RATIO, PROPORTION AND PERCENTAGE

Unit Structure:

- 1.0 Objectives
- 1.1 Ratio
- 1.2 Proportion
- 1.3 Percentage
- 1.4 Check your progress
- 1.5 Unsolved problems

1.0 OBJECTIVES:

After going through this unit, learner will be able to:

- Define and distinguish between concepts of Ratio, Proportion and Percentage.
- Apply ratio, proportion and percentage to solve real life problems.

1.1 RATIO

1.1.1 Introduction:

Ratio is defined as the relation between two quantities of the same kind, same type, which expresses one quantity (x) as a multiple or part of the other quantity (y).

i.e., one cannot compare quantity with unit as kilometre with the quantity with unit as litre. Both quantities must have same unit. Even, one quantity in kilo meter and another in centimetre are not comparable. We need to convert both quantities in same unit, either in kilometre or in centimetre.

Ratio has no unit. i.e., if we compare two quantities of 4 kg and 20 kg, then ratio will be 4:20 without unit. It means that, whenever there is ratio x: y, both x and y have same unit.

The ratio of quantities x and y is written as x: y or $\frac{x}{y}$ (read as x is to y), where, x (first quantity) is called as antecedent and y (second quantity) is called as consequent.

If terms in ratio i.e., antecedent and consequent multiplied or divided by same non-zero number then resulting ratio remains same as that of original ratio.

Ex. If original ratio is 3:7 then

$$\frac{3 \times 5}{7 \times 5} = \frac{15}{35} = \frac{3}{7}$$

- 1. If x = y, then ratio x: y is called ratio of equality.
- 2. If x > y, then ratio x: y is called ratio of greater inequality.
- 3. If x < y, then ratio x: y is called ratio of lesser inequality.

1.1.2 Compound Ratio:

Ratios are compounded by multiplying together their antecedents to form a new antecedent and their consequent to form a new consequent.

If a: b and c: d are two ratios then $ac: bdi.e., \frac{ac}{bd}$ is compound ratio of a: b and c: d.

Ex: 2:3 and 11:7 are two ratios then compound ratio is $2 \times 11:3 \times 7$ i.e., 22: 21

One can compound more than two ratios.

Ex. For ratios $a_1: b_1, a_2: b_2, a_3: b_3, \dots, a_n: b_n$ the compound ratio will be $\frac{a_1 \times a_2 \times a_3 \times \dots \times a_n}{b_1 \times b_2 \times b_3 \times \dots \times b_n}$

For any ratio *x*: *y*,

1. ratio $\frac{x^2}{y^2}$, is called duplicate ratio of x: y

- 2. ratio $\frac{x^3}{y^3}$ is called triplicate ratio of *x*: *y*
- 3. ratio $\frac{\sqrt{x}}{\sqrt{y}}$ is called sub-duplicate ratio of x: y
- 4. ratio $\frac{\sqrt[3]{x}}{\sqrt[3]{y}}$ is called sub-triplicate ratio of *x*: *y*

Ex. For ratio 4:5

- 1) duplicate ratio is 4^2 : 5^2 i.e., $4 \times 4 : 5 \times 5$ i.e., 16:25
- 2) triplicate ratio is 4^3 : 5^3 i.e., $4 \times 4 \times 4 : 5 \times 5 \times 5$ i.e., 64: 125
- 3) sub-duplicate ratio is $\sqrt{4}$: $\sqrt{5}$
- 4) sub-triplicate ratio is $\sqrt[3]{4}$: $\sqrt[3]{5}$

1.1.3 Inverse Ratio:

If *x*: *y* is a ratio then *y*: *x* is inverse ratio or reciprocal ratio of *x*: *y*.

Ex. original ratio is 17: 25 then inverse ratio is 25 : 17

1.1.4 Continued Ratio:

If more than two quantities of the same kind are compared, it is known as continued ratio.

Ex. quantities a, b, c, d is said to be in continued ratio then it is written as a:b:c:d.

1.1.5 Solved examples:

Ex: Divide a profit of Rs. 25,828 between two partners in the ratio 4:7.

Solution:

Since the ratio of profit is 4:7.

Let the shares of two partners in profit be 4x and 7x respectively.

$$\therefore 4x + 7x = 25,828$$
$$\therefore 11x = 25,828$$
$$\therefore x = \frac{25,8258}{11}$$
$$\therefore x = 2,348$$

Hence, share of first partner in profit is $4x = 4 \times 2348 = \text{Rs}.9392$

Share of second partner in profit is $7x = 7 \times 2348 = \text{Rs.} 16436$

Ex: Find three positive numbers in the ratio 3:5:2 such that the sum of their squares is equal to 2432.

Solution :

Given that three numbers are in ratio 3:5:2.

Let these numbers be 3x, 5x and 2x.

$$\therefore (3x)^{2} + (5x)^{2} + (2x)^{2} = 2432$$
$$\therefore 9x^{2} + 25x^{2} + 4x^{2} = 2432$$
$$\therefore 38x^{2} = 2432$$

 $\therefore x^2 = \frac{2432}{38}$ $\therefore x = 64$

 $\Rightarrow x = 8$

So, required positive numbers are

 $3x = 3 \times 8 = 24,$

 $5x = 5 \times 8 = 40$ and

 $2x = 2 \times 8 = 16.$

Ex: If the ratio of A:B is 4:5 and the ratio of B:C is 3:2, find the ratio of A:B:C.

Solution:

Since A: B is 4:5,B:C is 3:2

Since we have to find ratio A:B:C i.e., to make contribution of B same in both ratios.

Therefore, we multiply first ratio 4:5 by 3

i.e.,
$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

also, we multiply second ratio 3:2 by 5

i.e.,
$$\frac{3}{2} = \frac{3 \times 5}{2 \times 5} = \frac{15}{10}$$

Therefore, ratio A:B:C is 12: 15:10.

Ex: Ratio of the present ages of Suresh and his mother is 2:5. Suresh was born when his mother was 27 years old. Find their present ages.

Solution:

Since, ratio of the present ages of Suresh and his mother is 2:5

Let, their present ages be 2x and 5x respectively.

Before 2x years, Mothers age was 27

So, 5x - 2x = 27

3x = 27

x = 9.

So present age of Suresh = $2x = 2 \times 9 = 18$ yrs.

Present age of his mother = $5x = 5 \times 9 = 45$ yrs.

Ex: There are 30fruits in a basket, and the ratio of the number of apples to the number of oranges is 1:1. How many more orange to be added to the basket to make the ratio 1:2?

Solution:

Total fruits in a basket = 30

Current ratio of apples to orange is 1:1

- \therefore Number of apples = $\frac{1}{2} \times 30 = 15$ and
- \therefore Number of oranges = $\frac{1}{2} \times 30 = 15$

Let *x* more oranges to be added to basket,

Then as per given condition new ratio becomes 1:2

$$\therefore \frac{15}{15+x} = \frac{1}{2}$$

$$\therefore 15 \times 2 = 15 + x$$

$$\therefore 30 = 15 + x$$

$$\therefore x = 15.$$

Hence, 15 more oranges to be added to basket to make ratio 1:2

Ex: Monthly incomes of A and B are in the ratio 7:4 and their expenditures are in the ratio 9:5. Each of them saves Rs. 10,000. Find their incomes.

Solution:

Since, Monthly incomes of A and B are in the ratio 7:4

Let their incomes be Rs. 7x and 4x.

We know that, saving = income – expenditure

 \therefore expenditure = income - saving

Also, each of them saves Rs. 10,000

Therefore, their expenditures be 7x - 10,000 and 4x - 10,000

It is gives that; ratio of their expenditures is 9:5.

i.e.
$$\frac{7x-10,000}{4x-10,000} = \frac{9}{5}$$

 $\therefore 5(7x - 10,000) = 9(4x - 10,000)$
 $\therefore 35x - 50,000 = 36x - 90,000$
 $\therefore 90,000 - 50,000 = 36x - 35x$
 $\therefore x = 40,000$

Hence,

Income of $A=7x - 10,000 = 7 \times 40,000 = Rs. 2,80,000$

Income of $B = 4x - 10,000 = 4 \times 40,000 = Rs. 1,60,000$

Ex: The total marksof Simran and Romain examination are in the ratio 11:13. The difference between their marks is 56. What are their marks?

Solution:

Given that, marks of Simran and Roma are in ratio 11: 13

Let, their marks be 11x and 13x.

Since, the difference between their marks is 56.

$$\therefore 13x - 11x = 56$$
$$\therefore 2x = 56$$
$$\therefore x = 28$$

Hence,

Marks of Simran $=11 \times 28 = 308$ and

Marks of Roma = $13 \times 28 = 364$.

1.2 PROPORTION:

1.2.1 Introduction:

Proportion indicates equality between two ratios. It can also extend to equality of more than two ratios.

Let a, b, c, d be four quantities such that a and b of the same kind and c and d are of the same kind. We say that a, b, c, d is in proportion, if two ratios a:b and c: dis equal.

It is denoted by a:b::c:d or $\frac{a}{b} = \frac{c}{d}$ this implies ad = bci.e., product of last two terms is equal to product of middle two trems.

In this relation first term and last term are called 'Extremes' and middle terms are called 'Means'.

1.2.2 Continued proportion:

If three quantities a, b, c of same kind such that $a: b = b: ci.e. \frac{a}{b} = \frac{b}{c}$

$$\Rightarrow ac = b^2$$

$$\Rightarrow b = \sqrt{ac}$$

1.2.3 Direct proportion:

When two ratios are in direct proportion with each other means they are in same direction. i.e., the terms of ratio are increasing or decreasing simultaneously.

One can express this relation in the form

$$\frac{a}{b} = k \cdot \frac{c}{d}$$
 i.e., $\frac{a}{b}$ is k times of $\frac{c}{d}$

1.2.4 Inverse proportion:

When two ratios are in inverse proportion with each other means they are in opposite direction. i.e., the terms of ratio are increasing or decreasing inversely.

One can express this relation in the form

$$\frac{a}{b} = k \cdot \frac{d}{c}$$
 i.e., $\frac{a}{b}$ is k times of $\frac{d}{c}$

1.2.5 Solved Examples:

Ex: find fourth proportional to 5, 10 and 7.

Solution:

Let, fourth proportional be x.

Therefore, we have 5, 10, 7 and x in proportion.

$$\therefore \frac{5}{10} = \frac{7}{x}$$

$$\therefore 5x = 7 \times 10$$

$$\therefore x = \frac{70}{5}$$

$$\therefore x = 14$$

Hence, required forth term in given proportion is 14.

Ex: if cost of 9 toys is Rs. 180, how much would 15toys cost?

Solution:

There is ratio of chocolates to cost in both statements, so they are in proportion.

Let, cost of 15 toys be Rs. x.

Then we have 9, 180, 15, x are in proportion and we have to find x.

$$\therefore \frac{9}{180} = \frac{15}{x}$$
$$\therefore 9x = 15 \times 180$$

$$\therefore x = \frac{15 \times 180}{9} = 300$$

Hence the cost of 15 toys is Rs. 300.

Ex: (x + 12), (x + 4), (x + 5) and (x - 1) are in proportion. Find x.

Solution:

$$\therefore \frac{x+12}{x+4} = \frac{x+5}{x-1}$$

$$\therefore (x+12)(x-1) = (x+4)(x+5)$$

$$\therefore x^{2} + 11x - 12 = x^{2} + 9x + 20$$

$$\therefore 11x - 9x = 20 + 12$$

$$\therefore 2x = 32$$

$$\therefore x = 16$$

Ex: What number must be subtracted from each of the numbers 8, 16 and 40. So that resulting numbers are in continued proportion?

Solution:

Let x be that number to subtracted from each of 8, 16 and 40.

So let, resulting number be 8 - x, 16 - x, 40 - x which are in continued proportion.

$$\therefore (8 - x) \times (40 - x) = (16 - x)^{2}$$

$$\therefore 320 - 48x + x^{2} = 256 - 32x + x^{2}$$

$$\therefore 320 - 256 = 48x - 32x$$

$$\therefore 64 = 16x$$

$$\therefore x = 4$$

Hence 4 must be added from 8, 16 and 40 to get resulting numbers in continued proportion.

