ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, August 14, 2019 — 12:30 to 3:30 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 When the expression $(x + 2)^2 + 4(x + 2) + 3$ is rewritten as the product of two binomials, the result is

(1) $(x + 3)(x + 1)$
(2) $(x + 5)(x + 3)$
(3) $(x + 2)(x + 2)$
(4) $(x + 6)(x + 1)$

2 The first term of a geometric sequence is 8 and the fourth term is 216. What is the sum of the first 12 terms of the corresponding series?

(1) 236,192
(2) 708,584
(3) 2,125,760
(4) 6,377,288

3 Perry invested in property that cost him \$1500. Five years later it was worth \$3000, and 10 years from his original purchase, it was worth \$6000. Assuming the growth rate remains the same, which type of function could he create to find the value of his investment 30 years from his original purchase?

(1) exponential function
(2) linear function
(3) quadratic function
(4) trigonometric function

4 If $(a^3 + 27) = (a + 3)(a^2 + ma + 9)$, then m equals

(1) -9
(2) -3
(3) 3
(4) 6

Use this space for computations.

5 If $\cos\theta = -\frac{3}{4}$ and θ is in Quadrant III, then $\sin\theta$ is equivalent to

(1) $-\frac{\sqrt{7}}{4}$

(2) $\frac{\sqrt{7}}{4}$

(3) $-\frac{5}{4}$

(4) $\frac{5}{4}$

6 A veterinary pharmaceutical company plans to test a new drug to treat a common intestinal infection among puppies. The puppies are randomly assigned to two equal groups. Half of the puppies will receive the drug, and the other half will receive a placebo. The veterinarians monitor the puppies.

This is an example of which study method?

(1) census

(2) observational study

(3) survey

(4) controlled experiment

7 The expression $2 - \frac{x-1}{x+2}$ is equivalent to

(1) $1 - \frac{3}{x+2}$

(2) $1 + \frac{3}{x+2}$

(3) $1 - \frac{1}{x+2}$

(4) $1 + \frac{1}{x+2}$

8 Which description could represent the graph of $f(x) = 4x^2(x + a) - x - a$, if a is an integer?

(1) As $x \to -\infty, f(x) \to \infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 3 x-intercepts.

(2) As $x \to -\infty, f(x) \to -\infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 3 x-intercepts.

(3) As $x \to -\infty, f(x) \to \infty$, as $x \to \infty, f(x) \to -\infty$, and the graph has 4 x-intercepts.

(4) As $x \to -\infty, f(x) \to -\infty$, as $x \to \infty, f(x) \to \infty$, and the graph has 4 x-intercepts.

Use this space for computations.

9 After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days.

Day (n)	1	2	3	4
Dosage (m)	2000	1680	1411.2	1185.4

How can this sequence best be modeled recursively?

(1) $m_1 = 2000$
$m_n = m_{n-1} - 320$

(2) $m_n = 2000(0.84)^{n-1}$

(3) $m_1 = 2000$
$m_n = (0.84)m_{n-1}$

(4) $m_n = 2000(0.84)^{n+1}$

10 The expression $\frac{9x^2 - 2}{3x + 1}$ is equivalent to

(1) $3x - 1 - \frac{1}{3x + 1}$

(2) $3x - 1 + \frac{1}{3x + 1}$

(3) $3x + 1 - \frac{1}{3x + 1}$

(4) $3x + 1 + \frac{1}{3x + 1}$

11 If $f(x)$ is an even function, which function must also be even?

(1) $f(x - 2)$

(2) $f(x) + 3$

(3) $f(x + 1)$

(4) $f(x + 1) + 3$

12 The average monthly temperature of a city can be modeled by a cosine graph. Melissa has been living in Phoenix, Arizona, where the average annual temperature is 75°F. She would like to move, and live in a location where the average annual temperature is 62°F. When examining the graphs of the average monthly temperatures for various locations, Melissa should focus on the

(1) amplitude

(2) horizontal shift

(3) period

(4) midline

Use this space for computations.

