ALGEBRA

# II

**The University of the State of New York**
**REGENTS HIGH SCHOOL EXAMINATION**

# ALGEBRA II

**Wednesday,** August 16, 2023 — 12:30 to 3:30 p.m., only

Student Name _____

School Name _____

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**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

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Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

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**Notice …**

**A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

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**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

ALGEBRA II

**Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet.** [48]

**Use this space for computations.**

**1** A group of high school students wanted to collect information on how many times per week students exercised. If they want the *least* biased results they should survey every fifth student at the school who is

(1) entering the gym         (3) entering the library

(2) in the junior class       (4) entering the building

**2** Given $x \neq -3$, which expression is equivalent to $\dfrac{2x^3 + 3x^2 - 4x + 5}{x + 3}$?

(1) $2x^3 + 9x^2 + 23x + 74$      (3) $2x^3 - 3x^2 + 5x - 10$

(2) $2x^2 - 3x + 5 - \dfrac{10}{x + 3}$      (4) $2x^2 + 9x + 23 + \dfrac{74}{x + 3}$

**3** The table below shows the food preferences of sports fans whose favorite sport is football or baseball.

**Favorite Food to Eat While Watching Sports**

|          | Wings | Pizza | Hot Dogs |
|----------|-------|-------|----------|
| Football | 14    | 20    | 6        |
| Baseball | 6     | 12    | 42       |

The probability that a fan prefers pizza given that the fan prefers football is

(1) $\dfrac{1}{2}$                (3) $\dfrac{5}{8}$

(2) $\dfrac{1}{5}$                (4) $\dfrac{13}{25}$

**4** If $f(x) = 12x - 4$, then the inverse function $f^{-1}(x)$ is
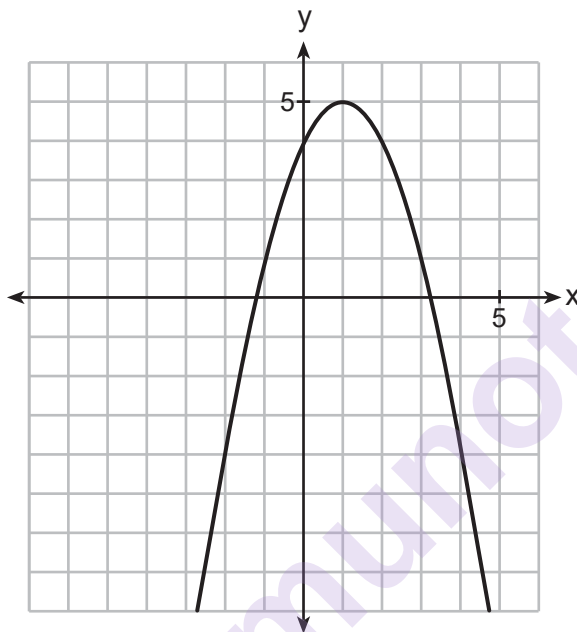
(1) $f^{-1}(x) = \dfrac{x+1}{3}$

(3) $f^{-1}(x) = \dfrac{x+4}{12}$

(2) $f^{-1}(x) = \dfrac{x}{3} + 1$

(4) $f^{-1}(x) = \dfrac{x}{12} + 4$

**5** The graph of a quadratic function is shown below.



When the graph of $x + y = 4$ is drawn on the same axes, one solution to this system is

(1) $(4,0)$

(3) $(2,2)$

(2) $(1,5)$

(4) $(3,1)$

**6** What is the solution of $2(3^{x+4}) = 56$?

(1) $x = \log_3(28) - 4$

(3) $x = \log(25) - 4$

(2) $x = -1$

(4) $x = \dfrac{\log(56)}{\log(6)} - 4$

**7** In a survey of people who recently bought a laptop, 45% said they were looking for a large screen, 31% said they were looking for a fast processor, and 58% said they wanted a large screen or a fast processor. If a survey respondent is selected at random, what is the probability that the respondent wanted both a large screen and a fast processor?