Ex: 15 workers can finish a project in 60days. If the project must be finished in 36 days, how many more worker will be required?

Solution:

Let, the number of workers required to finish project in 36 days be x.

number of workers and days are in inverse proportion.

i.e. $\frac{a}{b} = \frac{d}{c}$ and we have two ratios, 15 : 60 and x : 36

Therefore,

$$\therefore \frac{15}{x} = \frac{36}{60}$$
$$\therefore 15 \times 60 = 36x$$
$$\therefore \frac{15 \times 60}{36} = x$$
$$\therefore x = \frac{900}{36}$$
$$\therefore x = 25$$

i.e. 25 workers will finish work in 36 days.

So, needed 10 more workers(already 15 workers are there) to finish work in 36 days.

Ex: How to bearloss of Rs. 25,828 between two partners in the ratio 6:5.

Solution:

Let, their shares in loss be 6x and 5x.

$$\therefore 6x + 5x = 25828$$

.11x = 25828

$$\therefore x = \frac{25828}{11}$$
$$\therefore x = 2348$$

Therefore, partners have to bear loss of Rs.14,088 and Rs.11,740 respectively.

Ex: Manoj's saving box contains a total of Rs. 248 in the form of coins of Rs.10, Rs.5, Rs.2, Rs.1 in the ratio 9:3:7:5. Find the number of coins of each denomination.

Solution:

Let, number of coins be 9x, 3x, 7x, 5x respectively.

i.e., Rs. 10 coins be 9x, Rs. 5 coins be 3x, Rs. 2 coins be 7x and Rs. 1 coin be 5x.

Therefore, Amount of Rs.10 = $10 \times 9x = 90x$,

Amount of Rs.5 = $5 \times 3x = 15x$,

Amount of Rs.2 = $2 \times 7x = 14x$ and

Amount of Rs.1 = $1 \times 5x = 5x$

 $\therefore \text{Total amount} = 90x + 15x + 14x + 5x$

$$\therefore 248 = 90x + 15x + 14x + 5x$$

 $\therefore 124x = 248$ $\therefore x = \frac{248}{124} = 2$

Therefore,

coins of Rs.10 = 9x = 9(2) = 18Coins of Rs.5=3x = 3(2) = 6

Coins of Rs.2=7x = 7(2) = 14

Coins of Rs.1 = 5x = 5(2) = 10

Ex: A, B and C invested Rs.70,000, Rs.50,000 and Rs.80,000 respectively in a business. At the end of the year, C received Rs.16,000 as his share in the profit. Find A's and B's share in the profit.

Solution:

Since, ratio of their investment is 70,000: 50,000: 80,000

We can divide all terms by 10,000 as they in proportion.

 \therefore ration becomes 7: 5: 8

Profit is distributed in same ratio that of investment.

Let, share of partner's A, B, C in profit be 7x, 5x, 8x respectively.

Given that, share of C is 8x = 16,000

$$\therefore x = \frac{16,000}{8}$$
$$\therefore x = 2,000$$

Therefore, share of A = $7x = 7 \times 2000 = 14,000$

Share of B = $5x = 5 \times 2000 = 10,000$

1.3 PERCENTAGE:

1.3.1 Introduction:

Percentage means per cent (cent means hundreds) i.e., per hundred.

One can convert any ratio to percentage and vice versa.

1) Multiply fraction (ratio) by 100 it will get converted to percentage.

2) Divide percentage by 100 it will get converted to fraction.

Ex.:

1)
$$\frac{3}{4} = \left(\frac{3}{4} \times 100\right) \% = 75 \%$$

2) $\frac{1}{10} = 10 \%$
3) $25\% = \frac{25}{100} = \frac{1}{4}$
4) $33\% \text{ of } 240 = \frac{33}{100} \times 240 = 79.2$

1.3.2 Solved examples:

Ex: A person's monthly income increased from Rs. 8,000 to Rs. 8,560. Find the percentage increase.

Solution:

Income increased from Rs. 8000 to Rs. 8560, i.e. inceased by Rs. 560.

So we take ratio of incement to base. i.e. 560 : 8000

To convert in percentage we multiply it by 100.

$$\therefore \frac{560}{8000} \times 100 = 7\%$$

Hence, a person's monthly income increased by 7%.

Ex: 8% of certain amount is Rs.20. Find the amount.

Solution:

Let, that certain amount be *x*.

..8% of x = 20

$$\therefore \frac{8}{100} \times x = 20$$
$$\therefore x = 20 \times \frac{100}{8}$$
$$\therefore x = 250$$

Hence, required amount is Rs. 250.

Ex: Aniruddha spends 60% of his income. If in a month, saved Rs.20,928 then find his earning for that month.

Solution:

Let, Aniruddha's earning for that month be *x*.

He spends 60% of his income, it means he saves 40% of his income.

 \therefore As per given information,

Rs.20,928 is 40% of his income i.e. *x*

$$\therefore 40 \% of x = 20928$$
$$\therefore \frac{40}{100} \times x = 20928$$
$$\therefore x = 20928 \times \frac{100}{40}$$
$$\therefore x = 52320$$

Hence, aniruddha's income for that month is Rs.52,320.

Ex: If akshy's profit decreased from 28,000 to 26,880. find the percentage of decrease.

Solution:

We first find amount decreased and then convert it to percentage.

Since, akshy's profit decreased from 28,000 to 26,880, i.e. decreased by Rs. 1120.

We take ratio of amount decrease to base. i.e. 1120 : 28000

To convert to percentage, multiply by 100.

$$\therefore \frac{1120}{28000} \times 100 = 4\%$$

Hence, akshay's profit decreased by 4%.

1.4 CHECK YOUR PROGRESS:

1) If 4, 20, z is in continued proportion, then z =_____.

A) 100 B) 110 C) 120 D) 130

2) The	percentage of	f marks s	ecured	by a stu	ident sco	oring 43	38 out of	600 is
A)70	 B) 71		C) 72		D) 73			
3) The	duplicate rati	o of 4:3 i	S					
A)16: 9 B) 9: 16		16	C) 3: 4		D) 4: 9			
4) The	percentage of	f marks s	ecured	by a stu	ident sco	oring 30	50 out of	600 is
A) 70	 B) 60		C) 80		D) 90			
5) The	e compound ra A) 3/8	atio of 3/2 B) 3/8	2 , 1/5 a : 1/5: 2	nd 2/9 i /9	s C) 1/15		D) 8/3	
6) The	duplicate ration A) 6:4	o of 3:2 i B) 9:4	S		C) 5:2		D) 6:1	
7) The	sub-triplicate A) 64:1	ratio of B) 3:19	1:64 is 92		C) 1:16		D) 1:4	
8) The	inverse ratio A) 4:9	of 9:4 is B) 1/4	: 1/9		C) 3:2		D) 1:36	
9) If a:	b = 2:3 and $b:A) 2:4:3$	c = 2:3 t B) 4:6:	then a:b 9	o:c is	C) 2:6:9)	D) 9:6:4	
10) If de	the angels of gree measure A) 27, 72, 8	a triangl s are	le are i B) 32 ,	n the ra	tio 3:8:9	• then • C) 24,	their resp 64 , 92	oective
11) 209	% of 360 is A) 7.2	B) 720		C) 72				
12) Th	e fourth propo	ortion to 2	21, 30 a	and 35 is	5			
	A) 50	B) 18		C) 24.5	5			
13) Tw rat	to numbers at tio, the new ra A) 9 , 12	re in the atio is 4 : B) 18 ,	ratio 3 5. So, 24	: 4. If (the give C) 15 ,	6 is adde en numbe 20	ed to eaters are	ach term	of the
14) Pre	esent ages of	Soham a	nd Soh	a are in	the ration	07:1	2. If two	years

ago, the ratio of their ages was 3 : 8 , then their present ages are A) 7yrs , 12yrs B) 3yrs , 6yrs C) 3yrs 6 month, 6yrs

Answers:				
1) 100 2) 73	3) 16:9	4) 60	5) 1/15	6) 9:4
7) 1:4	8) 4:9	9) 4:6:9	10) 27, 72, 81	11) 72
12) 50	13) 18, 24	14) 3yrs 6mor	ths, 6yrs	

1.5 UNSOLVED PROBLEMS:

- 1) Find the value of 'x' if x: 3 :: 60: 15
- 2) A table costs a carpenter Rs. 720 to make. He sells it for Rs. 920. What percentage of profit does he earn?

Ans: 27.78 %

Ans: 12

3) The price of 15 suits is Rs.6750. How many such suits can be purchased by an amount of Rs. 4050?

Ans: 9

- 4) 6 men can paint a house in 4 days. How long it would take to paint the house if 3 men are employed?
- Ans: 8 days5) Sagar spends 25% of his income on house rent, 60% of the rest amount on household expenditure. If he saves Rs. 2100, what is his total income?

Ans: Rs. 7000

6) Ex: A college collected Rs. 6,02,136 for charity which is to be divided between an orphanage and a school in the ratio 7:11. what amount did each institution receive ?

Ans: Rs.2,34,164 &Rs.3,67,972

7) Three numbers are in the ratio 2: 3:5 and the sum of their squares is 950. Find the numbers.

Ans: 10, 15, 25

8) A ratio is equal to 5: 7. If its antecedent is 35, what is the consequent?

Ans: 49

9) Two numbers are in the ratio 3: 5. If 8 is added to each number, the ratio becomes 2:3. Find the numbers.

Ans: 24, 40

10) What number must be added to each term of the ratio 3 : 5 to make it 11 : 12 ?

Ans: 19

11) Divide 3,600 among A, B, C in the ratio 1/3:1/4:1/6.

Ans. 1600, 1200, 800

12) Divide 3,740 into three parts in such a way that half of the first part, one third of the second part, and one-sixth of the third part are equal.

Ans: 680, 1020, 2040

13) In a mixture of 63 litres, the ratio of milk and water is 5:2. How much water must be added to this mixture to make the ratio 3:2?

Ans: 12 litres

14) An employer reduces the number of employees in the ratio 10:7 and increases their wages in the ratio of 14:15. In what ratio, the wage bill is increased or decreased?

Ans: Decreased in ratio 4:3

15) A purse contains one rupee, 50 paise and 25 paise coins in the ratio 2: 3:4. If the total amount in the purse is Rs. 180, find the number of coins of each kind.

Ans: 80, 120, 160

16) Ritika reduces her weight in the ratio 3:2. What is her weight now, if originally it was 81 kg?

Ans: 54 kg

17) The length and width of a rectangle are in the ratio 5: 8. If the perimeter of the rectangle is 156 feet. What are the length and width of the rectangle?

Ans. length =30 feet, width =48 feet

18) A, B, C start a business by investing 2 20,000, 35,000 and 45,000 respectively and share the profit of 10,000 at the end of the year. Find the share in profit

Ans: A =2,000, B= 3,500, C=4,500

19) Mr. Manish, Mr. Gopal and Mr. Kumar started a transport business of investing 1 lakh each. Mr. Manish left after 5 month from the commencement of business and Mr. Gopal left 3 months later. At the end of the year the business realized a profit of 75,000. Find the share of profit of each partner.

Ans: Manish=15,000, Gopal= 24,000, Kumar=36,000

20) The difference of squares of two numbers which are in the ratio 3:5 is 144. Find the numbers.

