The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, January 24, 2018 — 1:15 to 4:15 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The operator of the local mall wants to find out how many of the mall's employees make purchases in the food court when they are working. She hopes to use these data to increase the rent and attract new food vendors. In total, there are 1023 employees who work at the mall. The best method to obtain a random sample of the employees would be to survey

(1) all 170 employees at each of the larger stores

(2) 50% of the 90 employees of the food court

(3) every employee

(4) every 30th employee entering each mall entrance for one week

2 What is the solution set for x in the equation below?

$$\sqrt{x + 1} - 1 = x$$

(1) $\{1\}$ (3) $\{-1,0\}$

(2) $\{0\}$ (4) $\{0,1\}$

3 For the system shown below, what is the value of z?

$$y = -2x + 14$$
$$3x - 4z = 2$$
$$3x - y = 16$$

(1) 5   (3) 6

(2) 2   (4) 4

4 The hours of daylight, y, in Utica in days, x, from January 1, 2013 can be modeled by the equation $y = 3.06\sin(0.017x - 1.40) + 12.23$. How many hours of daylight, to the *nearest tenth*, does this model predict for February 14, 2013?

(1) 9.4   (3) 12.1

(2) 10.4   (4) 12.2

5 A certain pain reliever is taken in 220 mg dosages and has a half-life of 12 hours. The function $A = 220\left(\dfrac{1}{2}\right)^{\frac{t}{12}}$ can be used to model this situation, where A is the amount of pain reliever in milligrams remaining in the body after t hours.

According to this function, which statement is true?

(1) Every hour, the amount of pain reliever remaining is cut in half.

(2) In 12 hours, there is no pain reliever remaining in the body.

(3) In 24 hours, there is no pain reliever remaining in the body.

(4) In 12 hours, 110 mg of pain reliever is remaining.

6 The expression $(x + a)(x + b)$ can *not* be written as

(1) $a(x + b) + x(x + b)$ (3) $x^2 + (a + b)x + ab$

(2) $x^2 + abx + ab$ (4) $x(x + a) + b(x + a)$

7 There are 440 students at Thomas Paine High School enrolled in U.S. History. On the April report card, the students' grades are approximately normally distributed with a mean of 79 and a standard deviation of 7. Students who earn a grade less than or equal to 64.9 must attend summer school. The number of students who must attend summer school for U.S. History is closest to

(1) 3   (3) 10

(2) 5   (4) 22

8 For a given time, x, in seconds, an electric current, y, can be represented by $y = 2.5(1 - 2.7^{-.10x})$. Which equation is *not* equivalent?

(1) $y = 2.5 - 2.5(2.7^{-.10x})$

(2) $y = 2.5 - 2.5((2.7^2)^{-.05x})$

(3) $y = 2.5 - 2.5\left(\dfrac{1}{2.7^{.10x}}\right)$

(4) $y = 2.5 - 2.5(2.7^{-2})(2.7^{.05x})$

9 What is the quotient when $10x^3 - 3x^2 - 7x + 3$ is divided by $2x - 1$?

(1) $5x^2 + x + 3$ (3) $5x^2 - x - 3$

(2) $5x^2 - x + 3$ (4) $5x^2 + x - 3$

10 Judith puts \$5000 into an investment account with interest compounded continuously. Which approximate annual rate is needed for the account to grow to \$9110 after 30 years?

(1) 2%   (3) 0.02%

(2) 2.2%   (4) 0.022%

11 If $n = \sqrt{a^5}$ and $m = a$, where $a > 0$, an expression for $\dfrac{n}{m}$ could be

(1) $a^{\frac{5}{2}}$ (3) $\sqrt[3]{a^2}$

(2) a^4 (4) $\sqrt{a^3}$

12 The solutions to $x + 3 - \dfrac{4}{x-1} = 5$ are

  (1) $\dfrac{3}{2} \pm \dfrac{\sqrt{17}}{2}$ (3) $\dfrac{3}{2} \pm \dfrac{\sqrt{33}}{2}$

  (2) $\dfrac{3}{2} \pm \dfrac{\sqrt{17}}{2}i$ (4) $\dfrac{3}{2} \pm \dfrac{\sqrt{33}}{2}i$