(1) 76%                         (3) 77%

(2) 14%                         (4) 18%

**8** In the quadratic formula, $b^2 - 4ac$ is called the discriminant. The function $f(x)$ has a discriminant value of 8, and $g(x)$ has a discriminant value of $-16$. The quadratic graphs, $h(x)$ and $j(x)$, are shown below.
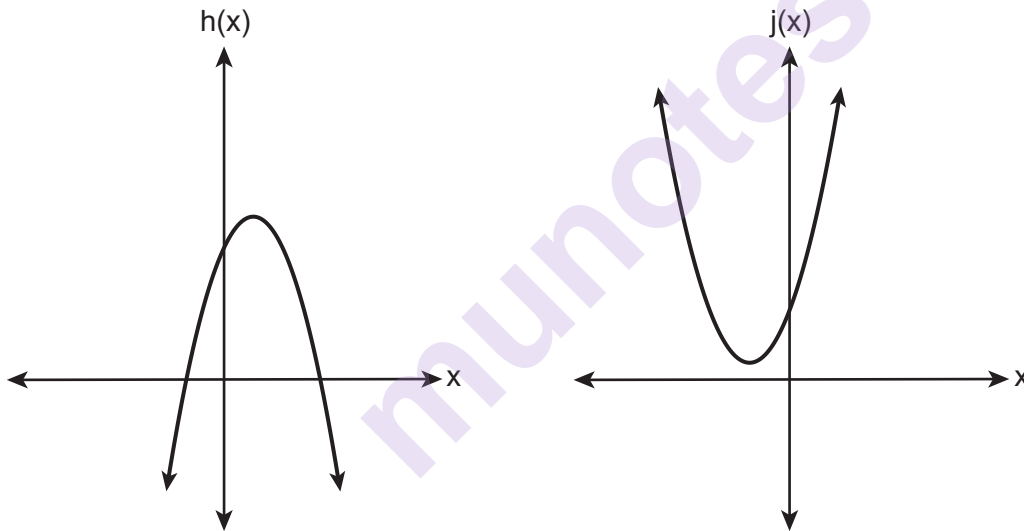


Which quadratic functions have imaginary roots?

(1) $g(x)$ and $h(x)$              (3) $f(x)$ and $h(x)$

(2) $g(x)$ and $j(x)$              (4) $f(x)$ and $j(x)$

**9** The element Americium has a half-life of 25 minutes. Given an initial amount, $A_0$, which expression could be used to determine the amount of Americium remaining after $t$ minutes?

(1) $A_0\left(\dfrac{1}{2}\right)^{\frac{t}{25}}$

(3) $25\left(\dfrac{1}{2}\right)^{t}$

(2) $A_0(25)^{\frac{t}{2}}$

(4) $A_0\left(\dfrac{1}{2}\right)^{25t}$

**10** Which function has the greatest $y$-intercept?

(1) $f(x) = 4\sin(2x)$

(3) $h(x) = 5e^{2x} + 3$

(2) $g(x) = 3x^4 + 2x^3 + 7$

(4) $j(x) = 6\log_2(3x + 4)$

**11** According to the USGS, an agency within the Department of Interior of the United States, the frog population in the U.S. is decreasing at the rate of 3.79% per year. A student created a model, $P = 12{,}150(0.962)^t$, to estimate the population in a pond after $t$ years. The student then created a model that would predict the population after $d$ decades. This model is best represented by

(1) $P = 12{,}150(0.461)^d$

(3) $P = 12{,}150(0.996)^d$

(2) $P = 12{,}150(0.679)^d$

(4) $P = 12{,}150(0.998)^d$

**12** What is the value of $\tan\theta$ when $\sin\theta = \dfrac{2}{5}$ and $\theta$ is in quadrant II?

(1) $\dfrac{-\sqrt{21}}{5}$

(3) $\dfrac{-2}{\sqrt{21}}$

(2) $\dfrac{-\sqrt{21}}{2}$

(4) $\dfrac{2}{\sqrt{21}}$

13 A population is normally distributed with a mean of 23 and a standard deviation of 1.2. The percentage of the population that falls below 21, to the *nearest hundredth*, is

(1) 0.05                                    (3) 8.29

(2) 4.78                                    (4) 91.30

14 Audra is interested in studying the number of students entering kindergarten in the Ahlville Central School District over the next several years. Using data dating back to 2015, she determines that the number of kindergarteners is decreasing at an exponential rate. She creates a formula to model this situation $y = a(b)^x$, where $x$ is the number of years since 2015 and $y$ is the number of students entering kindergarten. If there were 105 students entering kindergarten in Ahlville in 2015, which statement about Audra's formula is true?