Ans: 9 and 15

21) Find the fourth proportional to 6, 10 and 12. Ans: 120 22) Find the third proportional to 8 and 16. Ans: 32 23) Find the mean proportion of 3.6 and 4.9. Ans: 4.2 24) What least number must be added to each of the numbers 16, 7, 79 and 43, so that the resulting numbers are in proportion? Ans: 5 25) If (x-2), (X+2), (2X+1) and (2X+19) are in proportion, find the value of X. Ans:x = 426) The ratio of income to the expenditure of a company is 5: 3. If the company spends 2,700, what is its income? How much is its saving? Ans:Rs. 4,500 27) If 4 men can do a work in 2 days, in how many days 2 men can do this work? Ans: 4 days 28) If 48 men can dig a pit in 14 days, how long will it take 28 men to dig the similar pit. Ans: 24 days 29) A garrison is provided with food for 80 soldiers to last for 60 days. Find how long would the food last if 20 additional soldiers join them after 15 days. Ans: 36 days 30) The rent for 3 rooms for 9 months is 135. What will be the rent for 9 rooms for 3 months? Ans:135 31) If 10 horses consume 18 bushels in 36 days, how long will 24 bushels last for 30 horses? Ans: 16 days

32) There are 20 eggs in a fridge and 6 of them are brown. What percent of eggs are not brown?

33) 44% of the students of a class are girls. If the number of girls is 6 less than the number of boys, how many students are there in the class?

Ans: 50

34) A man donated 5% of his monthly income to a charity and deposited 12% of the rest in bank. If he has Rs. 11,704 left with him, what is his monthly income?

Ans:Rs. 14,000

35) The passing marks in a examination is 35%. If a student gets 190 marks, falls short by 20 marks, then find the total marks of the examination conducted.

Ans: 600

36) Samir bought a bicycle for 1.724 and after three months he sold it at a profit of 25% for what price did he sell the bicycle?

Ans:Rs. 2,155

37) By selling an article for 4,200, the shopkeeper lost 20%. At what price should he have sold it to gain 16%?

Ans: Rs.6,090

38) By selling a soft set for 3,825, a shopkeeper loses 15% on it. Find the price at which it was bought.

Ans:Rs. 4,500

39) Rajnish sells a pair of shoes at a profit of 30%. Find its cost price if the selling price is 650.

Ans: 500

40) In an examination a student scored 176 marks and failed by 34 marks. Find the maximum marks of the examination if a student must score at least 40% to pass the examination.

Ans: 525

41) A shopkeeper purchases two calculators A and B at a total cost of 550. He sells calculator A at 15% profit and calculator B at a loss of 20% and gets the same selling price for both the calculators. Find the cost price of each one of the two calculators.

Ans. A= 225.64, B=324.36

42) Soham bought grapes worth Rs. 500 and sold half of them at a gain of 8%. At whatgain percent must he sell the remaining grapes so as to get a gain of 20% on the whole?

Ans: 32%

43) A dealer buys a table listed at 1,800 and gets a discount of 25%. He spends 150 on transportation and sells it at a profit of 10%. Find the selling price of table.

Ans: 1,650



PROFIT, LOSS, DISCOUNT, COMMISSION AND BROKERAGE

Unit Structure :

- 2.1 Profit and Loss
- 2.2 Exercises
- 2.3 Discount
- 2.4 Trade discount and cash discount
- 2.5 Exercises
- 2.6 Commission and brokerage
- 2.7 Exercises

2.1 PROFIT AND LOSS

Traders, shopkeepers earn by selling goods generally. Traders earn by buying and selling goods. The price at which an item is purchased is called its cost price (CP). The price at which an item is sold is called its Net Selling Price (NSP).

If the Net Selling Price is greater than cost price, then a profit is earned. If the selling price is less than the cost price, then a loss is incurred. When the selling price is equal to cost price then neither profit nor loss is made. This is called a Break even point.

Profit = NSP - CP it NSP > CP

Loss = CP - NSP if NSP < CP

Profit percentage (loss percentage) is calculated when profits (or loss) is compared with the cost price.

Profit %
$$= \frac{Profit}{CP} \times 100$$

$$Loss \% = \frac{Loss}{CP} \times 100$$

This gives profit = $(Profit\%) \times \frac{CP}{100}$

Now, since, NSP = CP + Profit

Hence, NSP = CP +
$$(Profit\%) \times \frac{CP}{100}$$

$$NSP = \left(I + \frac{Profit\%}{100}\right) \times CP$$
$$NSP = \left(\frac{100 + Profit\%}{100}\right) \times CP$$

Similarly we can show that,

$$NSP = \left(\frac{100 - Loss\%}{100}\right) \times CP$$

$$NSP = \left(\frac{100 + Profit\%}{100}\right) \times CP \text{ when profit is earned}$$

$$NSP = \left(\frac{100 - Loss\%}{100}\right) \times CP \text{ when loss is incurred}$$

Solved Examples : (Profit and Loss)

1) Ms. Divya bought a property per Rs. 14,00,000 and sold it for Rs. 14,72,000. Find her percentage profit or loss.

Solution :

Cost Price = 14,00,000

Selling price = 14,72,000

Since selling price > cost price, profit is earned

Profit = Selling price - Cost Price

= 72,000

% Profit =
$$\frac{Profit}{CP} \times 100$$

= $\frac{72,000}{14,00,000} \times 100 = 5.14$

2) Mr. Harish bought a wardrobe for Rs. 4,50,000 and sold it for Rs. 4,36,000. Find his percentage gain or loss.

Solution :

Cost Price = 4,50,000

Selling Price = 4,36,000

Since cost price > selling price, Loss is incurred

Loss = Cost Price - Selling Price = 4,50,000 - 4,36,000 = 14,000

Loss % =
$$\frac{Loss}{CP} \times 100$$

= $\frac{14,000}{4,50,000} \times 100$
= 3.11

3) An article is bought for Rs. 4,800 and sold at 15% profit. What was the selling price?

Solution :

Selling price = ? Cost price = 4,800, Profit % = 15 Selling price = Cost Price $\left(\frac{100 + Profit\%}{100}\right)$ = $4800\left(\frac{100 + 15}{100}\right)$ = $4800\left(\frac{115}{100}\right)$ = 5520

4) An article was bought for Rs. 7,000 and sold at 2% loss. Find its selling price.

Solution :

Cost Price = 7,000, Loss % = 2 Selling Price = Cost Price $\left(\frac{100 - Loss\%}{100}\right)$

$$= 7000 \left(\frac{100 - 2}{100}\right)$$
$$= 7000 \left(\frac{98}{100}\right)$$
$$= 6860$$

5) Printer was sold at Rs. 8,980 at a 5% loss. Find its cost Price.

Solution :

Selling Price = 8,980, Loss % = 5
Selling Price = Cost Price
$$\left(\frac{100 - Loss\%}{100}\right)$$

 $8,980 = \text{Cost Price} \left(\frac{100 - 5}{100}\right)$
 $8,980 = \text{Cost Price} (0.95)$
 $\Rightarrow \quad \text{Cost Price} = \frac{8,980}{0.95} = 9,452.63$

6) Mr. Tushar earned a profit at 25% on cost by selling an article for Rs. 6,140. What would have been the percentage profit or loss if he had to sold the article for Rs. 7,040?

Solution :

Selling Price = 6,140, Profit % = 25
Selling Price = Cost Price
$$\left(\frac{100 + Profit\%}{100}\right)$$

 $6,140 = \text{Cost Price} \left(\frac{100 + 25}{100}\right)$
 $6,140 = \text{Cost Price} (1.25)$
 $\Rightarrow \quad \text{Cost Price} = \frac{6,140}{1.25} = 4,912$

Now Cost Price = 4,912, and selling price = 7,040 then profit % or Loss % = ?

Since selling price > cost price, profit is earned Profit = Selling Price - Cost Price = 7,040 - 4,912 = 2,128 % Profit = $\frac{Profit}{CP} \times 100 = \frac{2,128}{4,912} \times 100 = 43.32$ 7) Mr. Vinay made a 7% loss by selling the article for Rs. 13,625. What would have been his percentage loss or gain if he had sold it for Rs. 15,250?

Solution :

Selling Price = 13,625, Loss % = 7, CP = ?

Selling Price = Cost Price
$$\left(\frac{100 - Loss\%}{100}\right)$$

 $13,625 = \text{Cost Price}\left(\frac{100 - 7}{100}\right)$
 $13,625 = \text{Cost Price}\left(\frac{93}{100}\right)$
 $\Rightarrow \quad \text{Cost Price} = \frac{13,625}{0.93} = 14,650.54$

Selling Price = 15,250 and cost price = 14,650.54.

Since selling price > cost price, Profit is earned

% Profit =
$$\frac{Profit}{CP} \times 100 = \frac{599.46}{14,650.54} \times 100 = 4.09$$

8) A shopkeeper brought two tables at Rs. 8,800 each. He sold one table at 10% profit and the other at 10% loss. Find his total percentage gain or loss.

Solution :

For the first table, selling price =?
Selling price = Cost Price
$$\left(\frac{100 + Profit\%}{100}\right)$$

= 8,800 $\left(\frac{100 + 10}{100}\right)$
= 9,680

For Second table, Selling Price =?

Selling price = Cost Price
$$\left(\frac{100 - Loss\%}{100}\right)$$

= 8,800 $\left(\frac{100 - 10}{100}\right)$
= 7,920

Total Cost Price = 8,800 + 8,800 = 17,600

Total Selling Price = 9,680 + 7,920 = 17,600

Total Cost Price = Total Selling Price .: No Profit No Loss

9) A person sold two tables for Rs. 990 each and thus earned 10% profit on one and incurred a 10% loss on the other. If both tables are considered together, find his percentage profit or loss.

Solution :

Total selling price = 990 + 990 = 1980

For first table, CP = ?

Selling Price = Cost Price
$$\left(\frac{100 + Profit\%}{100}\right)$$

990 = Cost Price $\left(\frac{100 + 10}{100}\right)$
990 = Cost price (1.1)
 \Rightarrow Cost Price = $\frac{990}{101} = 900$

For Second table, CP = ?
Selling Price = Cost Price
$$\left(\frac{100 - Loss\%}{100}\right)$$

990 = Cost Price $\left(\frac{100 - 10}{100}\right)$
990

Cost Price $=\frac{990}{0.9}=1100$ \Rightarrow

Total Cost Price = 900 + 1100 = 2000

Total Selling Price = 990 + 990 = 1980

Since Total Selling Price < Total Cost Price, Loss is incurred.

Loss = Total Cost Price - Total Selling Price = 2000 - 1980= 20

Loss % =
$$\frac{Loss}{Total Cost Price} \times 100$$

= $\frac{20}{2000} \times 100 = 1$

10) When sold at a profit of 6% an article gives Rs. 715 more than when it is sold at a loss of 5%. Find its cost price?

Solution :

Let Cost Price = Rs. x

If the article is sold at 6% profit, then Selling Price = Cost Price $\left(\frac{100 + Profit\%}{100}\right)$ = $x\left(\frac{100 + 6}{100}\right)$ = 1.06x

If the article is sold at 5% loss, then Selling Price = Cost Price $\left(\frac{100 - Profit\%}{100}\right)$ = $x\left(\frac{100 - 5}{100}\right)$ = 0.95x

The difference = 1.06x - 0.95x = 0.11xBut it given as 715. 0.11x = 715 $\Rightarrow x = \frac{715}{0.11} = 6500$ \therefore Cost Price = 6500

2.2 EXERCISES

- 1) Ms. Sheena bought a property for Rs. 12,00,000 and sold it for Rs. 12,72,000. Find her percentage profit or loss.
- 2) Mr. Madhukar bought a sofa set for Rs. 3,50,000 and sold it for Rs. 3,36,000. Find his percentage profit or loss.
- 3) A Lamp shade was bought for Rs. 2,800 and sold at 15% profit, what was the selling price?
- 4) A cupboard was sold for Rs. 7,906 at 18% profit, what was the cost price?
- 5) An article was bought for Rs. 5,000 and sold at 5% loss. Find its selling price.
- 6) A printer was sold at Rs. 7,980 at 5% loss. Find its cost price.
- 7) By selling an article at Rs. 3,000, a person earned 20% profits. What would have been the percentage profit or loss is he has sold it at Rs. 2,750?

- 8) A person earned 12% profit by selling an article at Rs. 4,144. What would have been the selling price if he had sold it at 16% profit?
- 9) By selling an article at Rs. 24,288, a loss of 8% on cost was incurred. Had the article been sold at 8% profit, what would have been the selling price?
- 10) A shopkeeper bought 2 chairs at Rs. 4,400 each. He sold one chair at 10% profit and the other at 10% loss. Find his total percentage gain or loss.
- 11) When sold at a profit of 6%, an article fetches Rs. 715 more than when it is sold at a loss of 5%. Find its cost price.