13 If $ae^{bt} = c$, where a, b, and c are positive, then t equals

  (1) $\ln\!\left(\dfrac{c}{ab}\right)$ (3) $\dfrac{\ln\!\left(\dfrac{c}{a}\right)}{b}$

  (2) $\ln\!\left(\dfrac{cb}{a}\right)$ (4) $\dfrac{\ln\!\left(\dfrac{c}{a}\right)}{\ln b}$

14 For which values of x, rounded to the *nearest hundredth*, will $\left|x^2 - 9\right| - 3 = \log_3 x$?

  (1) 2.29 and 3.63   (3) 2.84 and 3.17

  (2) 2.37 and 3.54   (4) 2.92 and 3.06

15 The terminal side of θ, an angle in standard position, intersects the unit circle at $P\!\left(-\dfrac{1}{3}, -\dfrac{\sqrt{8}}{3}\right)$. What is the value of $\sec\theta$?

  (1) -3 (3) $-\dfrac{1}{3}$

  (2) $-\dfrac{3\sqrt{8}}{8}$ (4) $-\dfrac{\sqrt{8}}{3}$

16 What is the equation of the directrix for the parabola $-8(y - 3) = (x + 4)^2$?

  (1) $y = 5$ (3) $y = -2$

  (2) $y = 1$ (4) $y = -6$

17 The function below models the average price of gas in a small town since January 1st.

$G(t) = -0.0049t^4 + 0.0923t^3 - 0.56t^2 + 1.166t + 3.23$,

where $0 \leq t \leq 10$.

If $G(t)$ is the average price of gas in dollars and t represents the number of months since January 1st, the absolute maximum $G(t)$ reaches over the given domain is about

(1) $1.60

(3) $4.01

(2) $3.92

(4) $7.73

18 Written in simplest form, $\dfrac{c^2 - d^2}{d^2 + cd - 2c^2}$ where $c \neq d$, is equivalent to

(1) $\dfrac{c + d}{d + 2c}$

(3) $\dfrac{-c - d}{d + 2c}$

(2) $\dfrac{c - d}{d + 2c}$

(4) $\dfrac{-c + d}{d + 2c}$

19 If $p(x) = 2x^3 - 3x + 5$, what is the remainder of $p(x) \div (x - 5)$?

(1) -230

(3) 40

(2) 0

(4) 240

20 The results of simulating tossing a coin 10 times, recording the number of heads, and repeating this 50 times are shown in the graph below.
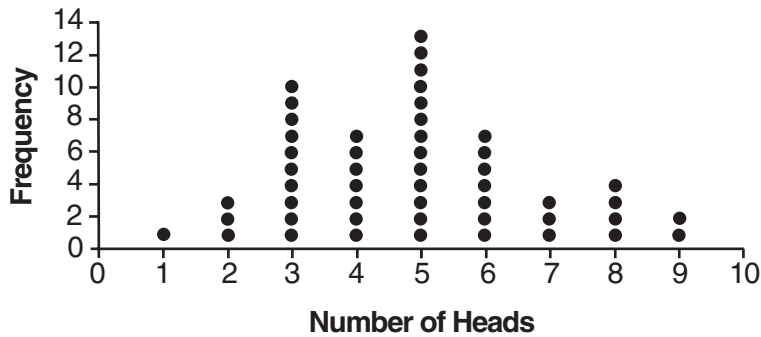
Based on the results of the simulation, which statement is *false*?

(1) Five heads occurred most often, which is consistent with the theoretical probability of obtaining a heads.

(2) Eight heads is unusual, as it falls outside the middle 95% of the data.

(3) Obtaining three heads or fewer occurred 28% of the time.

(4) Seven heads is not unusual, as it falls within the middle 95% of the data.

21 What is the inverse of $f(x) = -6(x - 2)$?

(1) $f^{-1}(x) = -2 - \frac{x}{6}$

(2) $f^{-1}(x) = 2 - \frac{x}{6}$

(3) $f^{-1}(x) = \frac{1}{-6(x - 2)}$

(4) $f^{-1}(x) = 6(x + 2)$

22 Brian deposited 1 cent into an empty non-interest bearing bank account on the first day of the month. He then additionally deposited 3 cents on the second day, 9 cents on the third day, and 27 cents on the fourth day. What would be the total amount of money in the account at the end of the 20th day if the pattern continued?