(1) $a$ is positive and $b$ is negative.

(2) $a$ is negative and $b$ is positive.

(3) Both $a$ and $b$ are positive.

(4) Both $a$ and $b$ are negative.

15 The solution set for the equation $\sqrt{3(x + 6)} = x$ is

(1) {6, –3}                                  (3) {6}

(2) {–6, 3}                                  (4) {–3}

**16** The George family would like to borrow $45,000 to purchase a new boat. They qualified for a loan with an annual interest rate of 6.75%. The monthly loan payment can be found using the formula below.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

$M$ = monthly payment
$P$ = amount borrowed
$r$ = annual interest rate
$n$ = number of monthly payments

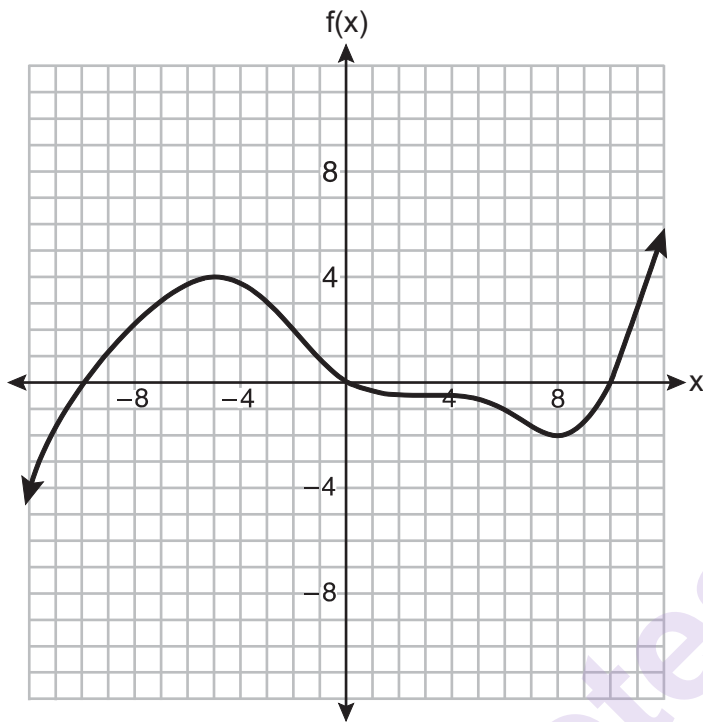What is the monthly payment if they would like to pay off the loan in five years?

(1) $262.99            (3) $915.24

(2) $252.13            (4) $885.76

**17** A retailer advertises that items will be discounted by 10% every Monday until they are sold. In how many weeks will an item costing $50 first be sold for under half price?

(1) 7                  (3) 5

(2) 6                  (4) 4

**18** The graph of the function $f(x)$ is shown below.



In which interval is $f(x)$ always positive?

(1) $(-2,4)$          (3) $(-12,-5)$

(2) $(0,10)$          (4) $(-10,0)$

**19** If $f(x) = (x^2 + 3x + 2)(x^2 - 4x + 3)$ and $g(x) = x^2 - 9$, then how many real solutions are there to the equation $f(x) = g(x)$?