Answers :

6% Profit 2) 4% loss 3) Rs. 3,220 4) Rs. 6,700 5) Rs. 4,900
 6) Rs. 8,400 7) 10% Profit 8) Rs. 4,292 9) Rs. 28,512
 10) No Profit No Loss 11) Rs. 6,500

2.3 DISCOUNT

In the market each good / item for sale has a marked price or printed price. The prices of all goods forms a list or catalogue, hence this rice is also called as the List Price or listed price or catalogue price.

 $LP \rightarrow List Price$

In order to attract customers, the seller (manufacturer or trader) offers a reduction in the list price. This reduction is called as Discount. After deducting the discount, an item is sold at the Net Selling Price (NSP). Thus,

NSP = LP - Discount

Usually discount is expressed as a percentage on the list price.

2.4 TRADE DISCOUNT AND CASH DISCOUNT

When a trader is selling goods to another trader (e.g. wholesaler to retailer) usually a two discounts structure is followed.

First at all, trade discount (TD) is given to all traders. T.D. is a percentage on the List price. List price minus the Trade Discount is called Invoice Price (IP) or reduced List Price.

If an item is purchased for immediate cash payment then an extra discount which is called as cash discount is offered. It is calculated against the invoice price. Thus, Trade discount = x% at List Price. Invoice Price = List Price - Trade Discount Cash discount = 9% at Invoice Price. Net Selling Price = Invoice Price - Cash discount Profit = NSP - CP Profit % = $\frac{Profit}{CP} \times 100$ Loss = CP - NSP Loss % = $\frac{Loss}{CP} \times 100$

Solved Examples : (Discount)

1) After giving a 15% discount on the list price, a pant is sold for Rs. 1,500, find the list price.

Solution :

Trade discount = 15% (on list price) Net selling price = 1500 Let List Price = Rs. x Trade discount = 15% of x = 0.15xNo Cash discount is given, \therefore Net Selling Price = Invoice Price Invoice Price = Net Selling Price = List Price - Trade discount 1500 = x - 0.15x1500 = 0.85x $\Rightarrow x = \frac{1500}{0.85} = 1764.71$

2) A trader bought an article for Rs. 4000 and listed it for Rs. 9000. He gave 10% discount on the list price. What was the his profit percentage?

Solution :

Cost price = 4000 List Price = 9000 Trade discount = 10% Invoice Price = List Price - Trade discount = 9000 - 10% at 9000 = 9000 - 900 = 8100 No cash discount is given \therefore Invoice Price = Net Selling Price Net Selling Price = 8100 Profit = Net Selling Price - Cost Price = 8100 - 4000 = 4100 Profit % = $\frac{Profit}{CP} \times 100$ = $\frac{4100}{4000} \times 100$ = 102.5

3) A trader purchased a cupboard and listed it two times his purchase price and then allowed 50% discount on the list price. Find the profit percentage.

Solution :

Let Cost Price = Rs. 100 \therefore List Price = 200 Trade discount = 50% on LP Invoice Price = List Price - Trade discount = 200 - 50% of 200 = 200 - 100 = 100

Cash discount is not given :. Invoice Price = Net Selling Price

Net Selling Price = 100

Profit = Net Selling Price - Cost Price= 100 - 100= 0

4) A trader purchased an item for Rs. 4000. After giving 20% discount on the list price, he made 44% Profit on his cost. Find the list price.

Solution :

Cost Price = 4000 Trade discount = 20% on List Price Let List Price = Rs. x Invoice Price = List Price - Trade discount = x - 20% of x = 0.8xProfit % = 44 Net Selling Price = Invoice Price

Net Selling Price = Cost Price
$$\left(\frac{100 + Profit\%}{100}\right)$$

 $0.8x = 4000 \left(\frac{100 + 44}{100}\right)$
 $0.8x = 4000 (1.44)$
 $0.8x = 5760$
 $x = \frac{5760}{0.8} = 7200$
∴ List Price = `7200

5) A trader gave 30% discount on List Price and made 50% profit on his cost. If his list price was Rs. 7200, find his cost price.

Solution :

Trade discount = 30% Profit % = 50 List Price = 7200 Invoice Price = List Price - Trade discount = 7200 - 30% of 7200 Invoice Price = 7200 - 2160 = 5040 Net Selling Price = Invoice Price Net Selling Price = Cost Price $\left(\frac{100 + Profit\%}{100}\right)$ $5040 = \text{Cost Price} \left(\frac{100 + 50}{100}\right)$ 5040 = Cost Price (1.5) $\text{Cost Price} = \frac{5040}{1.5} = 3360$

6) After giving a 20% discount, a pen was sold for Rs. 608 and 45% profit on cost was made. Find the list price and the cost price.

Solution :

```
Trade discount = 20%

Net Selling Price - 608

Profit % = 45

Let List Price = Rs. x

Invoice Price = List Price - Trade discount

= x - 20\% of x

= 0.8x

Net Selling Price = Invoice Price
```

 $\therefore \text{ Net Selling Price} = 0.8x$ $\therefore 0.8x = 608$ $x = \frac{608}{0.8} = 760$ $\therefore \text{ List Price} = \text{Rs. 760}$

Net Selling Price = Cost Price
$$\left(\frac{100 + Profit\%}{100}\right)$$

 $608 = \text{Cost Price} \left(\frac{100 + 45}{100}\right)$
 $\text{Cost Price} = \frac{608}{1.45} = 419.31$

7) A trader gave 20% discount on list price and a further 2% cash discount on the invoice price. If the list price was Rs. 3000 then find net selling price.

Solution :

List Price = 3000 Trade discount = 20% on List Price Invoice Price = List Price - Trade discount = 3000 - 20% of 3000 = 3000 - 600 = 2900 Cash discount is given on Invoice Price = 2% of 2400 = $2 \times \frac{1}{100} \times 2400 = 48$ Net Selling Price = Invoice Price - Cash discount

$$= 2400 - 48$$

= 2352

8) A trader gave 15% trade discount and further 1% cash discount for cash payment and sold article for Rs. 50000. Find the List Price.

Solution :

Let List Price = x Invoice Price = List Price - Trade discount = x - 15% of x = 0.85xCash discount = 1% of Invoice Price Net Selling Price = Invoice Price - Cash discount = 0.85x - 1% of 0.85x= 0.85x - 0.0085x $\therefore \text{ Net Selling Price} = 0.8415x$ But Net Selling Price = 50000 $\therefore 50000 = 0.8415x$ $\Rightarrow x = \frac{50000}{0.8415} = 59417.71$ $\therefore \text{ List Price} = \text{Rs. 59417.71}$

9) Shopkeeper purchased article for Rs. 4000 and listed it at 4 times the purchase price. He then allowed 30% trade discount and further 3% for cash payment. Find the profit percentage.

Solution:

Cost Price = Rs. 4000List Price = Rs. 16000Trade discount = 30% on List Price Invoice Price = List Price - Trade discount = 16000 - 30% of 16000 = 16000 - 4800= 11200Cash discount = 3% on invoice price Net Selling Price = Invoice Price - Cash discount = 11200 - 3% of 11200 = 11200 - 336 $Profit \Rightarrow Profit = Net Selling Price - Cost Price$ = 10864 - 4000= 6864 Profit % = $\frac{Profit}{Cost \ Price} \times 100$ $=\frac{6864}{4000}\times100$ =171.6

10) A trader gave 30% trade discount and 5% cash payment and made 20% Profit on his cost price of Rs. 8000. Find the list price.

Solution :

Cost Price = 8000 Let List Price = Rs. x Invoice Price = List Price - Trade discount = x - 30% of x= 0.7x

Cash discount is given on Invoice Price = 5%

$$\therefore \text{ Net Selling Price} = \text{Invoice Price} - \text{Cash discount}$$

$$= 0.7x - 5\% \text{ of } 0.7x$$

$$= 0.7x - 0.035x$$

$$= 0.665x$$
Net Selling Price = Cost Price $\left(\frac{100 + Profit\%}{100}\right)$

$$0.665x = 8000 \left(\frac{100 + 20}{100}\right)$$

$$0.665x = 8000 (1.2)$$

$$0.665x = 9600$$

$$\Rightarrow x = \frac{9600}{0.665} = 14436.09$$
List Price = 14436.09

11) A shopkeeper gave 30% trade discount and a further 2% cash discount on an item and sold it for a net price of Rs. 40000 and still made 25% profit on cost. Find the list price and cost price.

Solution :

...

Let List Price = Rs. xTrade discount = 30% on List Price : Invoice Price = List Price - Trade discount = x - 30% of x = 0.7xCash discount is 2% on Invoice Price Net Selling Price = Invoice Price - Cash discount = 0.7x - 2% of 0.7x= 0.7x - 0.014x= 0.686xBut Net Selling Price = 40000 \Rightarrow 40000 = 0.686x $\Rightarrow x = \frac{40000}{0.686} = 58309.04$ Net Selling Price = Cost Price $\left(\frac{100 + Profit\%}{100}\right)$ $40000 = \text{Cost Price}\left(\frac{100+25}{100}\right)$ \Rightarrow Cost Price = $\frac{40000}{1.25}$ = 32000

2.5 EXERCISES

- 1) After giving a 12% discount on the list price, a pant is sold for Rs. 1056, Find the List price.
- 2) A Trader bought gift item for Rs. 3,600 and listed it for Rs. 8,500. He gave 9% discount on the list price, what was his profit percentage?
- 3) A trader purchased a cupboard and listed it four times his purchase price and then allowed a 50% discount on the list price. Find the profit percentage.
- 4) A trader purchased an item for Rs. 4,000 and after giving 20% discount on the list price, he made 44% profit on his cost. Find the list price.
- 5) A trader gave 20% discount on list price made 44% profit on his cost. If the list price was Rs. 720, find his cost price.
- 6) After giving 20% discount, a pen was sold for Rs. 304 and 52% profit on cost was made. Find the list price and cost price.
- 7) A trader gave 10 trade discount on the list price and a further 1% cash discount on the invoice price, if the list price was Rs. 2430, then find net selling price.
- 8) A trader gave 10% trade discount and a further 1% discount for cash payment and sold a bathroom fitting for Rs. 216513. Find the list price.
- 9) A trader purchased a gift item for Rs. 4,000 and listed it 2.5 times the purchase price she then allowed 40% discount and further 4% for cash payment. Find the percentage profit.
- 10) After giving 25% trade discount and a further 4% discount for cash payment a merchant made 19.52% profit. If the list price was Rs. 16600 find the merchant's cost price.
- 11) A merchant allowed 30% trade discount and a further 2% cash payment discount on an item and sold it for a net price of Rs. 30870 and still made 20% profit on cost. Find the merchant's list price and cost price.

Answers :

1) Rs. 1200 5) Rs. 400	 2) 114.896% 6) Rs. 380, Rs. 200 	3) 100% 4) Rs. 7200 7) Rs. 2165.13
8) Rs. 243000	9) 44%	10) Rs. 10000
11) Rs. 45000	12) Rs. 25725	

2.6 COMMISSION AND BROKERAGE

Most of the manufacturers do not sell their goods directly to the customers. They employ other to sell goods for them. Traders who by and sell goods also sometimes employ others to buy and or / sell goods for them.

An agent or a commission agent is a person who buys and or / sells goods for another person. The person who employs the services of the agent is called the principal and the remuneration given by the principal to the agent is called commission.

A commission agent may sells goods for cash or on credit. If the sale takes place on credit, then there could be a risk of payment default.

A Del credere agent sells goods and guarantees for the collection of dues from the customers to the principal. for this he or she charges extra commission which is also known as del credere.

A broker is an agent who brings together buyer and seller and negotiated the sale. For this he or she charges a commission called brokerage from buyer and the seller.

An auctioneer sells the goods by public audition to the highest bidder.

Solved Examples : (Commission and Brokerage)

1) An agent charged 10% commission on sales and thus earned Rs. 5000. Find the value of sales.