13 Consider the probability statements regarding events A and B below.

$P(A \text{ or } B) = 0.3$;
$P(A \text{ and } B) = 0.2$, and
$P(A \mid B) = 0.8$

What is $P(B)$?

(1) 0.1
(2) 0.25
(3) 0.375
(4) 0.667

14 Given $y > 0$, the expression $\sqrt{3x^2y} \cdot \sqrt[3]{27x^3y^2}$ is equivalent to

(1) $81x^5y^3$
(2) $3^{1.5}x^2y$
(3) $3^{\frac{5}{2}}x^2y^{\frac{5}{3}}$
(4) $3^{\frac{3}{2}}x^2y^{\frac{7}{6}}$

15 What is the solution set of the equation $\frac{10}{x^2 - 2x} + \frac{4}{x} = \frac{5}{x - 2}$?

(1) {0,2}
(2) {0}
(3) {2}
(4) { }

Use this space for computations.

16 What are the solution(s) to the system of equations shown below?

$$x^2 + y^2 = 5$$
$$y = 2x$$

(1) $x = 1$ and $x = -1$
(2) $x = 1$
(3) (1, 2) and (−1, −2)
(4) (1, 2), only

17 If \$5000 is put into a savings account that pays 3.5% interest compounded monthly, how much money, to the *nearest ten cents*, would be in that account after 6 years, assuming no money was added or withdrawn?

(1) \$5177.80
(2) \$5941.30
(3) \$6146.30
(4) \$6166.50

18 The Fahrenheit temperature, $F(t)$, of a heated object at time t, in minutes, can be modeled by the function below. F_s is the surrounding temperature, F_0 is the initial temperature of the object, and k is a constant.

$$F(t) = F_s + (F_0 - F_s)e^{-kt}$$

Coffee at a temperature of 195°F is poured into a container. The room temperature is kept at a constant 68°F and $k = 0.05$. Coffee is safe to drink when its temperature is, at most, 120°F. To the *nearest minute*, how long will it take until the coffee is safe to drink?

(1) 7
(2) 10
(3) 11
(4) 18

Use this space for computations.

19 The mean intelligence quotient (IQ) score is 100, with a standard deviation of 15, and the scores are normally distributed. Given this information, the approximate percentage of the population with an IQ greater than 130 is closest to

(1) 2%
(2) 31%
(3) 48%
(4) 95%

20 After examining the functions $f(x) = \ln(x + 2)$ and $g(x) = e^{x-1}$ over the interval $(-2,3]$, Lexi determined that the correct number of solutions to the equation $f(x) = g(x)$ is

(1) 1
(2) 2
(3) 3
(4) 0

21 Evan graphed a cubic function, $f(x) = ax^3 + bx^2 + cx + d$, and determined the roots of $f(x)$ to be ± 1 and 2. What is the value of b, if $a = 1$?

(1) 1
(2) 2
(3) -1
(4) -2

Use this space for computations.

22 The equation $t = \frac{1}{0.0105}\ln\left(\frac{A}{5000}\right)$ relates time, t, in years, to the amount of money, A, earned by a \$5000 investment. Which statement accurately describes the relationship between the average rates of change of t on the intervals [6000, 8000] and [9000, 12,000]?

(1) A comparison cannot be made because the intervals are different sizes.

(2) The average rate of change is equal for both intervals.

(3) The average rate of change is larger for the interval [6000, 8000].

(4) The average rate of change is larger for the interval [9000, 12,000].

23 What is the inverse of $f(x) = \frac{x}{x+2}$, where $x \neq -2$?

(1) $f^{-1}(x) = \frac{2x}{x-1}$

(2) $f^{-1}(x) = \frac{-2x}{x-1}$

(3) $f^{-1}(x) = \frac{x}{x-2}$

(4) $f^{-1}(x) = \frac{-x}{x-2}$

24 A study of black bears in the Adirondacks reveals that their population can be represented by the function $P(t) = 3500(1.025)^t$, where t is the number of years since the study began. Which function is correctly rewritten to reveal the monthly growth rate of the black bear population?