(1) 1          (3) 6

(2) 2          (4) 4

**20** Which expression is a factor of $x^4 - x^3 - 11x^2 + 5x + 30$?

(1) $x + 2$                      (3) $x + 5$

(2) $x - 2$                      (4) $x - 5$

**21** The expression $\dfrac{x^2 + 6}{x^2 + 4}$ is equivalent to

(1) $\dfrac{6}{4}$                      (3) $1 - \dfrac{2}{x^2 + 4}$

(2) $1 + \dfrac{10}{x^2 + 4}$             (4) $1 + \dfrac{2}{x^2 + 4}$

**22** Stone Manufacturing has developed a cost model, $C(x) = 0.18x^3 + 0.02x^2 + 4x + 180$, where $x$ is the number of sprockets sold, in thousands. The sale price can be modeled by $S(x) = 95.4 - 6x$ and the company's revenue by $R(x) = x \bullet S(x)$. The company profits, $R(x) - C(x)$, could be modeled by

(1) $0.18x^3 + 6.02x^2 + 91.4x + 180$

(2) $0.18x^3 - 5.98x^2 - 91.4x + 180$

(3) $-0.18x^3 - 6.02x^2 + 91.4x - 180$

(4) $0.18x^3 + 5.98x^2 + 99.4x + 180$

**23** Which function is even?
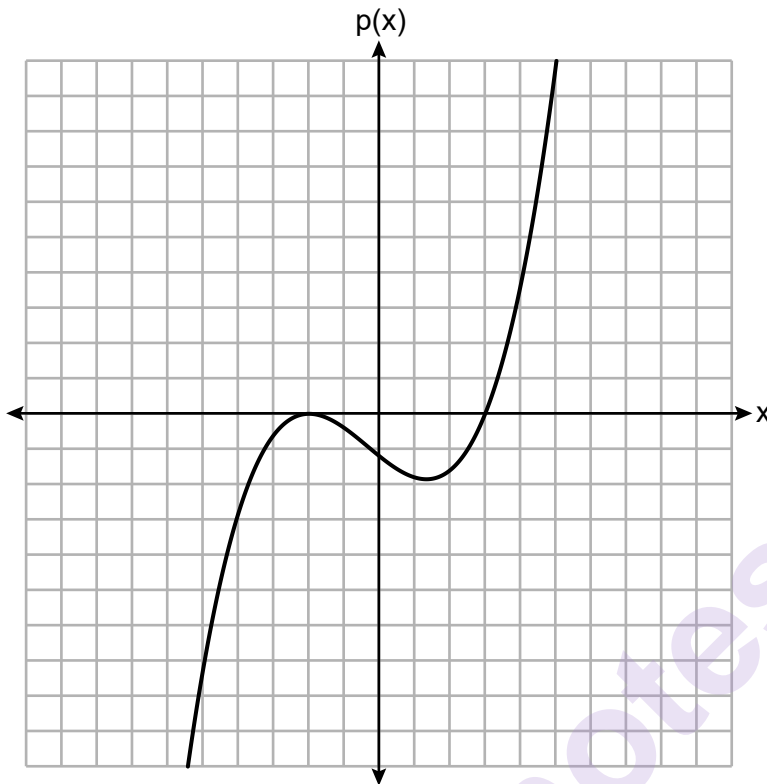
(1) $f(x) = x^3 + 2$             (3) $f(x) = |x + 2|$

(2) $f(x) = x^2 + 1$             (4) $f(x) = \sin(2x)$

**24** The graph of a cubic polynomial function $p(x)$ is shown below.



p(x)

If $p(x)$ is written as a product of linear factors, which factor would appear twice?

(1) $x - 2$                      (3) $x - 3$

(2) $x + 2$                      (4) $x + 3$

**Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [16]

**25** Factor the expression $2x^3 - 3x^2 - 18x + 27$ completely.

**26** Algebraically determine the values of $x$ that satisfy the system of equations shown below:
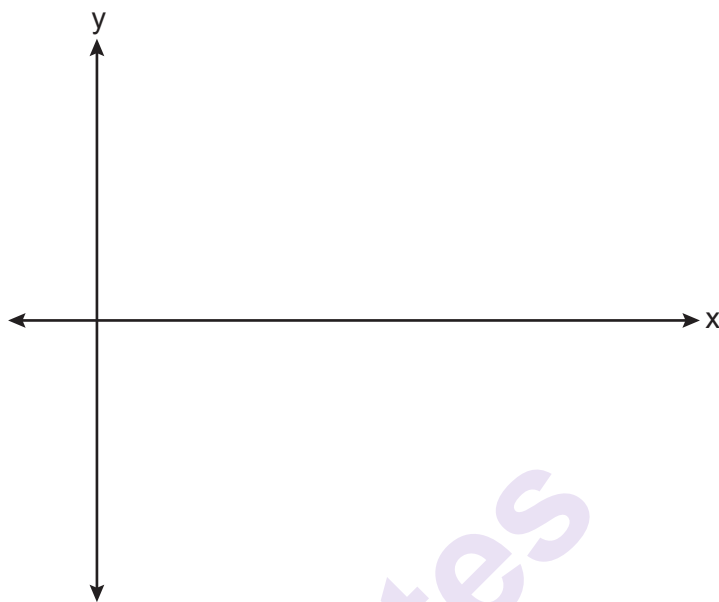
$$y = x^2 + 8x - 5$$
$$y = 8x - 4$$