Solution :

Sales = ?, Commission = 10% of sales

But commission = 5000 (Given)

Commission = 10% of sales

5000 = 10% of sales

$$5000 = \frac{1}{100} \times \text{sales}$$

$$\therefore \frac{5000 \times 100}{10} = \text{Sales}$$
$$\Rightarrow \text{Sales} = \text{Rs. } 50000$$

2) An agent sold goods worth Rs. 30000 and after deducting his commission, remitted Rs. 27125 to the principal. Find the rate of commission charged by the agent.

Solution :

Total Sales = Rs. 30000 Amount remitted = 27125 \therefore Commission = 30000 - 27125 = 2875Late rate of commission = x%Commission = x% of 30000 $\Rightarrow 2875 = x \times \frac{1}{100} \times 30000$ $\Rightarrow 2875 = 300x$ $\Rightarrow x = \frac{2875}{300} = 9.58$; 10%

4) A manufactured gave his agent 7% commission plus 4% del credere. The agent sold goods worth Rs. 25000. What amount should the agent give principal after deducting his commission?

Solution :

Total Sales = Rs. 25000 Ordinary Commission = 7% of 25000 $= 7 \times \frac{1}{100} \times 25000$ = 1750Del credere = 4% of 25000 $= 4 \times \frac{1}{100} \times 25000$ = 1000 $\therefore \text{ Total Commission} = \text{Ordinary Commission} + \text{Del credere}$ = 1750 + 1000 = 2750

Amount remitted after deducting his commission = 25000 - 2750 = 22250

4) A salesman receives 4% commission on sales up to Rs. 6000 and 5% commission on sales above Rs. 6000. If he sold goods worth Rs. 8000 in a week, find the commission earned by him.

Solution :

Commission at 4% on $=\frac{4}{100} \times 6000$ = 240

Additional Commission at 5% on (8000 - 6000) = 5% on 2000 = $5 \times \frac{1}{100} \times 2000$ = 100

Total Commission = 240 + 100 = 340

5) A company fixed the rate of commission to its salesman as follows : 5% on the first 12000, 9% on next 8000, 10% on next 9000 and 6% on balance.

Find the commission of a Salesman who received Sales worth Rs. 33000.

Solution :

Total Sales = 33000 = (12000 + 8000 + 9000 + 4000)Commission = 5% on 12000 + 9% on 8000 + 10% on 9000 + 6% on 4000 (balance) = $5 \times \frac{1}{100} \times 12000 + 9 \times \frac{1}{100} \times 8000 + 10 \times \frac{1}{100} \times 9000 + 6 \times \frac{1}{100} \times 4000$ = 600 + 720 + 900 + 240= `2460

6) A Company pays its salesman a monthly salary of Rs. 10000 and commission as follows : 2% on sales above Rs. 10000 and upto Rs. 16000 2.5% on sales over Rs. 16000 and upto Rs. 20000 3% on sales over Rs. 20000.

Find the total monthly remuneration of a salesman who sold goods worth Rs. 35000 in a month.

Solution :

Monthly Salary = Rs. 10000 Commission @2% between 10000 & 16000 = (16000 - 10000) $\Rightarrow 2.5\%$ on 4000 = 100 3% on sales over 20000 i.e. (35000 - 20000) = 3% on 15000 = 450 Total Commission = 120 + 100 + 450 = 670 Total monthly remuneration = 10000 + Total Commission = 10000 + 670 = 10670
A Salesman is allowed 8% commission on total sales plus a bonus of 2% on the sales exceeding Rs. 15000. If he earned Rs. 1600 on commission done, find his total earnings.

Solution :

Commission = Rs. 1600 Let Total Sales = Rs. x Commission = 8% of x $1600 = 8 \times \frac{1}{100} \times x$ 1600 = 0.08x $\Rightarrow x = \frac{1600}{0.08} = \text{Rs. 20,000}$ \therefore Total Sales = 20000 If sales exceeds Rs. 15000 then bonus will be @2% = 2% of (20000 - 15000) = 2% of 5000 $= 2 \times \frac{1}{100} \times 5000 = 100$ Total earnings = 1600 + 100 = 1700

8) A salesgirl is allowed 8% commission on total sales plus 3% bonus on sales above Rs. 24000. If her total earnings are Rs. 3440 then find the value of her sales.

Solution : Commission + bonus = 3440

Let total Sales be Rs. x Commission (a) 8% on x = 8% of x

$$= 8 \times \frac{1}{100} \times x$$
$$= 0.08x$$

Commission + bonus = 3440

Bonus = 3% of (x - 24000)

Commission + 3% of (x - 24000) = 3440

 \Rightarrow 3% of (x - 24000) = 3440 - Commission

$$\Rightarrow 3 \times \frac{1}{100} \times (x - 24000) = 3440 - 0.08x$$

$$\Rightarrow 3 \times \frac{1}{100} \times x - 3 \times \frac{1}{100} \times 24000 = 3440 - 0.08x$$

$$\Rightarrow 0.03x - 720 = 3440 - 0.08x$$

$$\Rightarrow 0.03x + 0.08x = 3440 + 720$$

$$\Rightarrow 0.11x = 4160$$

$$\Rightarrow x = \frac{4160}{0.11} = 37818.19$$

9) A house was sold through a broker for Rs. 90,00,000 who charged 2.5% from the buyer and 1.5% from the seller. Find the amount paid by the buyer. Also find the amounts received by the seller and the broker.

Solution : - Total Sales = Rs. **90,00,000**

Amount paid by buyer = Total purchase + brokerage

= 90,00,000 + 2.5% of 90,00,000

= 90,00,000 + 225000

= 92,25,000

Amount received by seller

= Total Sales - brokerage

= 90,00,000 - 1.5% of 90,00,000

= 90,00,000 - 1,35,000

Amount received by broker

= 2,25,000 + 1,35,000= 3,60,000

10) A Trader instructed his agent to buy 600 caps and sell them 50% above the purchase price. The agent charged 2% commission on the purchase and 3% commission on sales and thus earned Rs. 2000 as total commission what was the purchase price of a single cap?

Solution :

Let purchase price of a single cap = Rs. x Selling price of 600 caps = 600 x + 900 x= 900 x

 \therefore Commission of 2% on purchase = 2% of 600x

$$= 2 \times \frac{1}{100} \times 600x$$
$$= 12x$$

 $\therefore \text{ Commission of } 3\% \text{ on Sales} = 3\% \text{ of } 900x$ $= 3 \times \frac{1}{100} \times 900x$ = 27x $\therefore \text{ Total Commission} = 12x + 27x$ = 39xThis is given to be 2000 $\Rightarrow 39x = 2000$ $\Rightarrow x = \frac{2000}{39}$ $\Rightarrow x = 51.28 \text{ ; } 52$ Purchase price of a single cap = 52

11) A del credre agent charges 3% commission on cash sales and 5% commission on credit sales. In a particular year, he earned on an average 4.5% commission on total sales. Find the ratio of his cash sales to credit sales.

Solution :

Let Cash Sales = Rs. x Credit Sales = Rs. y Commission at 3% on Cash Sales = $\frac{3}{100} \times x$ = 0.03x Commission at 5% on Credit Sales = 0.05y Total Commission = 0.03x + 0.05y But given that Total Commission = 4.5% on Sales = $\frac{4.5}{100}$ (Cash Sales + Credit Sales) = 0.045 (x + y) = 0.045x + 0.045y

Thus we have,

Total Commission $\Rightarrow 0.03x + 0.05y = 0.045x + 0.045y$ $\Rightarrow 0.005y = 0.015x$ $\Rightarrow \frac{0.005}{0.015} = \frac{x}{y}$ $\Rightarrow \frac{1}{3} = \frac{x}{y}$ 12) After deducting his commission at 6% on first Rs. 90,000 and 9% on balance of sales made by him, an agent remitted Rs. 96,000 to his principal. Find the value of goods sold by him.

Solution :

Let Total Sales be Rs. x \therefore Commission at 6% on first Rs. 90000 $= \frac{6}{100} \times 90000 = 5400$ Commission at 9% on remaining Sales i.e. (x - 90000) $= \frac{9}{100} \times (x - 90000)$ = 0.09x - 8100 \therefore Total Commission = 5400 + 0.09x - 8100 = 0.09x - 2700Agent remitted to the principal = Sales Value - Total Commission = x - (0.09x - 2700) = x - 0.09x + 2700 = 0.91x + 2700This is given to be 96000 $\Rightarrow 0.91x + 2700 = 96000$

 $\Rightarrow 0.91x = 96000 - 2700$ $\Rightarrow x = \frac{93300}{0.91}$ $\Rightarrow x = 102527.47$

13) A Salesman is paid a fixed monthly salary plus a commission at a certain rate on sales. The salesman received Rs. 1130 and Rs. 1360 as remuneration for two successive months and his sales were Rs. 17100 and Rs. 21700 respected. Find the fixed monthly salary and the rate of commission.

Solution :

Remuneration = Salary + Commission Let Salary Rs. x Commission rate = y% $\Rightarrow 1130 = x + y\%$ of 17100 $\Rightarrow 1130 = x + 171y$ (1) 1360 = x + y% of 21700 $\Rightarrow 1360 = x + 217y$ (2) (1) - (2) 1130 = x + 171y -1360 = x - 217y -230 = -46y $\Rightarrow y = \frac{230}{46} = 5\%$ Rate of commission = 5% Putting y = 5 in (1), 1130 = x + 5% of 17100 i.e. 1130 = x + 5(171) 1130 = x + 855 $\Rightarrow x = 1130 - 855$ $\Rightarrow x = 275$ \Rightarrow Salary = `275

2.7 EXERCISES

- 1) An agent earned 4794 after charging 8.5% commission on sales. Find the value of sales.
- 2) An agent sold goods worth Rs. 25000 and after deducting his commission, remitted Rs. 24125 to the principal. Find the rate of commission charged by the agent.
- 3) A manufacturer gave his agent 6% commission plus 3% del credere. One agent sold goods worth Rs. 22000. What amount should the agent give the principal after deducting his commission?
- 4) A salesgirl receives 3.5% commission on sales upto Rs. 5000. If she sold goods worth Rs. 7500 in a week, find the commission earned by her.
- 5) A company fixed the rate of commission to its salesman as follows :

3% on first 1000

4% on next Rs. 9000

5% on next Rs. 8000

6% on balance

Find the remuneration of a salesman who secured sales worth Rs. 32500.

6) A Salesman is allowed 7% commission on total sales plus a bonus at 2.5% on the sales above Rs. 15000. If he earned Rs. 1400 on commission, alone, Find his total earnings.

- 7) A sales girl is allowed 8% commission on total sales plus 2% bonus on sales above Rs. 24000. If her total earnings are 2420 than find the value of her sales.
- 8) A house was sold through a broker for 80,00,000 who charges 2.5% from the buyer and 1.5% from the seller. Find the amount paid by the buyer. Also find the amount received by the seller and the broker.
- 9) A trader instructed his agent to buy 400 caps and sell them 50% above the purchase price. The agent charged 1% commission on the purchase and 2% commission on the sales and thus earned Rs. 1000 as total commission what was the purchase price of a single cap?
- 10) A del credere agent charges 3% commission on cash sales and 5% commission or credit sales. In a particular year, he earned on an average 4.5% commission on total sales find the ratio of his cash sales to credit sales.
- 11) An agent was paid Rs. 22275 as commission on total sales. If the rate of commission was 12% and the price of each TV was Rs. 6875, find the number of TV's he sold.
- 12) A Company fixed the rate of commission to its salesman as follows

3% on first Rs. 5000, 5% on the next Rs. 8000, 8% on next Rs. 10000 and 11% on the balance. The company had agreed to pay 114% of total sales as bonus it the sales crossed Rs. 30000. A salesman of the company secured sales worth Rs. 32000. Calculate total earnings of the salesman.