(1) $P(t) = 3500(1.00206)^{12t}$

(2) $P(t) = 3500(1.00206)^{\frac{t}{12}}$

(3) $P(t) = 3500(1.34489)^{12t}$

(4) $P(t) = 3500(1.34489)^{\frac{t}{12}}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 At Andrew Jackson High School, students are only allowed to enroll in AP U.S. History if they have already taken AP World History or AP European History. Out of 825 incoming seniors, 165 took AP World History, 66 took AP European History, and 33 took both. Given this information, determine the probability a randomly selected incoming senior is allowed to enroll in AP U.S. History.

26 Explain what a rational exponent, such as $\frac{5}{2}$ means. Use this explanation to evaluate $9^{\frac{5}{2}}$.

27 Write $-\frac{1}{2}i^3\left(\sqrt{-9}-4\right)-3i^2$ in simplest $a+bi$ form.

28 A person's lung capacity can be modeled by the function $C(t) = 250\sin\left(\frac{2\pi}{5}t\right) + 2450$, where $C(t)$ represents the volume in mL present in the lungs after t seconds. State the maximum value of this function over one full cycle, and explain what this value represents.

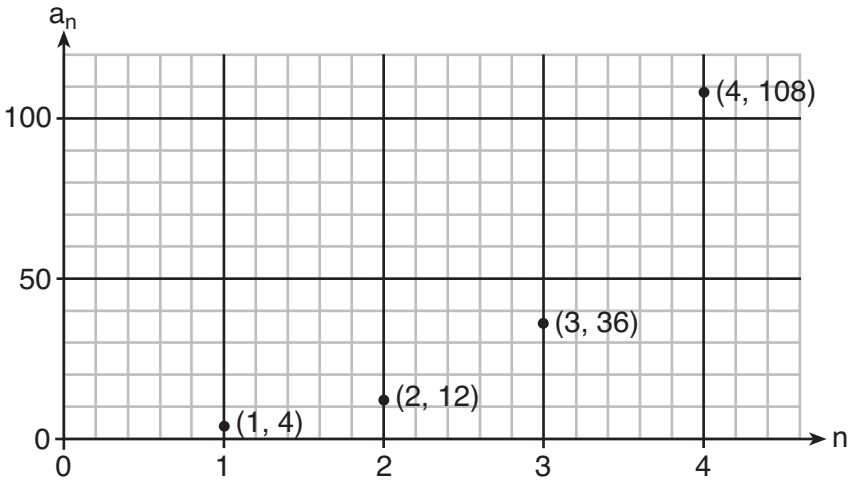
29 Determine for which polynomial(s) $(x + 2)$ is a factor. Explain your answer.