(1) $11,622,614.67   (3) $116,226,146.80

(2) $17,433,922.00   (4) $1,743,392,200.00

23 If the function $g(x) = ab^x$ represents exponential growth, which statement about $g(x)$ is *false*?

(1) $a > 0$ and $b > 1$ (3) The asymptote is $y = 0$.

(2) The y-intercept is $(0,a)$.   (4) The x-intercept is $(b,0)$.

24 At her job, Pat earns $25,000 the first year and receives a raise of $1000 each year. The explicit formula for the nth term of this sequence is $a_n = 25,000 + (n - 1)1000$. Which rule best represents the equivalent recursive formula?

(1) $a_n = 24,000 + 1000n$ (3) $a_1 = 25,000$, $a_n = a_{n-1} + 1000$

(2) $a_n = 25,000 + 1000n$ (4) $a_1 = 25,000$, $a_n = a_{n+1} + 1000$

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Elizabeth tried to find the product of $(2 + 4i)$ and $(3 - i)$, and her work is shown below.

$$(2 + 4i)(3 - i)$$
$$= 6 - 2i + 12i - 4i^2$$
$$= 6 + 10i - 4i^2$$
$$= 6 + 10i - 4(1)$$
$$= 6 + 10i - 4$$
$$= 2 + 10i$$

Identify the error in the process shown and determine the correct product of $(2 + 4i)$ and $(3 - i)$.

26 A runner is using a nine-week training app to prepare for a "fun run." The table below represents the amount of the program completed, *A*, and the distance covered in a session, *D*, in miles.

A	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{8}{9}$	1
D	2	2	2.25	3	3.25

Based on these data, write an exponential regression equation, rounded to the *nearest thousandth*, to model the distance the runner is able to complete in a session as she continues through the nine-week program.

27 A formula for work problems involving two people is shown below.

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_b}$$

t_1 = the time taken by the first person to complete the job

t_2 = the time taken by the second person to complete the job

t_b = the time it takes for them working together to complete the job

Fred and Barney are carpenters who build the same model desk. It takes Fred eight hours to build the desk while it only takes Barney six hours. Write an equation that can be used to find the time it would take both carpenters working together to build a desk.

Determine, to the *nearest tenth of an hour*, how long it would take Fred and Barney working together to build a desk.

28 Completely factor the following expression:

$$x^2 + 3xy + 3x^3 + y$$

29 Researchers in a local area found that the population of rabbits with an initial population of 20 grew continuously at the rate of 5% per month. The fox population had an initial value of 30 and grew continuously at the rate of 3% per month.

Find, to the *nearest tenth of a month*, how long it takes for these populations to be equal.

30 Consider the function $h(x) = 2\sin(3x) + 1$ and the function q represented in the table below.

x	q(x)
−2	−8
−1	0
0	0
1	−2
2	0

Determine which function has the *smaller* minimum value for the domain $[-2,2]$. Justify your answer.