Answers :

1) Rs. 56,400	2)	3.5%	3)	Rs. 20000 4) Rs. 287.5
5) Rs. 1,390	6)	Rs. 1,525	7)	Rs. 29,000	
8) Rs. 82,00,000, Rs.	78,80	,000,Rs. 3,20,00	09)	100	10) 1:3
11) 27	12) I	Rs. 2,420			

Glossary :

Agent or Commission Agent : A person or a firm that buys and / or sells goods for another person or firm, for a remuneration (which is called commission and which is usually a percentage on the sales value.)

Broker : An agent who brings together prospective buyer and seller and negotiates a deal, charging a commission from both the buyer and the seller.

Cash Discount (C.D) : A reduction given on the invoice price for cash payment, usually by a manufacturer or a trader to another trader, thereby lowering the invoice price to the net selling price. Cash discount is specified as a percentage on the invoice price.

Cost Price (C.P.) : The price at which an article is purchased.

Del Credere agent : A commission agent who guarantees the principal the collection of dues from the customers for an extra remuneration (above the usual commission) which is called as del credere commission.

Discount : A reduction in the price given to attract customers. Discount is specified as a percentage on the price.

Invoice Price (1. P) or Reduced List Price : The list price minus the trade discount.

List Price (L.P.) or Printed Price or Marked Price or Catalogue Price: It is the price printed on goods, which is found in the price list or catalogue, before giving any discount.

Loss : A negative profit.

Net Selling Price (N.S.P.) : The price at which an article is actually sold.

Principal : The person or firm which employs the commission agent.

Profit : The difference between the net selling price and the cost price. If the profit is negative, it is called a loss.

Trade Discount (T.D.) : A reduction on the list price given by a manufacturer or a trader to a trader, thereby lowering the list Price to the invoice price. Trade Discount is specified as a percentage on the list price.

SIMPLE INTEREST AND COMPOUND INTEREST

Unit Structure :

- 3.0 Objectives
- 3.1 Introduction
- 3.2 Definitions of Terms Used In This Chapter
- 3.3 Simple Interest
- 3.4 Compound Interest

3.0 OBJECTIVES

After reading this chapter you will be able to:

- Define interest, principal, rate of interest, period.
- Find simple interest (SI), rate of S.I., period of investment.
- Find Compound Interest (CI), rate of C.I., Amount accumulated at the end of a period.
- Compound interest compounded yearly, half-yearly, quarterly or monthly.

3.1 INTRODUCTION

In every day life individuals and business firms borrow money from various sources for different reasons. This amount of money borrowed has to be returned from the lender in a stipulated time by paying some *interest* on the amount borrowed. In this chapter we are going to study the two types of interests viz. simple and compound interest. We start with some definitions and then proceed with the formula related to both the types of interests.

3.2 DEFINITIONS OF TERMS USED IN THIS CHAPTER

Principal: The sum borrowed by a person is called its *principal*. It is denoted by **P**.

Period: The time span for which money is lent is called *period*. It is denoted by n.

Interest: The amount paid by a borrower to the lender for the use of money borrowed for a certain period of time is called *Interest*. It is denoted by *I*.

Rate of Interest: This is the interest to be paid on the amount of Rs. 100 per annum (i.e. per year). This is denoted by *r*.

Total Amount: The sum of the principal and interest is called as the total amount (*maturity value*) and is denoted by *A*. Thus, A = P + I. i.e. Interest paid I = A - P.

3.3 SIMPLE INTEREST

The interest which is payable on the principal only is called as *simple interest* (S.I.). For example the interest on Rs. 100 at 11% after one year is Rs.11 and the amount is 100 + 11 = Rs. 111.

It is calculated by the formula: $I = \frac{Pnr}{100} = P \ge n \ge \frac{r}{100}$

Simple Interest = Prinicpal x period x rate of interest

Amount at the end of
$$n^{\text{th}}$$
 year = $\mathbf{A} = \mathbf{P} + \mathbf{I} = \mathbf{P} + \frac{Pnr}{100} = \mathbf{P} \left(1 + \frac{nr}{100} \right)$

Remark: The period *n* is always taken in 'years'. If the period is given in months/days, it has to be converted into years and used in the above formula. For example, if period is 4 months then we take n = 4/12 = 1/3 or if period is 60 days then n = 60/365.

Example 1: If Mr. Sagar borrows Rs. 500 for 2 years at 10% rate of interest, find (i) simple interest and (ii) total amount. Ans: Given P = Rs. 500, n = 2 and r = 10 %

(i) $I = \frac{Pnr}{100} = \frac{500 \times 2 \times 10}{100} = \text{Rs. 100}$ (ii) A = P + I = 500 + 100 = Rs. 600

3.3.1 Problems involving unknown factors in the formula I = $\frac{Pnr}{100}$

The formula I = $\frac{Pnr}{100}$ remaining the same, the unknown factor in the formula is taken to the LHS and its value is computed. For example, if rate of interest is unknown then the formula is rewritten as $r = \frac{I \times 100}{P \times n}$.

Example **2:** If Mr. Prashant borrows Rs. 1000 for 5 years and pays an interest of Rs. 300, find rate of interest.

Ans: Given P = 1000, n = 5 and I = Rs. 300

Now,
$$I = \frac{Pnr}{100}$$
 $\Rightarrow r = \frac{I \times 100}{P \times n} = \frac{300 \times 100}{1000 \times 5} = 6$

Thus, the rate of interest is 6%.

Example **3**: Find the period for Rs. 2500 to yield Rs. 900 in simple interest at 12%.

Ans: Given P = Rs. 2500, I = 900, r = 12%Now, $I = \frac{Pnr}{100} \implies n = \frac{I \ge 100}{P \ge r} = \frac{900 \ge 100}{2500 \ge 12} = 3$

Thus, the period is 3 years.

Example **4:** Find the period for Rs. 1000 to yield Rs. 50 in simple interest at 10%.

Ans: Given P = Rs. 1000, I = 50, r = 10%Now, $I = \frac{Pnr}{100} \implies n = \frac{I \ge 100}{P \ge r} = \frac{50 \ge 100}{1000 \ge 10} = 0.5$ Thus, the period is 0.5 years i.e. 6 months.

Example 5: Mr. Akash lent Rs. 5000 to Mr. Prashant and Rs. 4000 to Mr. Sagar for 5 years and received total simple interest of Rs. 4950. Find (i) the rate of interest and (ii) simple interest of each.

Ans: Let the rate of interest be r. S.I. for Prashant = $\frac{5000 \times 5 \times r}{100} = 250r$... (1) and S.I. for Sagar = $\frac{4000 \times 5 \times r}{100} = 200r$... (2) from (1) and (2), we have, total interest from both = 250r + 200r = 450rBut total interest received be Mr. Akash = Rs. 4950 $\therefore 450r = 4950$ $\Rightarrow r = \frac{4950}{450} = 11$ \therefore the rate of interest = 11%

Example 6: The S.I. on a sum of money is one-fourth the principal. If the period is same as that of the rate of interest then find the rate of interest.

Ans: Given
$$I = \frac{P}{4}$$
 and $n = r$
Now, we know that $I = \frac{Pnr}{100}$
 $\therefore \frac{P}{4} = \frac{P \ge r \ge r}{100} \implies \frac{100}{4} = r^2$
 $\therefore r^2 = 25 \implies r = 5.$
 \therefore the rate of interest = 5%

Example 7: If Rs. 8400 amount to Rs. 11088 in 4 years, what will Rs. 10500 amount to in 5 years at the same rate of interest?

Ans:

(*i*) Given n = 4, P = Rs. 8400, A = Rs. 11088 $\therefore I = A - P = 11088 - 8400 = \text{Rs. 2688}$ Let *r* be the rate of interest.

Now,
$$I = \frac{Pnr}{100}$$
 $\Rightarrow 2688 = \frac{8400 \times 4 \times r}{100}$
 $\therefore r = 8\%$

(*ii*) To find A when
$$n = 5$$
, $P = \text{Rs. } 10500$, $r = 8$
A = $P\left(1 + \frac{nr}{100}\right) = 10500 \text{ x} \left(1 + \frac{5 \text{ x 8}}{100}\right) = 10500 \text{ x} \frac{140}{100} = 14700$
∴ the required amount = Rs. 14,700

Example 8: Mr. Shirish borrowed Rs. 12,000 at 9% interest from Mr. Girish on January 25, 2007. The interest and principal is due on August 10, 2007. Find the interest and total amount paid by Mr. Shirish.

Ans: Since the period is to be taken in years, we first count number of days from 25th January to 10th August, which is **197** days.

D 107 0	January	6
Now, $I = \frac{Pnr}{1} = 12000 \text{ x} \frac{197}{1} \text{ x} \frac{9}{1}$	February	28
100 365 100	March	31
$\therefore I = \text{Rs. } 582.9$	April	30
Total amount = $P + I = 12000 + 582.9$	May	31
$\therefore A = \text{Rs. 12,582.9}$	June	30
	July	31
	August	10
	Total	197

Check your progress 10.1

- Find the SI and amount for the following data giving principal, rate of interest and number of years:

 (i) 1800, 6%, 4 years.
 (ii) 4500, 8%, 5 years
 (iii) 7650, 5.5%, 3 years.
 (iv) 6000, 7.5%, 6 years
 (v) 25000, 8%, 5 years
 (vi) 20000, 9.5%, 10 years.
- Ans: (i) 432, 2232(ii) 1800, 6300,(iii) 1262.25, 8912.25(iv) 2700, 8700(v) 10000, 35000(vi) 19000, 39000
- 2. Find the S.I. and the total amount for a principal of Rs. 6000 for 3 years at 6% rate of interest.

Ans: 1080, 7080

3. Find the S.I. and the total amount for a principal of Rs. 3300 for 6 years at 3¹/₂ % rate of interest.

Ans: 693, 3993

4. Find the S.I. and the total amount for a principal of Rs. 10550 for 2 years at 10¹/₄ % rate of interest.

Ans: 2162.75, 12712.75

5. Find the rate of interest if a person invests Rs. 1000 for 3 years and receives a S.I. of Rs. 150.

Ans: 5%

6. Find the rate of interest if a person invests Rs. 1200 for 2 years and receives a S.I. of Rs. 168.

Ans: 7%

- A person invests Rs. 4050 in a bank which pays 7% S.I. What is the balance of amount of his savings after (*i*) six months, (*ii*) one year?
 Ans: 141.75, 283.5
- A person invests Rs. 3000 in a bank which offers 9% S.I. After how many years will his balance of amount will be Rs. 3135?
 Ans: 6 months
- 9. Find the principal for which the SI for 4 years at 8% is 585 less than the SI for $3\frac{1}{2}$ years at 11%.

Ans: 9000

10. Find the principal for which the SI for 5 years at 7% is 250 less than the SI for 4 years at 10%.

Ans: 5000

11. Find the principal for which the SI for 8 years at 7.5% is 825 less than the SI for $6\frac{1}{2}$ years at 10.5%.

Ans: 10000

12. Find the principal for which the SI for 3 years at 6% is 230 more than the SI for $3\frac{1}{2}$ years at 5%.

Ans: 46000

13. After what period of investment would a principal of Rs. 12,350 amount to Rs. 17,043 at 9.5% rate of interest?

Ans: 4 years

14. A person lent Rs. 4000 to Mr. X and Rs. 6000 to Mr. Y for a period of 10 years and received total of Rs. 3500 as S.I. Find (*i*) rate of interest, (*ii*) S.I. from Mr. X, Mr. Y.

Ans: 3.5%, 1400, 2100

15. Miss Pankaj Kansra lent Rs. 2560 to Mr. Abhishek and Rs. 3650 to Mr. Ashwin at 6% rate of interest. After how many years should he receive a total interest of Rs. 3726?

Ans: 10 years

16. If the rate of S.I. on a certain principal is same as that of the period of investment yields same interest as that of the principal, find the rate of interest.

Ans: 10%

17. If the rate of S.I. on a certain principal is same as that of the period of investment yields interest equal to one-ninth of the principal, find the rate of interest.

Ans:
$$3\frac{1}{3}$$
 years

18. Find the principal and rate of interest if a certain principal amounts to Rs. 2250 in 1 year and to Rs. 3750 in 3 years.