$$P(x) = x^4 - 3x^3 - 16x - 12$$
$$Q(x) = x^3 - 3x^2 - 16x - 12$$

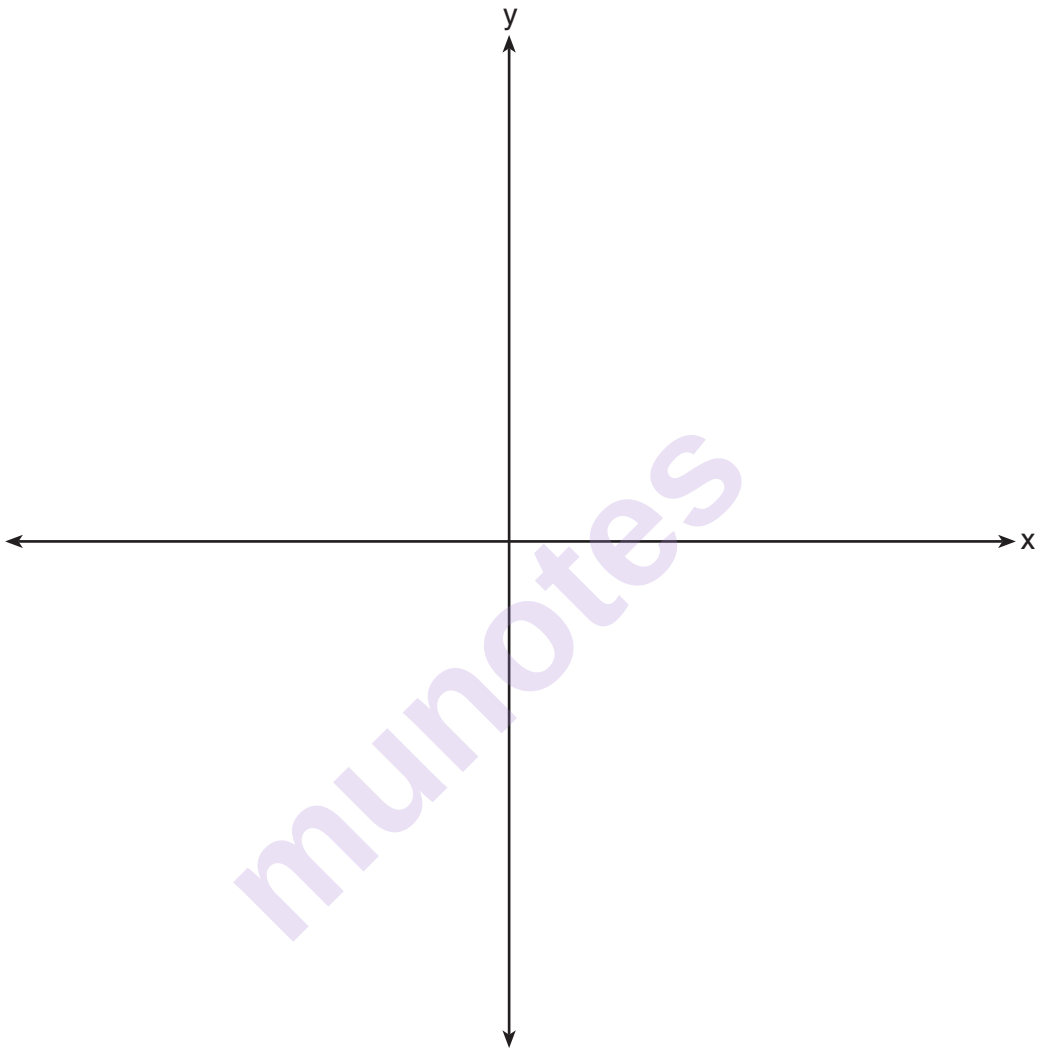
30 On July 21, 2016, the water level in Puget Sound, WA reached a high of 10.1 ft at 6 a.m. and a low of −2 ft at 12:30 p.m. Across the country in Long Island, NY, Shinnecock Bay's water level reached a high of 2.5 ft at 10:42 p.m. and a low of −0.1 ft at 5:31 a.m.

The water levels of both locations are affected by the tides and can be modeled by sinusoidal functions. Determine the difference in amplitudes, in feet, for these two locations.

31 Write a recursive formula, a_n, to describe the sequence graphed below.



32 Sketch the graphs of $r(x) = \frac{1}{x}$ and $a(x) = |x| - 3$ on the set of axes below. Determine, to the *nearest tenth*, the positive solution of $r(x) = a(x)$.



Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

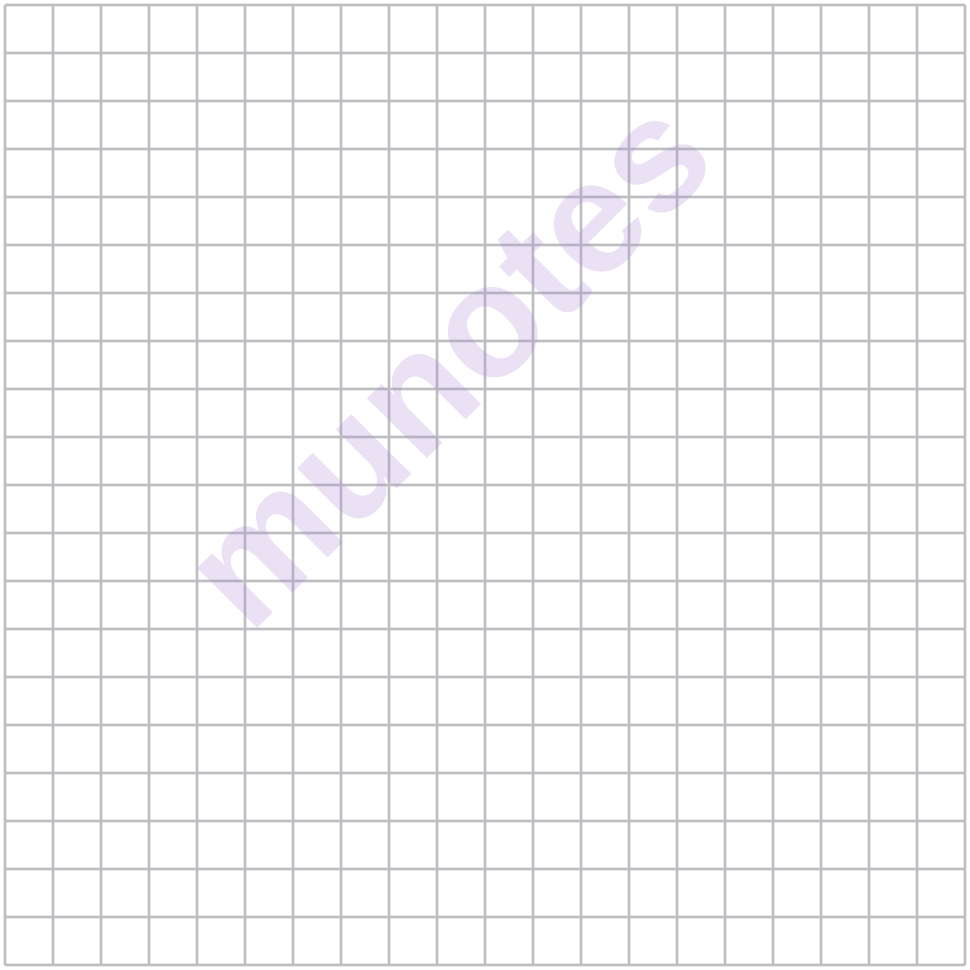
33 A population of 950 bacteria grows continuously at a rate of 4.75% per day.

Write an exponential function, $N(t)$, that represents the bacterial population after t days and explain the reason for your choice of base.

