The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, January 22, 2020 — 9:15 a.m. to 12:15 p.m., only

Student Name: _____

School Name: _____

> **The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

> **Notice…**
> **A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In the diagram below, $\overleftrightarrow{FAD} \parallel \overleftrightarrow{EHC}$, and $\overline{ABH}$ and $\overline{BC}$ are drawn.
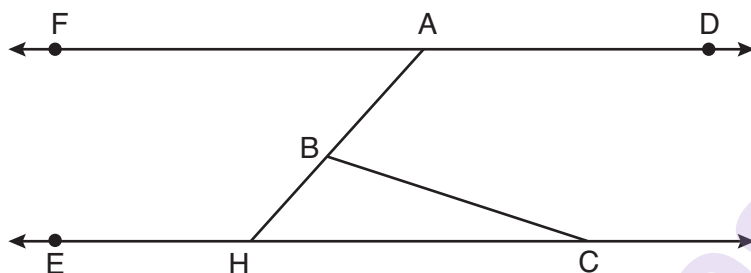
If m∠FAB = 48° and m∠ECB = 18°, what is m∠ABC?

(1) 18°   (3) 66°

(2) 48°   (4) 114°

2 A cone has a volume of 108π and a base diameter of 12. What is the height of the cone?

(1) 27   (3) 3

(2) 9   (4) 4

3 Triangle JGR is similar to triangle MST. Which statement is *not* always true?

(1) ∠J ≅ ∠M (3) ∠R ≅ ∠T

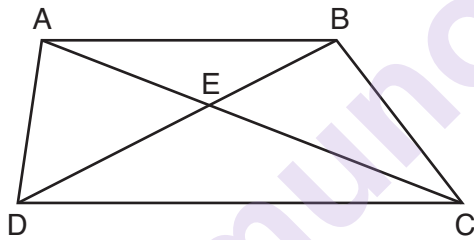(2) ∠G ≅ ∠T (4) ∠G ≅ ∠S

4 In parallelogram *ABCD*, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at *E*. Which statement proves *ABCD* is a rectangle?

(1) $\overline{AC} \cong \overline{BD}$ (3) $\overline{AC} \perp \overline{BD}$

(2) $\overline{AB} \perp \overline{BD}$ (4) $\overline{AC}$ bisects $\angle BCD$

5 The endpoints of directed line segment *PQ* have coordinates of $P(-7,-5)$ and $Q(5,3)$. What are the coordinates of point *A*, on $\overline{PQ}$, that divide $\overline{PQ}$ into a ratio of 1:3?

(1) $A(-1,-1)$ (3) $A(3,2)$

(2) $A(2,1)$ (4) $A(-4,-3)$

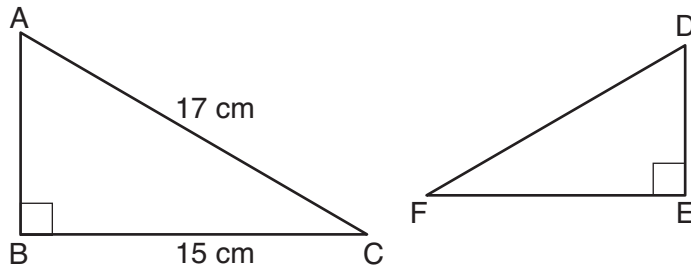**6** In trapezoid *ABCD* below, $\overline{AB} \parallel \overline{CD}$.

If $AE = 5.2$, $AC = 11.7$, and $CD = 10.5$, what is the length of $\overline{AB}$, to the *nearest tenth*?

(1) 4.7   (3) 8.4

(2) 6.5   (4) 13.1

7 Kayla was cutting right triangles from wood to use for an art project. Two of the right triangles she cut are shown below.


If $\triangle ABC \sim \triangle DEF$, with right angles B and E, $BC = 15$ cm, and $AC = 17$ cm, what is the measure of $\angle F$, to the *nearest degree*?

(1) 28°   (3) 62°

(2) 41°   (4) 88°

8 The line represented by $2y = x + 8$ is dilated by a scale factor of k centered at the origin, such that the image of the line has an equation of $y - \frac{1}{2}x = 2$. What is the scale factor?

