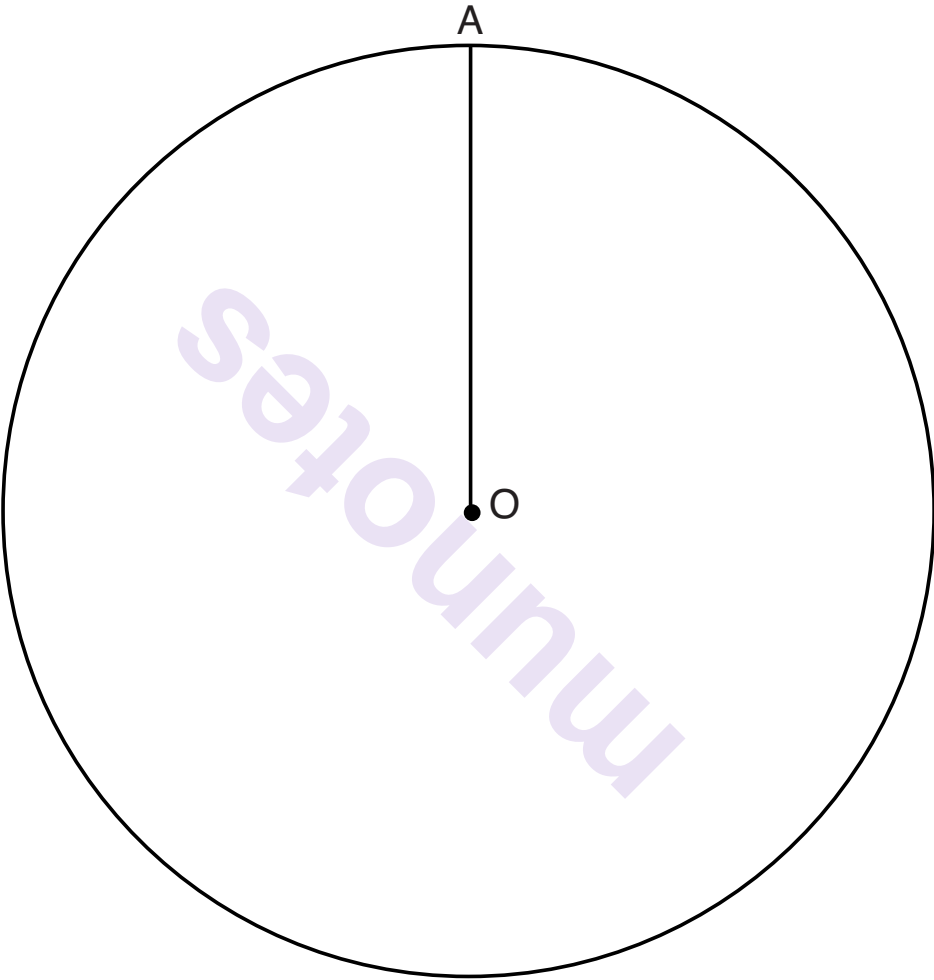
Question 30 continued

31 Given circle O with radius $\overline{OA}$, use a compass and straightedge to construct an equilateral triangle inscribed in circle O. [Leave all construction marks.]

A

O

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley's quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on page 37 is optional.]

Question 32 continued

Question 32 is continued on the next page.
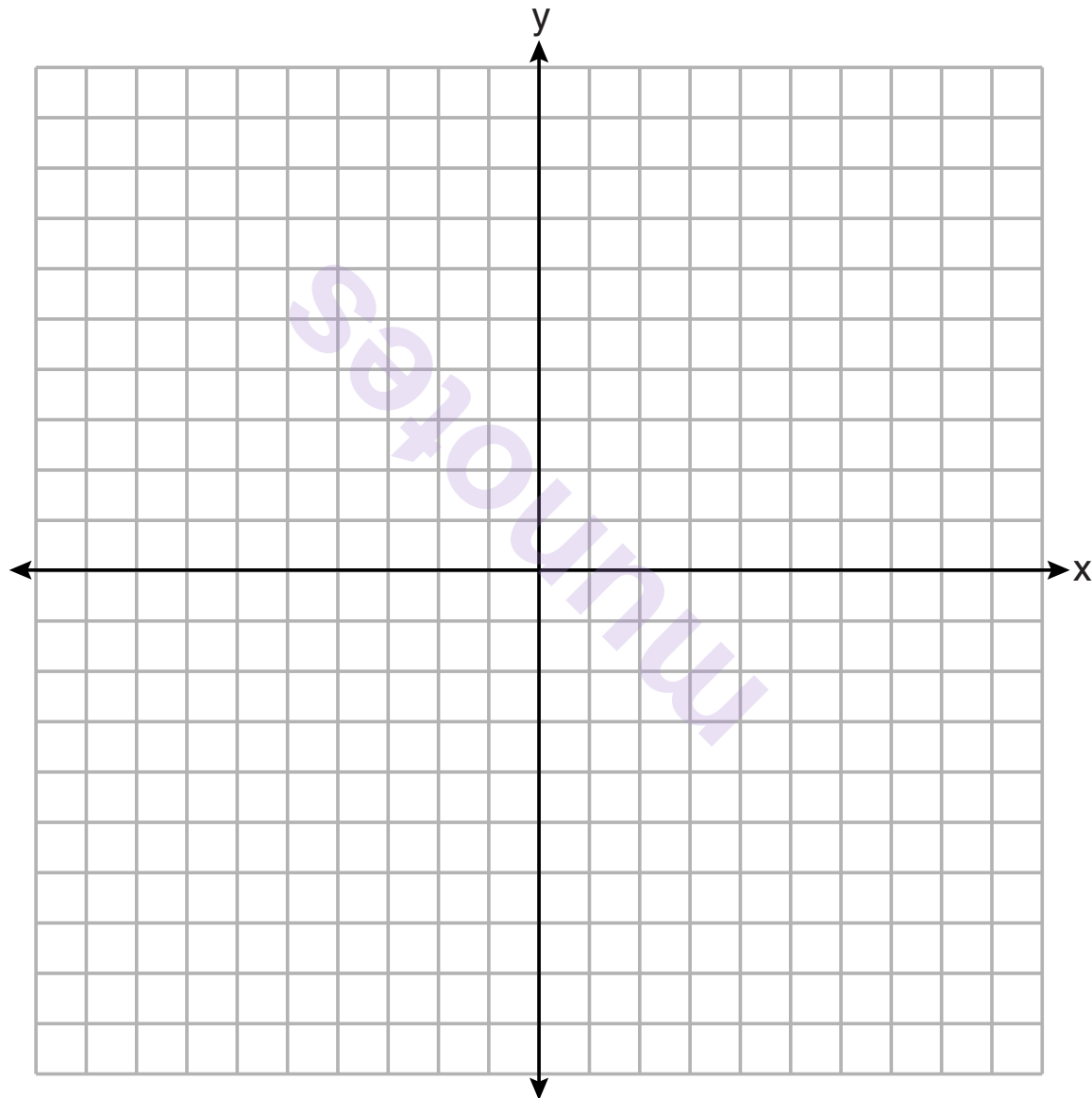
35

Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley's definition to prove that *ABCD* is *not* an isosceles trapezoid.

Question 32 continued

33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of 12 inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the *nearest cubic foot*.
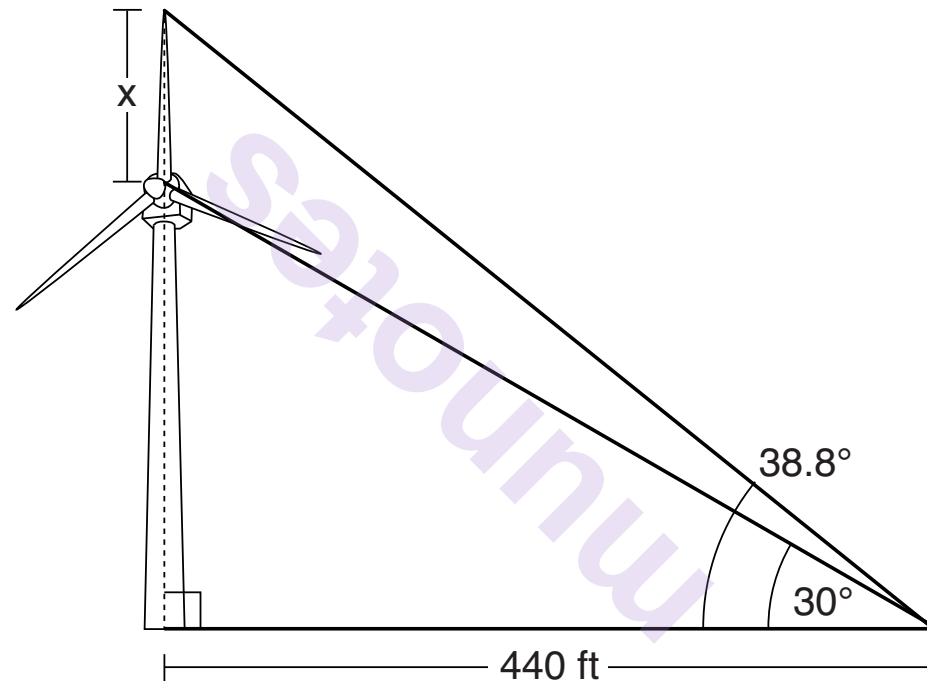
Question 33 continued

One cubic foot equals 7.48 gallons of water. Determine and state, to the *nearest gallon*, the number of gallons of water in the pool.

34 Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill's base. Using surveyor's tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°.

Determine and state a blade's length, *x*, to the *nearest foot*.

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for the question to determine your answer. Note that diagrams are not necessarily drawn to scale. For the question in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Quadrilateral $MATH$, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$
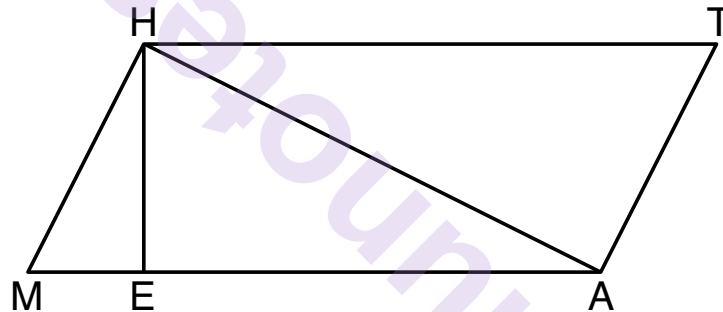
Prove: $TA \bullet HA = HE \bullet TH$

Work space for question 35 is continued on the next page.

Question 35 continued

High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$

The Reference Sheet is continued on the next page.

Cylinder	$V = \pi r^2 h$	Radians	$1 \text{ radian} = \dfrac{180}{\pi} \text{ degrees}$
Sphere	$V = \dfrac{4}{3}\pi r^3$	Degrees	$1 \text{ degree} = \dfrac{\pi}{180} \text{ radians}$
Cone	$V = \dfrac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$
Pyramid	$V = \dfrac{1}{3}Bh$		