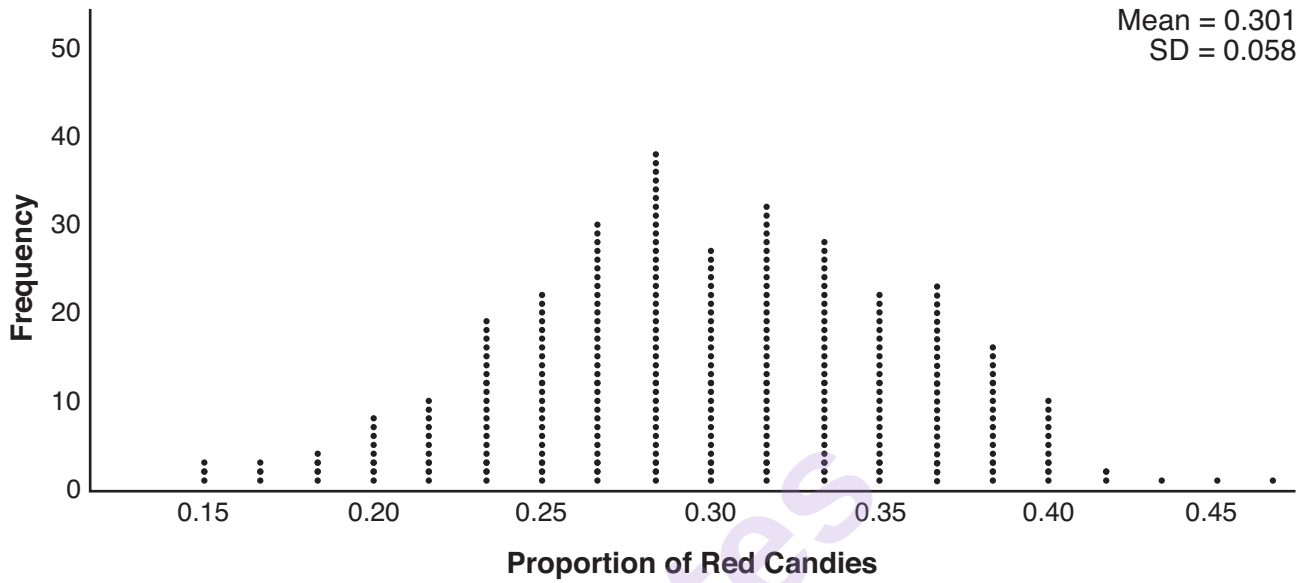
Determine the bacterial population after 36 hours, to the *nearest bacterium*.

34 Write an equation for a sine function with an amplitude of 2 and a period of $\frac{\pi}{2}$.

On the grid below, sketch the graph of the equation in the interval 0 to 2π.



35 Mary bought a pack of candy. The manufacturer claims that 30% of the candies manufactured are red. In her pack, 14 of the 60 candies are red. She ran a simulation of 300 samples, assuming the manufacturer is correct. The results are shown below.



Based on the simulation, determine the middle 95% of plausible values that the proportion of red candies in a pack is within.

Based on the simulation, is it unusual that Mary's pack had 14 red candies out of a total of 60? Explain.

36 a) Algebraically determine the roots, in simplest $a + bi$ form, to the equation below.

$$x^2 - 2x + 7 = 4x - 10$$

b) Consider the system of equations below.

$$y = x^2 - 2x + 7$$
$$y = 4x - 10$$

The graph of this system confirms the solution from part *a* is imaginary. Explain why.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The Beaufort Wind Scale was devised by British Rear Admiral Sir Francis Beaufort, in 1805 based upon observations of the effects of the wind. Beaufort numbers, B, are determined by the equation $B = 1.69\sqrt{s + 4.45} - 3.49$, where s is the speed of the wind in mph, and B is rounded to the nearest integer from 0 to 12.

Beaufort Wind Scale

Beaufort Number	Force of Wind
0	Calm
1	Light air
2	Light breeze
3	Gentle breeze
4	Moderate breeze
5	Fresh breeze
6	Steady breeze
7	Moderate gale
8	Fresh gale
9	Strong gale
10	Whole gale
11	Storm
12	Hurricane

Using the table above, classify the force of wind at a speed of 30 mph. Justify your answer.

Question 37 is continued on the next page.

Question 37 continued

In 1946, the scale was extended to accommodate strong hurricanes. A strong hurricane received a B value of exactly 15. Algebraically determine the value of s, to the *nearest mph*.

Any B values that round to 10 receive a Beaufort number of 10. Using technology, find an approximate range of wind speeds, to the *nearest mph*, associated with a Beaufort number of 10.