**27** Solve the equation $3x^2 + 5x + 8 = 0$. Write your solution in $a + bi$ form.

**28** On the coordinate plane below, sketch *at least one cycle* of a cosine function with a midline at $y = -2$, an amplitude of 3, and a period of $\frac{\pi}{2}$.

**29** Given $i$ is the imaginary unit, simplify $(5xi^3 - 4i)^2$ as a polynomial in standard form.

**30** Consider the parabola given by $y = \dfrac{1}{4}x^2 + x + 8$ with vertex $(-2,7)$ and focus $(-2,8)$. Use this information to explain how to determine the equation of the directrix.

**31** Write $\dfrac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$ as a single term in simplest form, with a rational exponent.

**32** A fruit fly population can be modeled by the equation $P = 10(1.27)^t$, where $P$ represents the number of fruit flies after $t$ days. What is the average rate of change of the population, rounded to the *nearest hundredth*, over the interval [0,10.5]? Include appropriate units in your answer.

**Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [16]

**33** Sketch $p(x) = -\log_2(x + 3) + 2$ on the axes below.
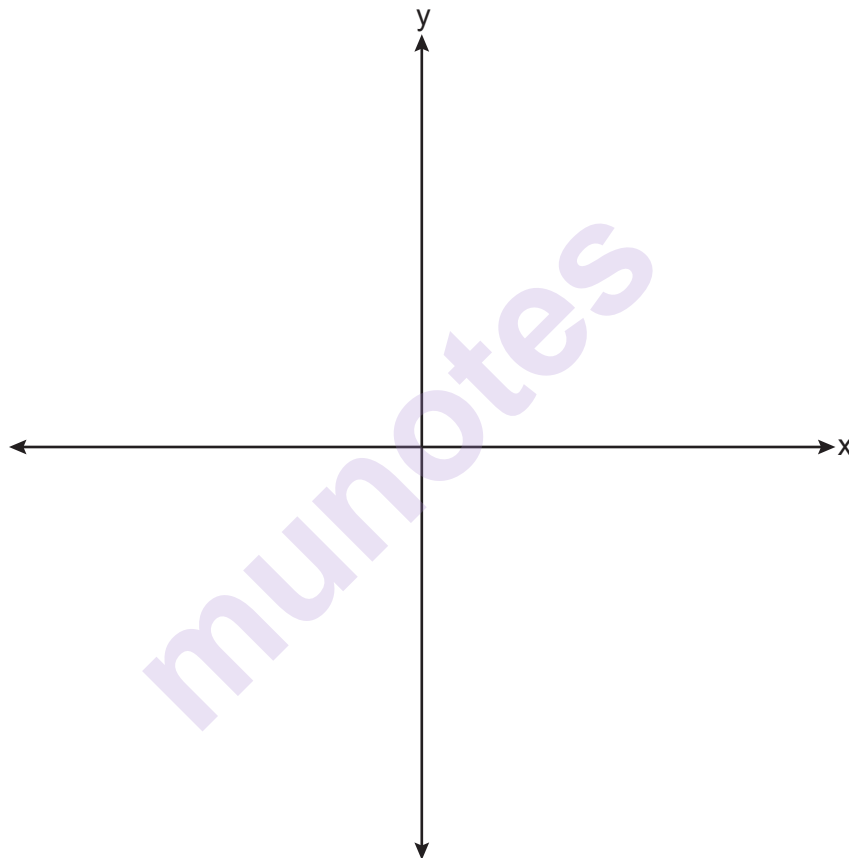
Describe the end behavior of $p(x)$ as $x \rightarrow -3$.