(1) $k = \frac{1}{2}$ (3) $k = \frac{1}{4}$

(2) $k = 2$ (4) $k = 4$

9 In quadrilateral $ABCD$ below, $\overline{AB} \parallel \overline{CD}$, and E, H, and F are the midpoints of $\overline{AD}$, $\overline{AC}$, and $\overline{BC}$, respectively.
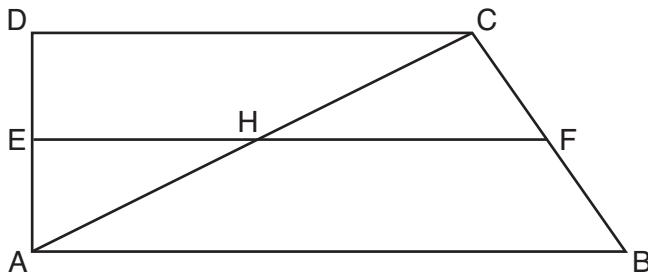
If $AB = 24$, $CD = 18$, and $AH = 10$, then FH is

(1) 9   (3) 12

(2) 10   (4) 21

10 Jaden is comparing two cones. The radius of the base of cone *A* is twice as large as the radius of the base of cone *B*. The height of cone *B* is twice the height of cone *A*. The volume of cone *A* is

(1) twice the volume of cone *B*

(2) four times the volume of cone *B*

(3) equal to the volume of cone *B*

(4) equal to half the volume of cone *B*

11 A regular hexagon is rotated about its center. Which degree measure will carry the regular hexagon onto itself?
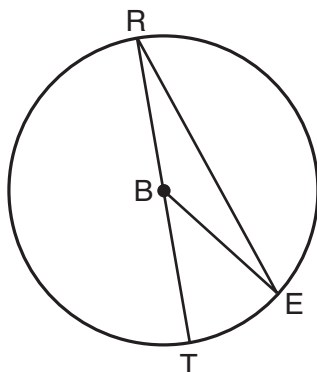
(1) 45° (3) 120°

(2) 90° (4) 135°

12 In triangle *MAH* below, $\overline{MT}$ is the perpendicular bisector of $\overline{AH}$.
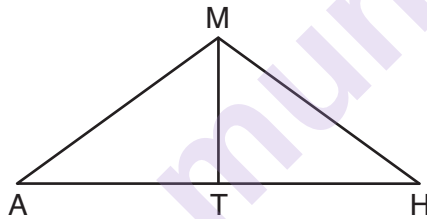
Which statement is *not* always true?

(1) △*MAH* is isosceles.

(2) △*MAT* is isosceles.

(3) $\overline{MT}$ bisects ∠*AMH*.

(4) ∠*A* and ∠*TMH* are complementary.

13 In circle B below, diameter $\overline{RT}$, radius $\overline{BE}$, and chord $\overline{RE}$ are drawn.

If m$\angle TRE = 15°$ and $BE = 9$, then the area of sector EBR is
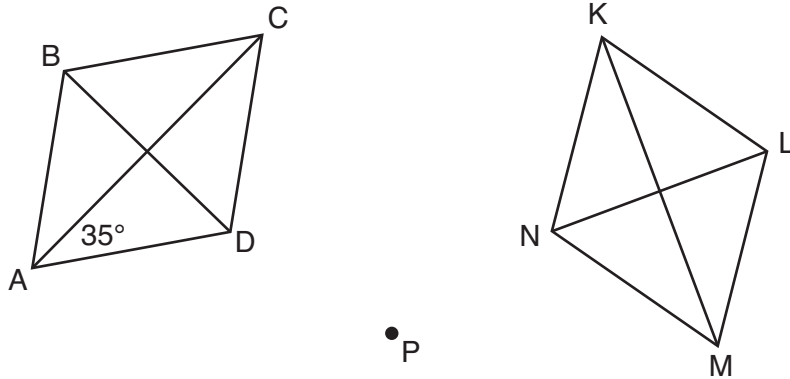
(1) 3.375π (3) 33.75π

(2) 6.75π (4) 37.125π

14 Lou has a solid clay brick in the shape of a rectangular prism with a length of 8 inches, a width of 3.5 inches, and a height of 2.25 inches. If the clay weighs 1.055 oz/in^3, how much does Lou's brick weigh, to the *nearest ounce*?
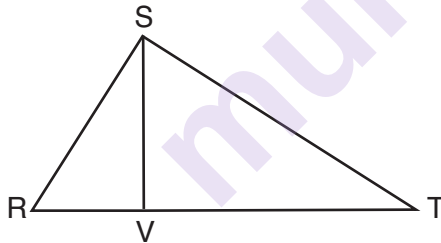
(1) 66   (3) 63

(2) 64   (4) 60

15 Rhombus *ABCD* can be mapped onto rhombus *KLMN* by a rotation about point *P*, as shown below.

What is the measure of $\angle KNM$ if the measure of $\angle CAD = 35°$?