31 The zeros of a quartic polynomial function h are -1, ± 2, and 3.

   Sketch a graph of $y = h(x)$ on the grid below.
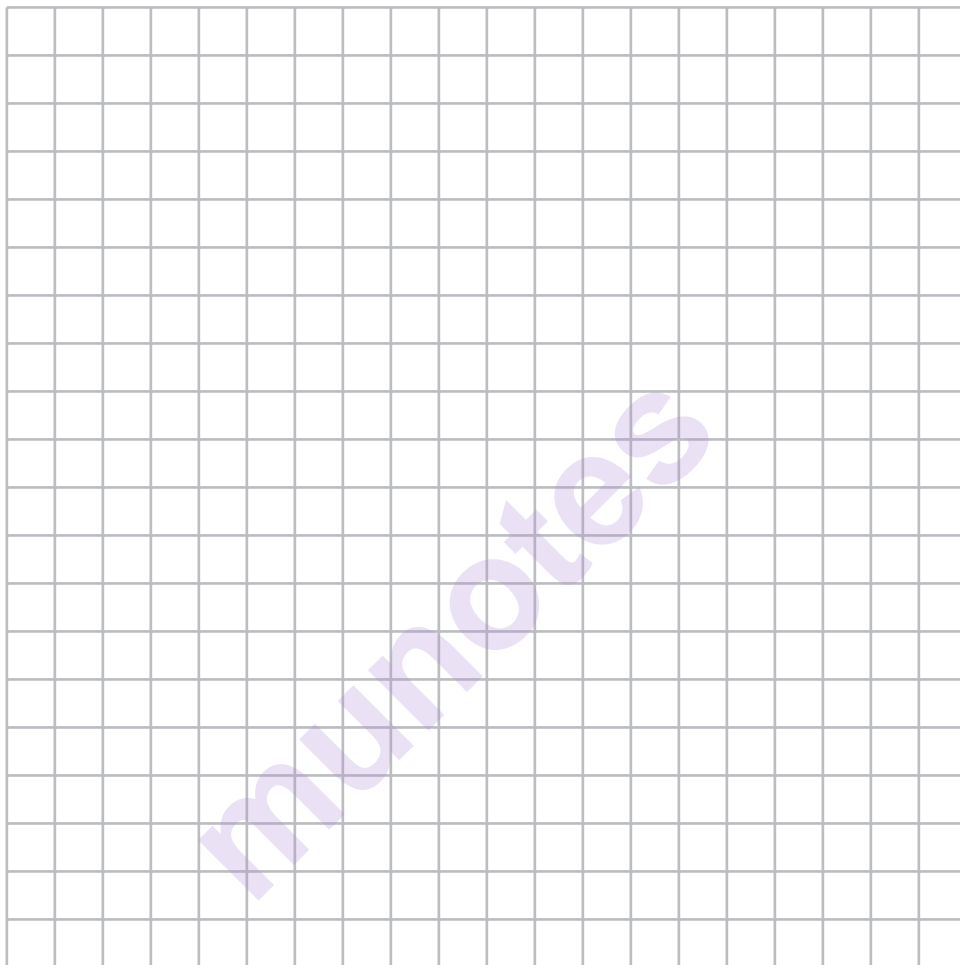
32 Explain why $81^{\frac{3}{4}}$ equals 27.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.   [16]

33 Given: $f(x) = 2x^2 + x - 3$ and $g(x) = x - 1$

Express $f(x) \cdot g(x) - \left[f(x) + g(x) \right]$ as a polynomial in standard form.

34 A student is chosen at random from the student body at a given high school. The probability that the student selects Math as the favorite subject is $\frac{1}{4}$. The probability that the student chosen is a junior is $\frac{116}{459}$. If the probability that the student selected is a junior or that the student chooses Math as the favorite subject is $\frac{47}{108}$, what is the exact probability that the student selected is a junior whose favorite subject is Math?

Are the events "the student is a junior" and "the student's favorite subject is Math" independent of each other? Explain your answer.

35 In a random sample of 250 men in the United States, age 21 or older, 139 are married. The graph below simulated samples of 250 men, 200 times, assuming that 139 of the men are married.
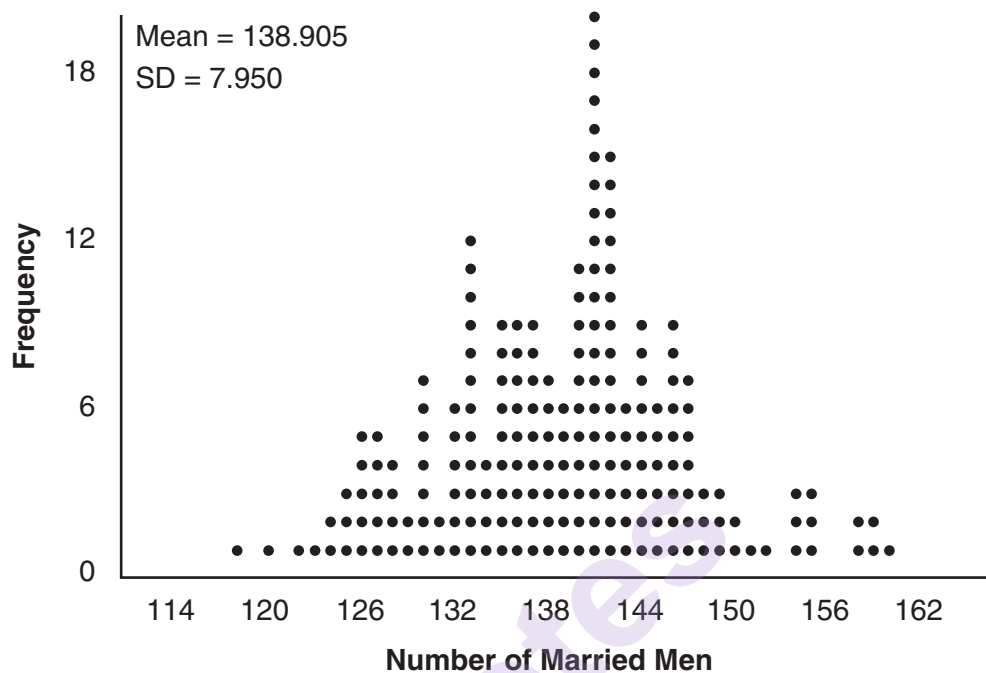
a) Based on the simulation, create an interval in which the middle 95% of the number of married men may fall. Round your answer to the *nearest integer.*