Describe the end behavior of $p(x)$ as $x \rightarrow \infty$.

**34** Solve for $x$ algebraically: $\dfrac{1}{x-6} + \dfrac{x}{x-2} = \dfrac{4}{x^2 - 8x + 12}$

**35** Solve the following system of equations algebraically for $x$, $y$, and $z$.

$$2x + 4y - 3z = 12$$
$$3x - 2y + 2z = -9$$
$$-x + y - 3z = 0$$

**36** Two classes of students were entered into an experiment to see whether using an interactive whiteboard leads to better grades. It was observed that the mean grade of students in the class with the interactive whiteboard was 0.6 points higher than the class without it. To determine if the observed difference is statistically significant, the classes were rerandomized 5000 times to study these random differences in the mean grades. The output of the simulation is summarized in the histogram below.
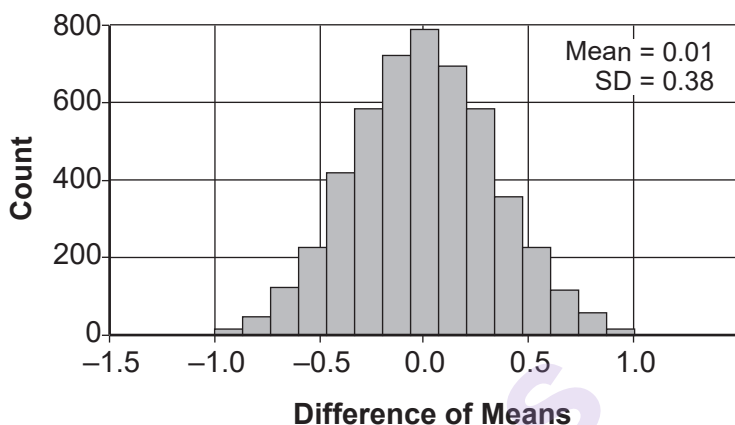


Determine an interval containing the middle 95% of the simulation results. Round your answer to the *nearest hundredth*.

Does the interval indicate that the difference between the classes' grades is significant? Explain.

**Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [6]

**37** The Manford family started savings accounts for their twins, Abby and Brett, on the day they were born. They invested $8000 in an account for each child. Abby's account pays 4.2% annual interest compounded quarterly. Brett's account pays 3.9% annual interest compounded continuously.

Write a function, $A(t)$, for Abby's account and a function, $B(t)$, for Brett's account that calculates the value of each account after $t$ years.

Determine who will have more money in their account when the twins turn 18 years old, and find the difference in the amounts in the accounts to the *nearest cent*.

**Question 37 is continued on the next page.**

Algebraically determine, to the *nearest tenth of a year*, how long it takes for Brett's account to triple in value.

# High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

| Triangle | $A = \frac{1}{2}bh$ |
|---|---|
| Parallelogram | $A = bh$ |
| Circle | $A = \pi r^2$ |
| Circle | $C = \pi d$ or $C = 2\pi r$ |
| General Prisms | $V = Bh$ |
| Cylinder | $V = \pi r^2 h$ |
| Sphere | $V = \frac{4}{3}\pi r^3$ |
| Cone | $V = \frac{1}{3}\pi r^2 h$ |
| Pyramid | $V = \frac{1}{3}Bh$ |

| Pythagorean Theorem | $a^2 + b^2 = c^2$ |
|---|---|
| Quadratic Formula | $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ |
| Arithmetic Sequence | $a_n = a_1 + (n - 1)d$ |
| Geometric Sequence | $a_n = a_1 r^{n-1}$ |
| Geometric Series | $S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$ |
| Radians | 1 radian = $\dfrac{180}{\pi}$ degrees |
| Degrees | 1 degree = $\dfrac{\pi}{180}$ radians |
| Exponential Growth/Decay | $A = A_0 e^{k(t - t_0)} + B_0$ |