(1) 35°   (3) 70°

(2) 55°   (4) 110°

16 In right triangle *RST* below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$.

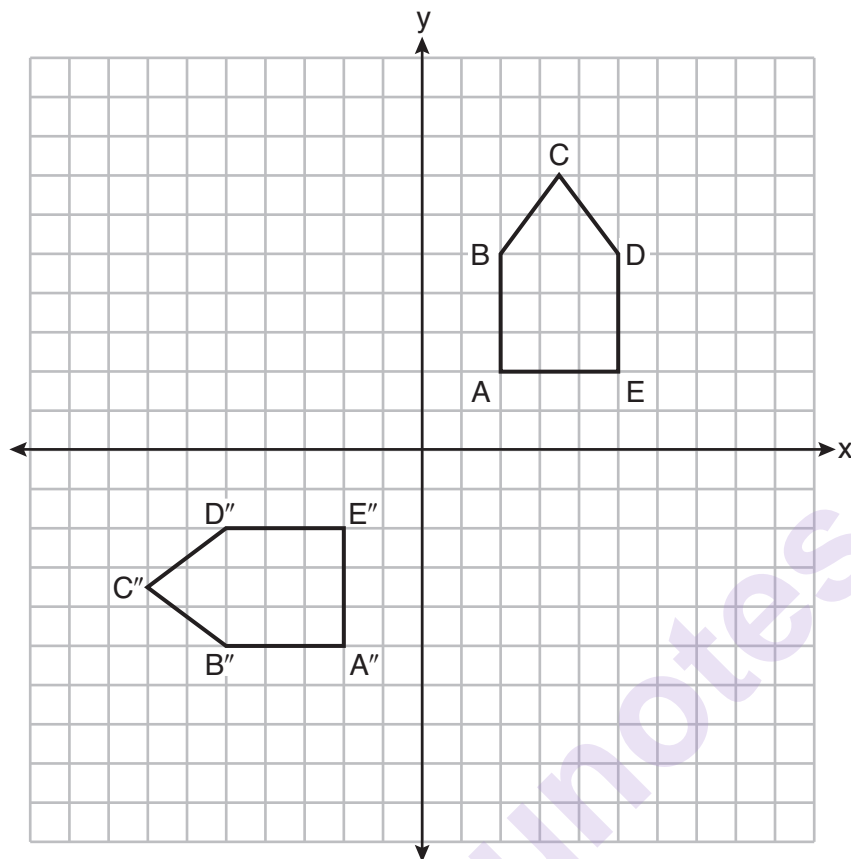If $RV = 4.1$ and $TV = 10.2$, what is the length of $\overline{ST}$, to the *nearest tenth*?

(1) 6.5   (3) 11.0

(2) 7.7   (4) 12.1

**17** On the set of axes below, pentagon *ABCDE* is congruent to *A″B″C″D″E″*.

Which describes a sequence of rigid motions that maps *ABCDE* onto *A″B″C″D″E″*?

(1) a rotation of 90° counterclockwise about the origin followed by a reflection over the *x*-axis

(2) a rotation of 90° counterclockwise about the origin followed by a translation down 7 units

(3) a reflection over the *y*-axis followed by a reflection over the *x*-axis

(4) a reflection over the *x*-axis followed by a rotation of 90° counterclockwise about the origin

18 On the set of axes below, rhombus *ABCD* has vertices whose coordinates are *A*(1,2), *B*(4,6), *C*(7,2), and *D*(4,–2).
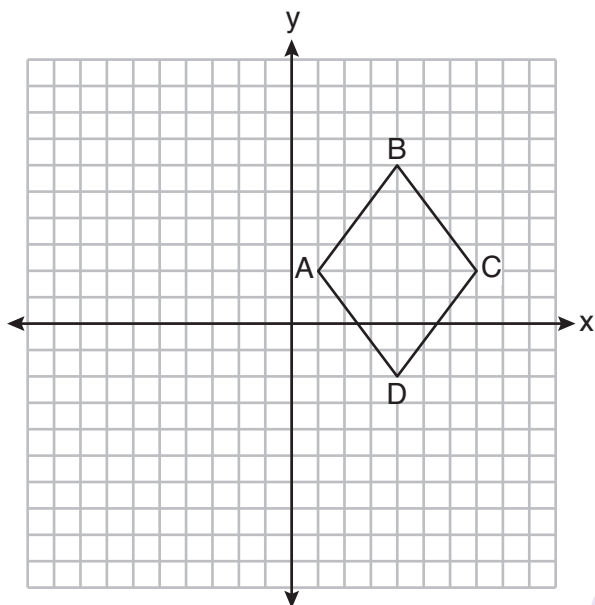
What is the area of rhombus *ABCD*?