b) A study claims "50 percent of men 21 and older in the United States are married." Do your results from part *a* contradict this claim? Explain.

36 The graph of $y = f(x)$ is shown below. The function has a leading coefficient of 1.
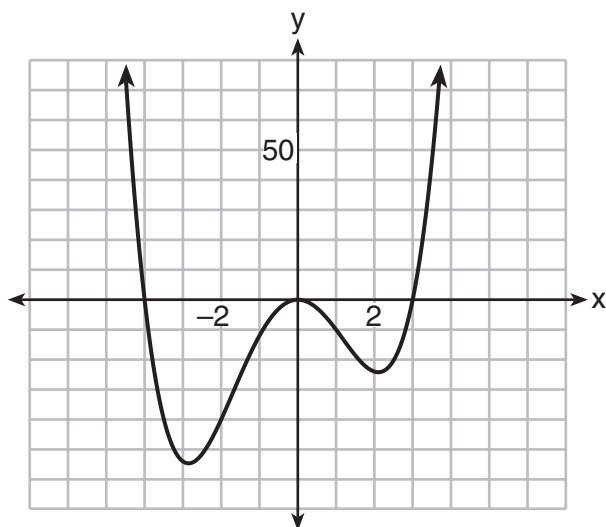
Write an equation for $f(x)$.

The function g is formed by translating function f left 2 units. Write an equation for $g(x)$.

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The resting blood pressure of an adult patient can be modeled by the function P below, where $P(t)$ is the pressure in millimeters of mercury after time t in seconds.

$$P(t) = 24\cos(3\pi t) + 120$$

On the set of axes below, graph $y = P(t)$ over the domain $0 \le t \le 2$.
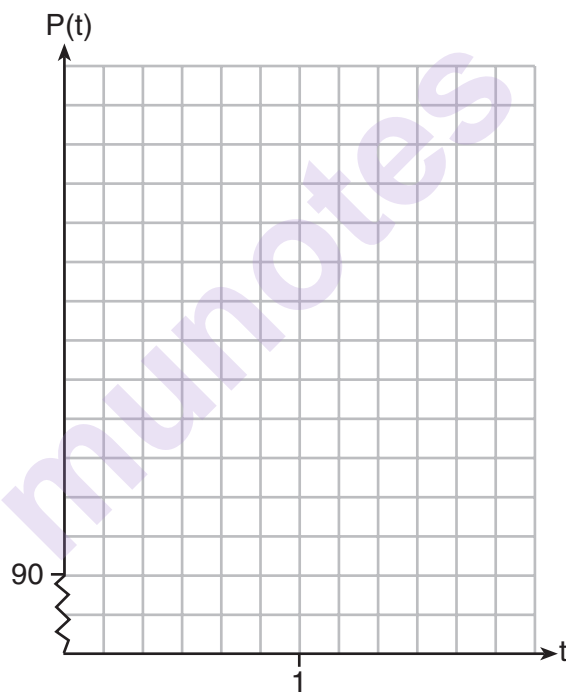
**Question 37 is continued on the next page.**

Question 37 continued.

Determine the period of *P*. Explain what this value represents in the given context.