Ans: 1500, 50%

- **19.** Find the principal and rate of interest if a certain principal amounts to Rs. 3340 in 2 years and to Rs. 4175 in 3 years. **Ans:** 1670, 50%
- 20. If Rs. 2700 amount Rs. 3078 in 2 years at a certain rate of interest, what will Rs. 7200 amount to in 4 years at the same rate on interest?
 Ans: 7%, 9216
- **21.** At what rate on interest will certain sum of money amount to three times the principal in 20 years?

Ans: 15%

22. Mr. Chintan earns as interest Rs. 1020 after 3 years by lending Rs. 3000 to Mr. Bhavesh at a certain rate on interest and Rs. 2000 to Mr. Pratik at a rate on interest 2% more than that of Mr. Bhavesh. Find the rates on interest.

Ans: 6%, 8%

23. Mr. Chaitanya invested a certain principal for 3 years at 8% and received an interest of Rs. 2640. Mr. Chihar also invested the same amount for 6 years at 6%. Find the principal of Mr. Chaitanya and the interest received by Mr. Chihar after 6 years.

Ans: 11000, 3960

- 24. Mr. Ashfaque Khan invested some amount in a bank giving 8.5% rate of interest for 5 years and some amount in another bank at 9% for 4 years. Find the amounts invested in both the banks if his total investment was Rs. 75,000 and his total interest was Rs. 29,925.
 Ans: 45000, 30000
- **25.** Mrs. Prabhu lent a total of Rs. 48,000 to Mr. Diwakar at 9.5% for 5 years and to Mr. Ratnakar at 9% for 7 years. If she receives a total interest of Rs. 25,590 find the amount she lent to both.

Ans: 18000, 30000

3.4 COMPOUND INTEREST

The interest which is calculated on the amount in the previous year is called *compound interest*.

For example, the compound interest on Rs. 100 at 8% after one year is Rs. 8 and after two years is 108 + 8% of 108 = Rs. 116.64

If *P* is the principal, *r* is the rate of interest p.a. then the amount at the end of n^{th} year called as *compound amount* is given by the formula:

$$\boldsymbol{A} = \boldsymbol{P} \left(1 + \frac{\boldsymbol{r}}{100} \right)^n$$

The *compound interest* is given by the formula:

$$\mathbf{CI} = \mathbf{A} - \mathbf{P}$$

<u>Note</u>:

1. The interest may be compounded annually (yearly), semi-annually (half yearly), quarterly or monthly. Thus, the general formula to calculate the amount at the end of n years is as follows:

$$A = P \left(1 + \frac{r}{p \ge 100} \right)^{np}$$

Here *p*: number of times the interest is compounded in a year.

p = 1 if interest is compounded **annually**

p = 2 if interest is compounded semi-annually (half-yearly)

p = 4 if interest is compounded quarterly

p = 12 if interest is compounded monthly

2. It is easy to calculate amount first and then the compound interest as compared with finding interest first and then the total amount in case of simple interest.

Example **9**: Find the compound amount and compound interest of Rs. 1000 invested for 10 years at 8% if the interest is compounded annually.

Ans: Given P = 1000, r = 8, n = 10. Since the interest is compounded annually, we have

$$A = P\left(1 + \frac{r}{100}\right)^n = 1000 \text{ x} \left(1 + \frac{8}{100}\right)^{10} = 1000 \text{ x} 2.1589 = \text{Rs. } 2158.9$$

Example **10**: Find the principal which will amount to Rs. 11,236 in 2 years at 6% compound interest compounded annually.

Ans: Given
$$A = \text{Rs. 11236}, n = 2, r = 6 \text{ and } P = ?$$

Now, $A = P\left(1 + \frac{r}{100}\right)^n$
 $\therefore 11236 = P\left(1 + \frac{6}{100}\right)^2 = P \times 1.1236$
 $\therefore P = \frac{11236}{1.1236} = 10,000$

 \therefore the required principal is Rs. 10,000.

Example 11: Find the compound amount and compound interest of Rs. 1200 invested for 5 years at 5% if the interest is compounded *(i)* annually, *(ii)* semi annually, *(iii)* quarterly and *(iv)* monthly.

Ans: Given
$$P = \text{Rs. } 1200, r = 5, n = 5$$

Let us recollect the formula $A = P \left(1 + \frac{r}{p \ge 100} \right)^{np}$
(*i*) If the interest is compounded annually, $p = 1$:
 $A = P \left(1 + \frac{r}{100} \right)^n = 1200 \ge 1200 \ge 1.2763 = \text{Rs. } 1531.56$
 $CI = A - P = 1531.56 - 1200 = \text{Rs. } 331.56$

(*ii*) If the interest is compounded semi-annually,
$$p = 2$$
:
 $A = P \left(1 + \frac{r}{2 \times 100} \right)^{2n} = 1200 \times \left(1 + \frac{5}{200} \right)^{10} = 1200 \times 1.28 =$ Rs. 1536
 $CI = A - P = 1536 - 1200 =$ Rs. 336.

(*iii*) If the interest is compounded quarterly,
$$p = 4$$
:
 $A = P\left(1 + \frac{r}{4 \times 100}\right)^{4n} = 1200 \times \left(1 + \frac{5}{400}\right)^{20} = 1200 \times 1.2820 =$
Rs. 1538.4
 $CI = A - P = 1538.4 - 1200 =$ **Rs. 338.4**

(*iv*) If the interest is compounded monthly,
$$p = 12$$
:
 $A = P \left(1 + \frac{r}{12 \times 100} \right)^{12n} = 1200 \times \left(1 + \frac{5}{1200} \right)^{60} = 1200 \times 1.2834 =$ Rs. 1540
 $CI = A - P = 1540 - 1200 =$ Rs. 340

Example 12: Mr. Santosh wants to invest some amount for 4 years in a bank. Bank *X* offers 8% interest if compounded half yearly while bank *Y* offers 6% interest if compounded monthly. Which bank should Mr. Santosh select for better benefits?

Ans: Given n = 4. Let the principal Mr. Santosh wants to invest be P = Rs. 100 From Bank X: r = 8 and interest is compounded half-yearly, so p = 2. $\therefore A = P \left(1 + \frac{r}{2 \times 100} \right)^{2n} = 100 \times \left(1 + \frac{8}{200} \right)^4 = 116.9858 \dots (1)$

<u>From Bank *Y*</u>: r = 6, p = 12

$$\therefore A = P \left(1 + \frac{r}{12 \times 100} \right)^{12n} = 100 \times \left(1 + \frac{6}{1200} \right)^{48} = 127.0489 \qquad \dots (2)$$

Comparing (1) and (2), Dr. Ashwinikumar should invest his amount in bank Y as it gives more interest at the end of the period.

Example **13**: In how many years would Rs. 75,000 amount to Rs. 1,05,794.907 at 7% compound interest compounded semi-annually?

Ans: Given A = Rs. 105794.907, P = Rs. 75000, r = 7, p = 2

$$A = P \left(1 + \frac{r}{2 \times 100} \right)^{2n}$$

$$\therefore 105794.907 = 75000 \times \left(1 + \frac{7}{200} \right)^{2n}$$

$$\therefore \frac{105794.907}{75000} = (1.035)^{2n}$$

$$\therefore 1.41059876 = (1.035)^{2n}$$

$$\therefore (1.035)^{10} = (1.035)^{2n} \Rightarrow 2n = 10$$

Thus, $n = 5$

Example 14: A certain principal amounts to Rs. 4410 after 2 years and to Rs.4630.50 after 3 years at a certain rate of interest compounded annually. Find the principal and the rate of interest.

Ans: Let the principal be *P* and rate of interest be *r*.

Now, we know that
$$A = P\left(1 + \frac{r}{100}\right)^n$$

From the given data we have,
 $4410 = P\left(1 + \frac{r}{100}\right)^2$ and $4630.5 = P\left(1 + \frac{r}{100}\right)^3$
 $\therefore 4410 = P(1 + 0.01r)^2 \qquad \dots (1)$
Do not write '1 + 0.01r'
as 1.01r

$$4630.5 = P(1+0.01r)^3 \qquad \dots (2)$$

Dividing (2) by (1), we have

$$\frac{4630.5}{4410} = \frac{P(1+0.01r)^3}{P(1+0.01r)^2} \implies 1.05 = 1+0.01r$$

$$\therefore \quad 0.05 = 0.01r$$

Thus, $r = 5\%$

Example 15: Find the rate of interest at which a sum of Rs. 2000 amounts to Rs. 2690 in 3 years given that the interest is compounded half yearly. $(\sqrt[6]{1.345} = 1.05)$

Ans: Given
$$P = \text{Rs. } 2000, A = \text{Rs. } 2680, n = 3, p = 2$$

Now, $A = P\left(1 + \frac{r}{2 \times 100}\right)^{2n}$
 $\therefore 2690 = 2000 \times \left(1 + \frac{r}{200}\right)^{6}$
 $\therefore \frac{2690}{2000} = \left(1 + \frac{r}{200}\right)^{6} \Rightarrow 1.345 = \left(1 + \frac{r}{200}\right)^{6}$
 $\therefore \sqrt[6]{1.345} = 1 + \frac{r}{200} \Rightarrow 1.05 = 1 + \frac{r}{200}$

 $\therefore r = 0.05 \text{ x } 200 = 10\%$ Thus, the rate of compound interest is **10 %**.

Example 16: If the interest compounded half yearly on a certain principal at the end of one year at 8% is Rs. 3264, find the principal.

Ans: Given
$$CI = Rs$$
. 3264, $n = 1$, $p = 2$ and $r = 8$
Now, $CI = A - P = P\left(1 + \frac{8}{200}\right)^2 - P$
i.e. 3264 = $P[(1.04)^2 - 1] = 0.0816P$
 $\therefore P = \frac{3264}{0.0816} = 40000$

Thus, the principal is **Rs. 40,000**.

Check your progress 10.2

1. Compute the compound amount and interest on a principal of Rs. 21,000 at 9% p.a. after 5 years.

Ans: 32,311.10, 11,311.10

2. Compute the compound amount and interest on a principal of Rs. 6000 at 11% p.a. after 8 years.

Ans: 13827.23, 7827.23

- 3. Compute the compound amount and compound interest of Rs. 5000 if invested at 11% for 3 years and the interest compounded *i*) annually, (*ii*) semi annually, (*iii*) quarterly and (*iv*) monthly.
 Ans: (*i*) 6838.16, 1838.16 (*ii*) 6894.21, 189421 (*iii*) 6923.92, 1923.92 (*iv*) 6944.39, 1944.39
- 4. Compute the compound amount and compound interest of Rs. 1200 if invested at 9% for 2 years and the interest compounded *i*) annually, (*ii*) semi annually, (*iii*) quarterly and (*iv*) monthly.
 Ans: (*i*) 1425.72, 225.72 (*ii*) 1431.02, 231.02 (*iii*) 1433.8, 233.8 (*iv*) 1435.7, 235.7
- Miss Daizy invested Rs. 25,000 for 5 years at 7.5% with the interest compounded semi-annually. Find the compound interest at the end of 5 years.
 Ans: 11,126.10
- 6. Mr. Dayanand borrowed a sum of Rs. 6500 from his friend at 9% interest compounded quarterly. Find the interest he has to pay at the end of 4 years? Ans: 2779.54
- 7. Mr. Deepak borrowed a sum of Rs. 8000 from his friend at 8% interest compounded annually. Find the interest he has to pay at the end of 3 years? Ans: 2077.70
- Mr. Deshraj borrowed Rs. 1,25,000 for his business for 3 years at 25% interest compounded half yearly. Find the compound amount and interest after 3 years. Ans: 2,53,410.82; 12,8410.82
- **9.** Mrs. Hemlata bought a Sony Digital Camera for Rs. 15,800 from Vijay Electronics by paying a part payment of Rs. 2,800. The remaining amount was to be paid in 3 months with an interest of 9% compounded monthly on the due amount. How much amount did Mrs. Hemlata paid and also find the interest.