(1) 20

(3) 25

(2) 24

(4) 48

19 Which figure(s) below can have a triangle as a two-dimensional cross section?

  I. cone

  II. cylinder

  III. cube

  IV. square pyramid

(1) I, only

(2) IV, only

(3) I, II, and IV, only

(4) I, III, and IV, only

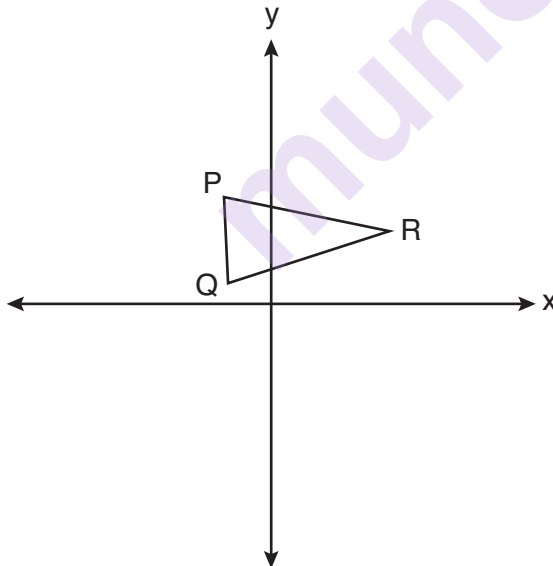**20** What is an equation of a circle whose center is at $(2,-4)$ and is tangent to the line $x = -2$?

(1) $(x - 2)^2 + (y + 4)^2 = 4$

(2) $(x - 2)^2 + (y + 4)^2 = 16$

(3) $(x + 2)^2 + (y - 4)^2 = 4$

(4) $(x + 2)^2 + (y - 4)^2 = 16$

21 For the acute angles in a right triangle, $\sin (4x)° = \cos (3x + 13)°$. What is the number of degrees in the measure of the *smaller* angle?

(1) $11°$ (3) $44°$

(2) $13°$ (4) $52°$

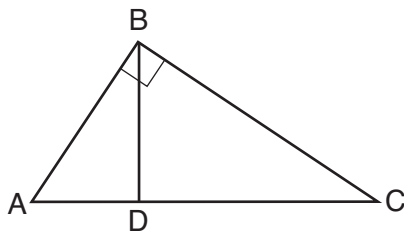**22** Triangle *PQR* is shown on the set of axes below.

Which quadrant will contain point R'', the image of point R, after a 90° clockwise rotation centered at $(0,0)$ followed by a reflection over the *x*-axis?

(1) I   (3) III

(2) II   (4) IV

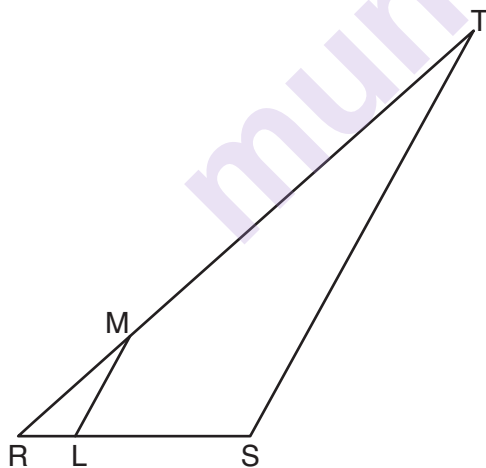**23** In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn.

Which ratio is always equivalent to cos A?

(1) $\dfrac{AB}{BC}$ (3) $\dfrac{BD}{AB}$

(2) $\dfrac{BD}{BC}$ (4) $\dfrac{BC}{AC}$

24 In the diagram below of $\triangle RST$, L is a point on $\overline{RS}$, and M is a point on $\overline{RT}$, such that $\overline{LM} \parallel \overline{ST}$.


If $RL = 2$, $LS = 6$, $LM = 4$, and $ST = x + 2$, what is the length of $\overline{ST}$?

(1) 10   (3) 14

(2) 12   (4) 16

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 In the diagram below, right triangle *PQR* is transformed by a sequence of rigid motions that maps it onto right triangle *NML*.
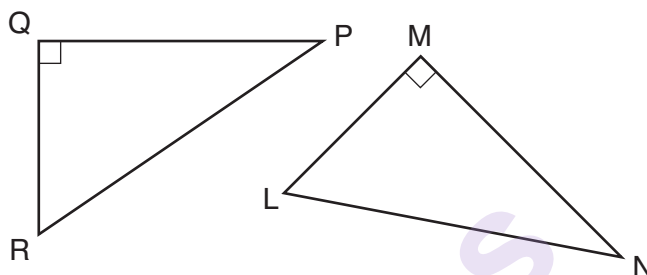
Write a set of three congruency statements that would show *ASA* congruency for these triangles.