Normal resting blood pressure for an adult is 120 over 80. This means that the blood pressure oscillates between a maximum of 120 and a minimum of 80. Adults with high blood pressure (above 140 over 90) and adults with low blood pressure (below 90 over 60) may be at risk for health disorders. Classify the given patient's blood pressure as low, normal, or high and explain your reasoning.
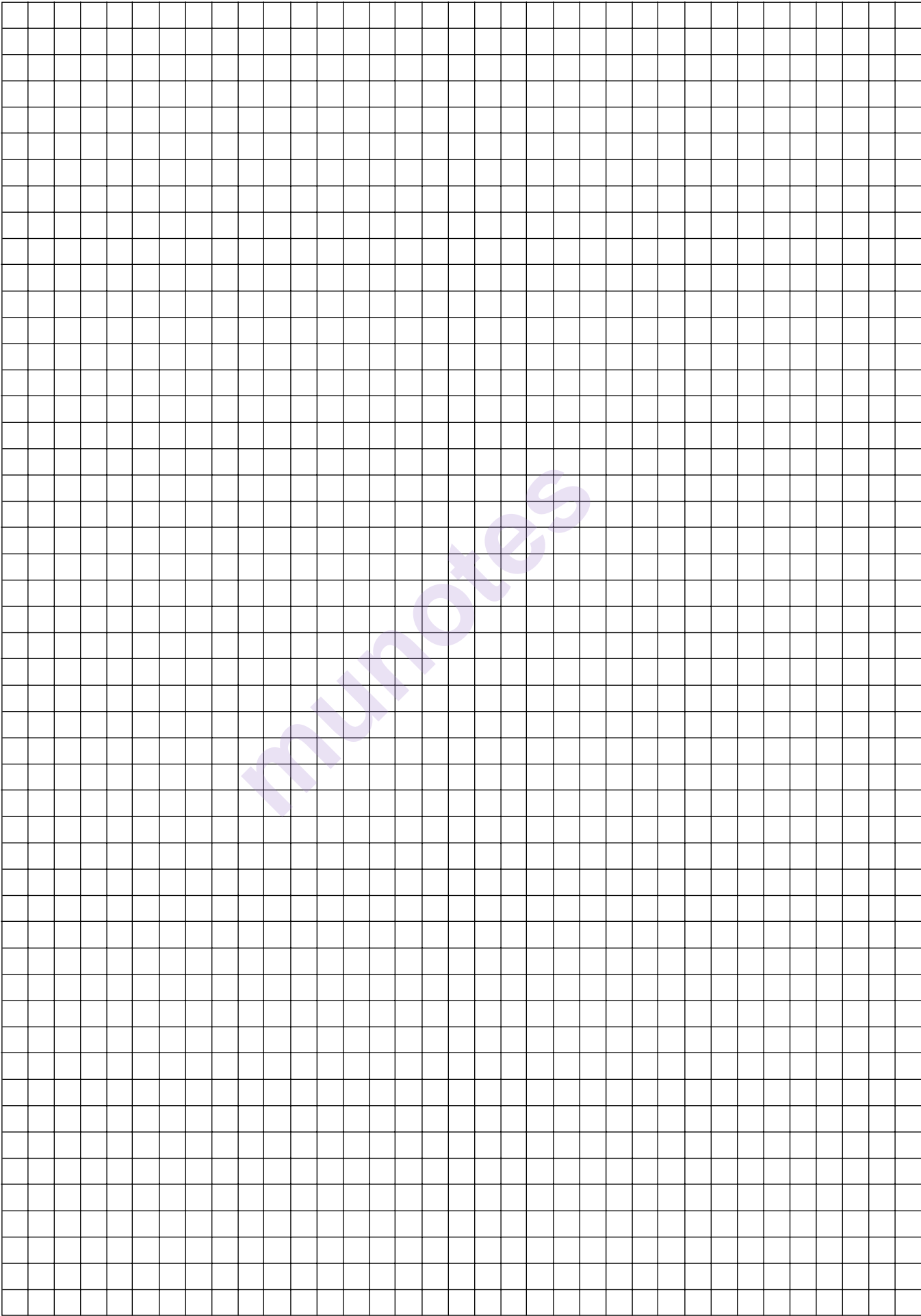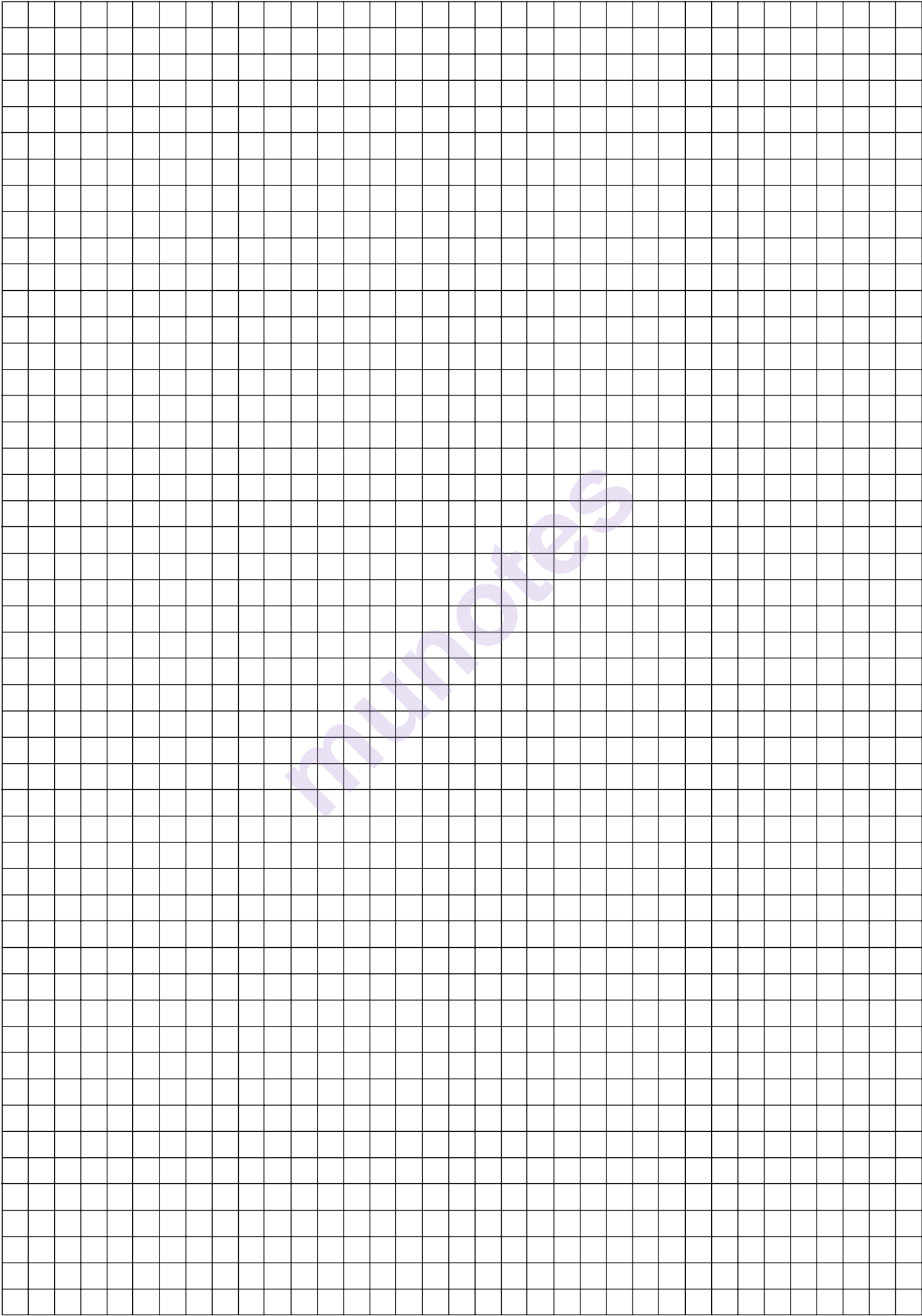
High School Math Reference Sheet

1 inch = 2.54 centimeters   1 kilometer = 0.62 mile   1 cup = 8 fluid ounces
1 meter = 39.37 inches   1 pound = 16 ounces   1 pint = 2 cups
1 mile = 5280 feet   1 pound = 0.454 kilogram   1 quart = 2 pints
1 mile = 1760 yards   1 kilogram = 2.2 pounds   1 gallon = 4 quarts
1 mile = 1.609 kilometers   1 ton = 2000 pounds   1 gallon = 3.785 liters
   1 liter = 0.264 gallon
   1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian = $\dfrac{180}{\pi}$ degrees
Degrees	1 degree = $\dfrac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$