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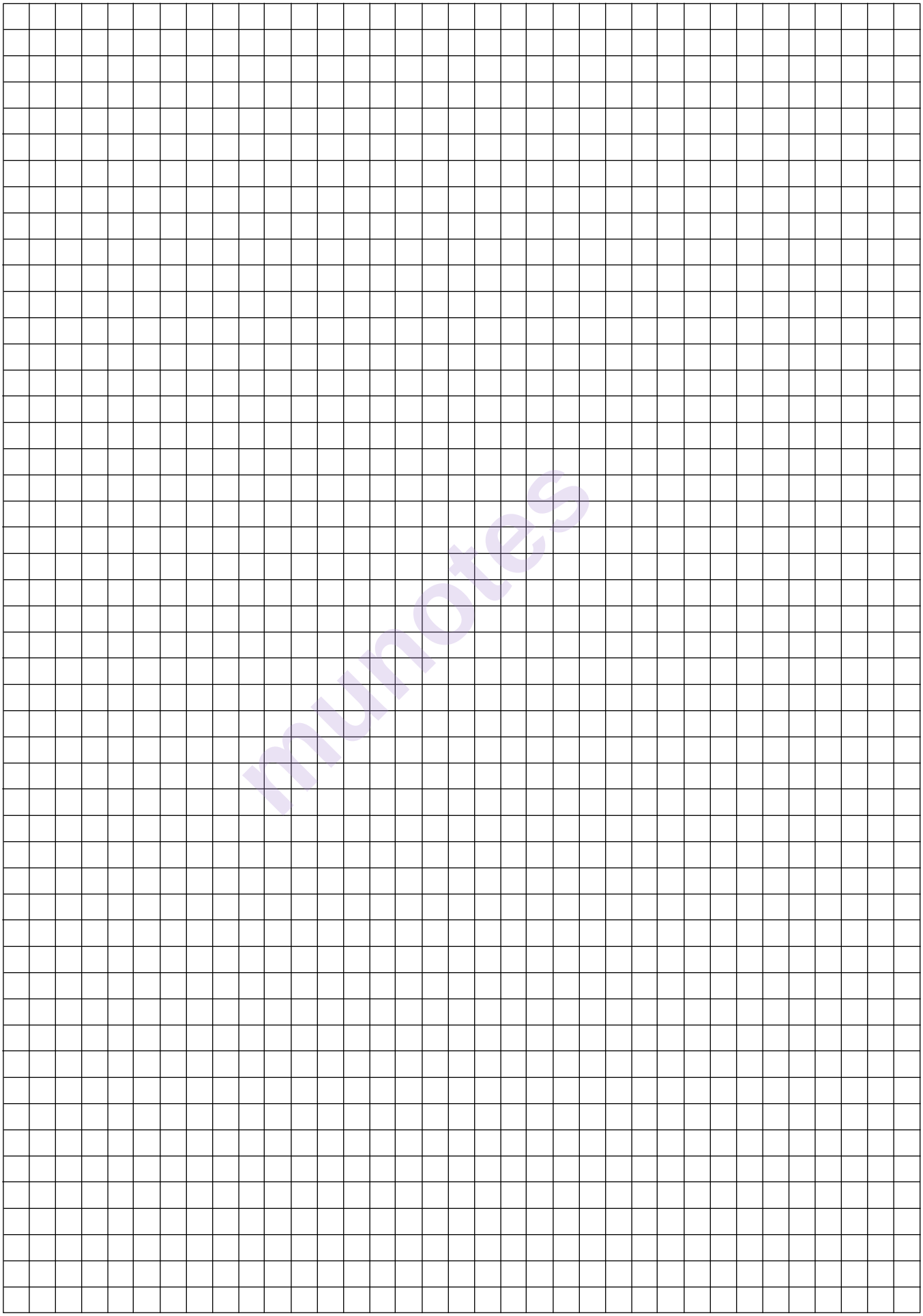

Scrap Graph Paper — This sheet will *not* be scored.

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High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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