26 Diego needs to install a support beam to hold up his new birdhouse, as modeled below. The base of the birdhouse is $24\frac{1}{2}$ inches long. The support beam will form an angle of 38° with the vertical post. Determine and state the approximate length of the support beam, x, to the *nearest inch*.
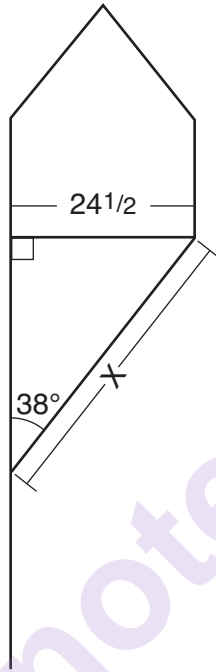
27 A rectangular tabletop will be made of maple wood that weighs 43 pounds per cubic foot. The tabletop will have a length of eight feet, a width of three feet, and a thickness of one inch. Determine and state the weight of the tabletop, in pounds.

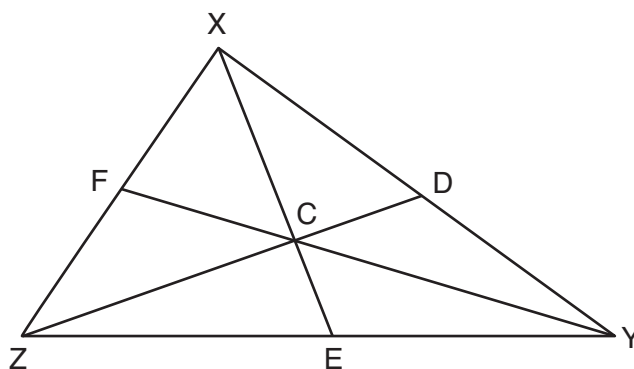**28** In the diagram below of circle O, secant $\overline{ABC}$ and tangent $\overrightarrow{AD}$ are drawn.
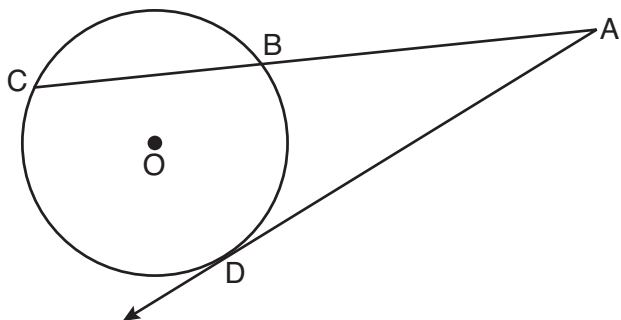
If $CA = 12.5$ and $CB = 4.5$, determine and state the length of $\overline{DA}$.

29 Given $\overline{MT}$ below, use a compass and straightedge to construct a 45° angle whose vertex is at point *M*.
[Leave all construction marks.]

M _____ T

30 In △*XYZ* shown below, medians $\overline{XE}$, $\overline{YF}$, and $\overline{ZD}$ intersect at *C*.

If *CE* = 5, *YF* = 21, and *XZ* = 15, determine and state the perimeter of triangle *CFX*.

31 Determine and state an equation of the line perpendicular to the line $5x - 4y = 10$ and passing through the point $(5,12)$.

**Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [12]

32 Quadrilateral *NATS* has coordinates $N(-4,-3)$, $A(1,2)$, $T(8,1)$, and $S(3,-4)$.

Prove quadrilateral *NATS* is a rhombus.
[The use of the set of axes below is optional.]
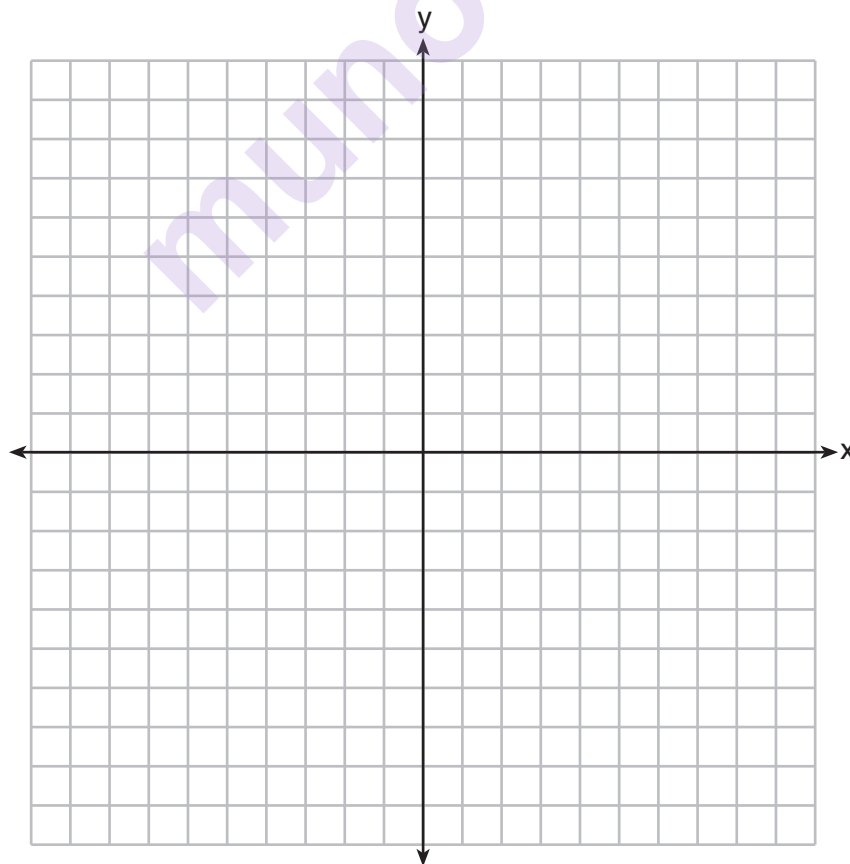
33 David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground.
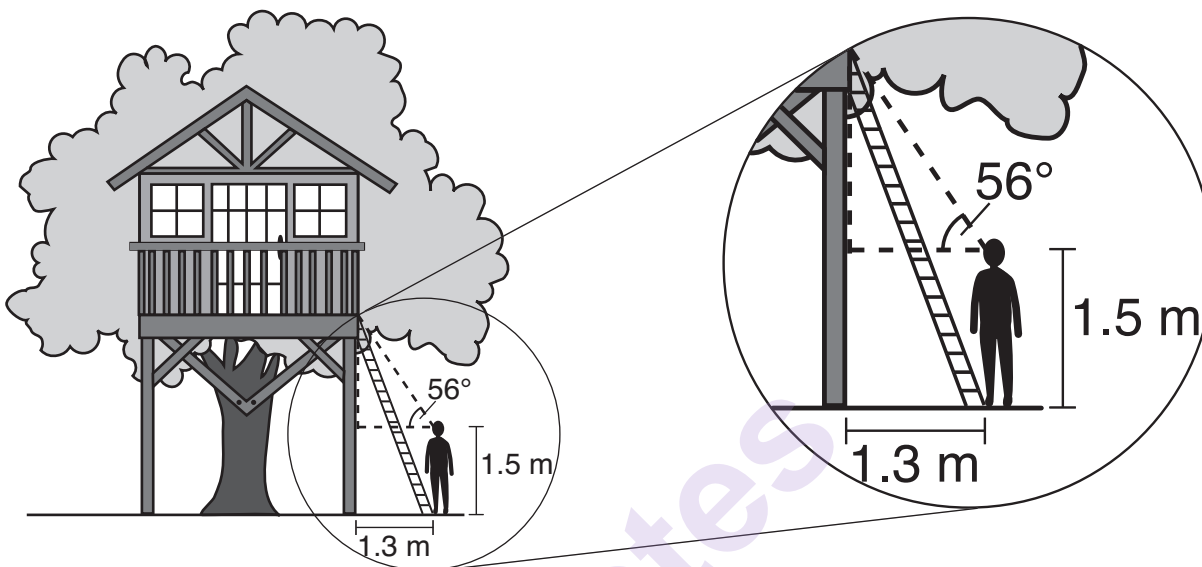
Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

34 A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds.
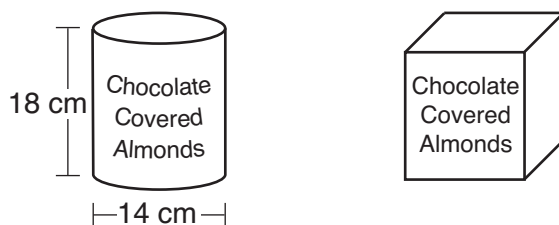

If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds.

A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for the question to determine your answer. Note that diagrams are not necessarily drawn to scale. For the question in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 In quadrilateral $ABCD$, E and F are points on $\overline{BC}$ and $\overline{AD}$, respectively, and $\overline{BGD}$ and $\overline{EGF}$ are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.
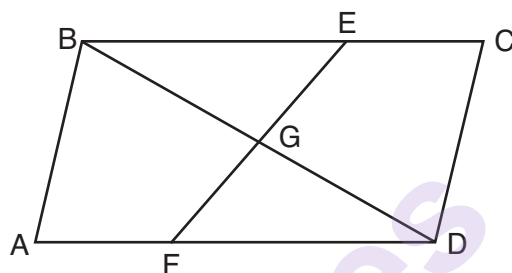
Prove: $\overline{FG} \cong \overline{EG}$
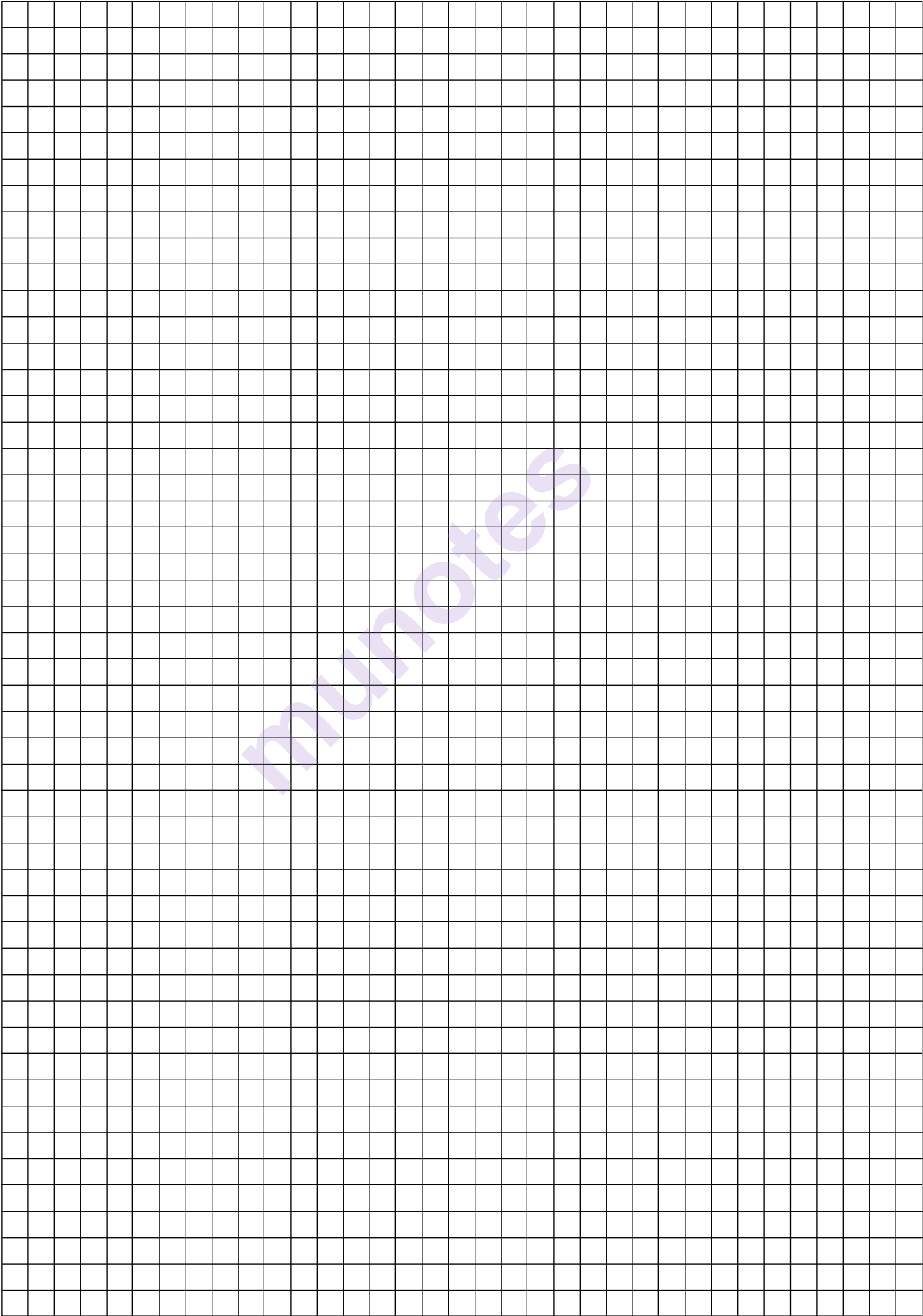
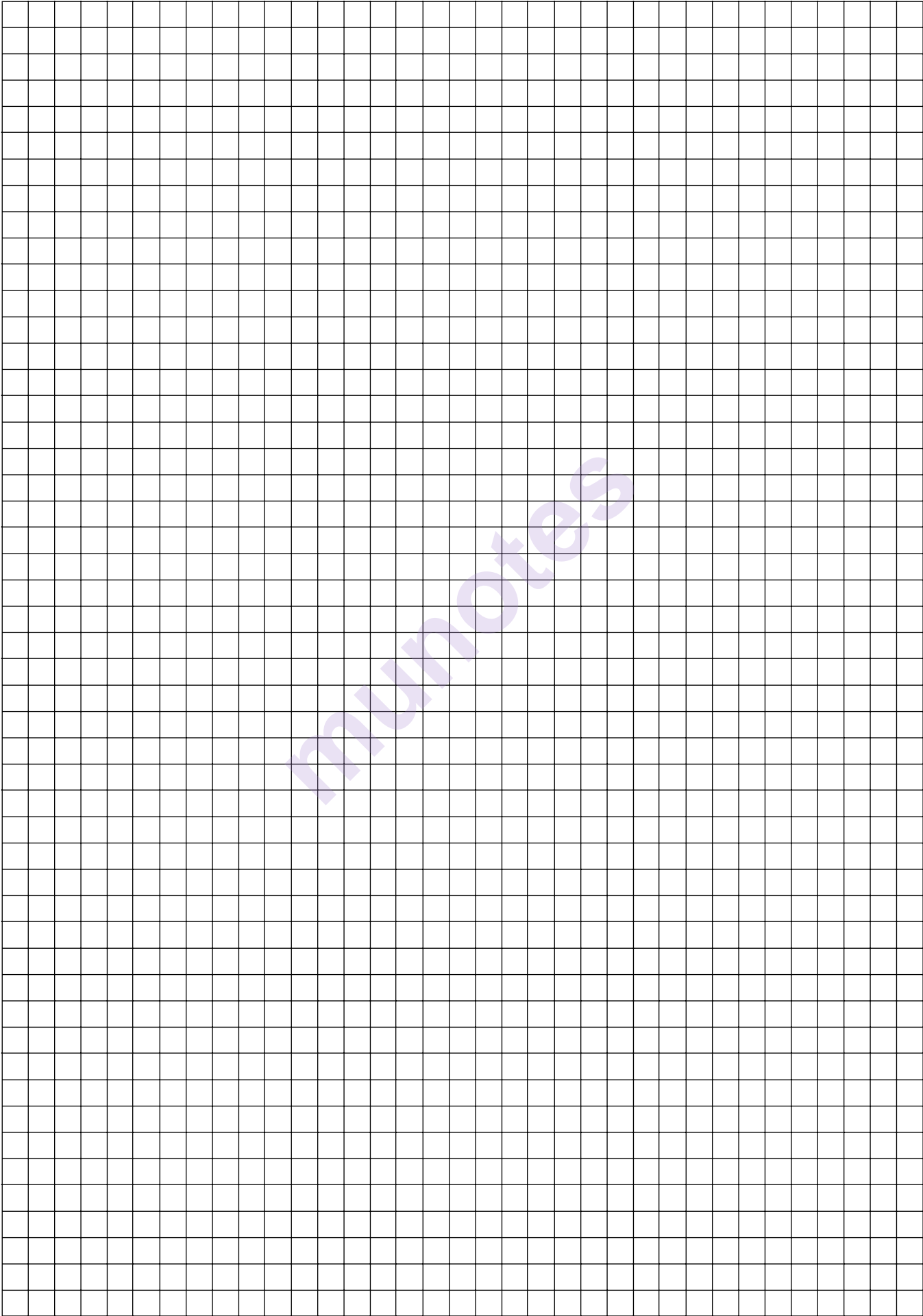**Work space for question 35 is continued on the next page.**

Question 35 continued

High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$	Pythagorean Theorem	$a^2 + b^2 = c^2$
Parallelogram	$A = bh$	Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle	$A = \pi r^2$	Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Circle	$C = \pi d$ or $C = 2\pi r$	Geometric Sequence	$a_n = a_1 r^{n-1}$
General Prisms	$V = Bh$	Geometric Series	$S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Cylinder	$V = \pi r^2 h$	Radians	1 radian = $\dfrac{180}{\pi}$ degrees
Sphere	$V = \frac{4}{3}\pi r^3$	Degrees	1 degree = $\dfrac{\pi}{180}$ radians
Cone	$V = \frac{1}{3}\pi r^2 h$	Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$
Pyramid	$V = \frac{1}{3}Bh$		