Ans: 13294.70, 294.70

10. Mr. Irshad bought a Kisan Vikas Patra for Rs. 10000, whose maturing value is Rs. 21,000 in $4\frac{1}{2}$ years. Calculate the rate of interest if the compound interest is compounded quarterly.

Ans: 16.8%

- 11. What sum of money will amount to Rs. 11236 in 2 years at 6% p.a. compound interest?Ans: 10,000
- 12. Find the principal which will amount to Rs. 13468.55 in 5 years at 6% interest compounded quarterly. [$(1.015)^{20} = 1.346855$]

Ans: 10000

- 13. Find the principal which will amount to Rs. 30626.075 in 3 years at 7% interest compounded yearly. Ans: 25000
- 14. Find the principal if the compound interest payable annually at 8% p.a. for 2 years is Rs. 1664. Ans: 10000
- 15. If Mr. Sagar wants to earn Rs. 50000 after 4 years by investing a certain amount in a company at 10% rate of interest compounded annually, how much should he invest? Ans: 34150.67
- 16. Find after how many years will Rs. 4000 amount to Rs. 4494.40 at 6% rate of interest compounded yearly. Ans: n = 2
- 17. Find after how many years Rs. 10,000 amount to Rs. 12,155 at 10% rate of interest compounded half-yearly. Ans: n = 1
- 18. Find the rate of interest at which a principal of Rs.10000 amounts to Rs. 11236 after 2 years.Ans: 6%
- **19.** Find the rate of interest at which a principal of Rs.50000 amounts to Rs. 62985.6 after 3 years. $(\sqrt[3]{1.259712} = 1.08)$ Ans: 8%
- **20.** Mrs. Manisha Lokhande deposited Rs. 20,000 in a bank for 5 years. If she received Rs.3112.50 as interest at the end of 2 years, find the rate of interest p.a. compounded annually. **Ans:** 7.5%
- 21. A bank X announces a super fixed deposit scheme for its customers offering 10% interest compounded half yearly for 6 years. Another bank Y offers 12% simple interest for the same period. Which bank's scheme is more beneficial for the customers? Ans: Bank X
- **22.** *ABC* bank offers 9% interest compounded yearly while *XYZ* bank offers 7% interest compounded quarterly. If Mr. Arunachalam wants to invest Rs. 18000 for 5 years, which bank should he choose?

Ans: Bank ABC

- 23. Mangesh borrowed a certain amount from Manish at a rate of 9% for 4 years. He paid Rs. 360 as simple interest to Manish. This amount he invested in a bank for 3 years at 11% rate of interest compounded quarterly. Find the compound interest Mangesh received from the bank after 3 years. Ans: 1384.78
- **24.** On a certain principal for 3 years the compound interest compounded annually is Rs. 1125.215 while the simple interest is Rs. 1050, find the principal and the rate of interest.

Ans: 5000, 7%

25. On a certain principal for 4 years the compound interest compounded annually is Rs. 13923 while the simple interest is Rs. 12000, find the principal and the rate of interest.Ans: 30000, 10%.

26. Which investment is better for Mr. Hariom Sharma (i) 6% compounded half yearly or (ii) 6.2% compounded quarterly?

Ans:

27. Which investment is better for Mr. Suyog Apte (i) 9% compounded yearly or (ii) 8.8% compounded quarterly?

Ans:

28. A bank X offers 7% interest compounded semi-annually while another bank offers 7.2% interest compounded monthly. Which bank gives more interest at the end of the year?

Ans:

29. Mr. Nitin Tare has Rs. 10000 to be deposited in a bank. One bank offers 8% interest p.a. compounded half yearly, while the other offers 9% p.a. compounded annually. Calculate the returns in both banks after 3 years. Which bank offers maximum return after 3 years?

Ans:

ANNUITIES AND EMI

Unit Structure :

- 4.0 Objectives
- 4.1 Introduction
- 4.2 Annuity
- 4.3 Types of Annuities
- 4.4 Sinking Fund
- 4.5 Equated Monthly Installment (Emi)

4.0 OBJECTIVES

After reading this chapter you will be able to:

- Define annuity, future value, present value, EMI, Sinking Fund.
- Compute Future Value of annuity due, Present Value of an ordinary annuity.
- Compute EMI of a loan using reducing balance method and flat interest method.
- Compute Sinking Fund (periodic payments).

4.1 INTRODUCTION

In the previous chapter we have seen how to compute compound interest when a lump sum amount is invested at the beginning of the investment. But many a time we pay (or are paid) a certain amount not in lump sum but in periodic installments. This series of equal payments done at periodic intervals is called as *annuity*.

Let us start the chapter with the definition of an *annuity*.

4.2ANNUITY

A series of equal periodic payments is called *annuity*. The payments are of *equal size* and *at equal time interval*.

The common examples of annuity are: monthly recurring deposit schemes, premiums of insurance policies, loan installments, pension installments etc. Let us understand the terms related to annuities and then begin with the chapter.

Periodic Payment:

The amount of payment made is called as *periodic payment*.

Period of Payment:

The time interval between two payments of an annuity is called as the *period of payment*.

Term of an annuity:

The time of the beginning of the first payment period to the end of the last payment period is called as *term of annuity*. An annuity gets *matured* at the end of its term.

4.3 **TYPES OF ANNUITIES**

Though we will be discussing two types of annuities in detail, let us understand different types of annuities based on the duration of the term or on the time when the periodic payments are made. On the basis of the closing of an annuity, there are three types of annuities:

1. Certain Annuity:

Here the duration of the annuity is fixed (or certain), hence called *certain annuity*. We will be learning such annuities in detail.

2. Perpetual Annuity:

Here the annuity has no closing duration, i.e. it has indefinite duration. Practically there are rarely any perpetuities.

3. Contingent Annuity:

Here the duration of the annuity depends on an event taking place. An example of contingent annuity is *life annuity*. Here the payments are to be done till a person is alive, like, pension, life insurance policies for children (maturing on the child turning 18 years) etc.

On the basis of when the periodic payments are made we have two types of annuities: ordinary annuity and annuity due.

4.3.1 Immediate (Ordinary) Annuity:

The annuity which is paid at the *end of each period* is called as *immediate* (*ordinary*) *annuity*. The period can be monthly, quarterly or yearly etc. For example, stock dividends, salaries etc.

Let us consider an example of an investment of Rs. 5000 each year is to be made for four years. If the investment is done at the end of each year then we have the following diagrammatic explanation for it:



4.3.2 Present Value:

The sum of all periodic payments of an annuity is called its *present value*. In simple words, it is that sum which if paid *now* will give the same amount which the periodic payments would have given at the end of the decided period. It is the one time payment of an annuity. The formula to find the present value (PV) is as follows:



Where *P*: periodic equal payment

r: rate of interest p.a.

p: period of annuity

Let $i = \frac{r}{p \times 100}$, the rate per period, then the above formula can be

rewritten as follows:

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4.3.3 Future Value (Accumulated value):

The sum of all periodic payments along with the interest is called the *future value (accumulated amount)* of the annuity.

The formula to find the future value (*A*) of an immediate annuity is as follows:

$$A = \frac{P}{\frac{r}{p \ge 100}} \left[\left(1 + \frac{r}{p \ge 100} \right)^{np} - 1 \right]$$

$$A = \frac{P}{i} \left[\left(1 + i \right)^{np} - 1 \right]$$

$$A = \frac{P}{i} \left[\left(1 + i \right)^{np} - 1 \right]$$
Here,

$$P: \text{ periodic equal payment}$$

$$r: \text{ rate of interest p.a.}$$

$$p: \text{ period of annuity i.e. yearly}$$
half yearly, quarterly on monthly
and $i = \frac{r}{p \ge 100}$

Example 1: Find the future value after 2 years of an immediate annuity of Rs. 5000, the rate of interest being 6% p.a compounded annually.

Ans: Given
$$n = 2$$
, $P = \text{Rs. 5000}$, $r = 6$ and $p = 1 \implies i = \frac{6}{100} = 0.06$
 $A = \frac{P}{i} \left[(1+i)^{np} - 1 \right] = \frac{5000}{0.06} \left[(1+0.06)^2 - 1 \right] = 5000 \left[\frac{1.1236 - 1}{0.06} \right]$
∴ $A = 5000 \ge 2.06 = \text{Rs. 10300}$

Example 2: Find the amount for an ordinary annuity with periodic payment of Rs. 3000, at 9% p.a. compounded semi-annually for 4 years.

Ans: Given
$$n = 4$$
, $P = \text{Rs. } 3000$, $r = 9$ and $p = 2 \implies i = \frac{9}{2 \times 100} = 0.045$
Now, $A = \frac{P}{i} \left[(1+i)^{np} - 1 \right] = \frac{3000}{0.045} \left[(1+0.045)^{2 \times 4} - 1 \right] = \frac{3000}{0.045} \times 0.4221$
Thus, $A = \text{Rs. } 28,140$

Example 3: Mr. Ravi invested Rs. 5000 in an annuity with quarterly payments for a period of 2 years at the rate of interest of 10%. Find the accumulated value of the annuity at the end of 2^{nd} year.

Ans: Given
$$n = 2$$
, $P = \text{Rs.5000}$, $r = 10$ and $p = 4 \implies i = \frac{10}{4 \times 100} = 0.025$
Now, $A = \frac{P}{i} \left[(1+i)^{np} - 1 \right] = \frac{5000}{0.025} \left[(1.025)^{2 \times 4} - 1 \right] = \frac{5000}{0.025} \times 0.2184$
Thus, $A = \text{Rs. 43,680}$

Example 4: Mr. Ashok Rane borrowed Rs. 20,000 at 4% p.a. compounded annually for 10 years. Find the periodic payment he has to make. Ans: Given PV = Rs. 20,000; n = 10; p = 1 and $r = 4 \Rightarrow i = 0.04$ Now to find the periodic payments P we use the following formula:

$$PV = \frac{P}{i} \left[1 - \frac{1}{(1+i)^{np}} \right]$$

$$\therefore 20000 = \frac{P}{0.04} \left[1 - \frac{1}{(1+0.04)^{10}} \right] = \frac{P}{0.04} \ge 0.3244$$

$$\therefore P = \frac{20000 \ge 0.04}{0.3244} = 2466.09$$

Thus, Mr. Rane has to make the periodic payments of Rs. 2466.09

Example 5: Find the future value of an immediate annuity after 3 years with the periodic payment of Rs. 12000 at 5% p.a. if the period of payments is (i) yearly, (ii) half-yearly, (iii) quarterly and (iv) monthly.

Ans: Given
$$P = \text{Rs. } 1200, n = 3, r = 5$$

(i) period $p = 1$ then $i = \frac{5}{100} = 0.05$
 $A = \frac{P}{i} \left[(1+i)^n - 1 \right] = \frac{12000}{0.05} \left[(1+0.05)^3 - 1 \right] = \frac{12000}{0.05} \left[1.1576 - 1 \right]$
 $\therefore A = 12000 \text{ x } 3.1525 = \text{Rs. } 37,830$

(ii) period
$$p = 2$$
 then $i = \frac{5}{2 \times 100} = 0.025$

$$A = \frac{P}{i} \left[(1+i)^{2n} - 1 \right] = \frac{12000}{0.025} \left[(1+0.025)^6 - 1 \right] = \frac{12000}{0.025} \times 0.1597$$

$$\therefore A = 12000 \times 6.388 =$$
Rs. 76,656

(iii) period
$$p = 4$$
 then $i = \frac{5}{4 \times 100} = 0.0125$

$$A = \frac{P}{i} \left[\left(1+i \right)^{4n} - 1 \right] = \frac{12000}{0.0125} \left[\left(1+0.0125 \right)^{12} - 1 \right] = \frac{12000}{0.0125} \ge 0.16075$$

$$\therefore A = 12000 \text{ x } 12.86 = \text{Rs. 1,54,320}$$

(iv) period
$$p = 12$$
 then $i = \frac{5}{12 \times 100} = 0.00417$

$$A = \frac{P}{i} \left[\left(1+i \right)^{12n} - 1 \right] = \frac{12000}{0.00417} \left[\left(1+0.00417 \right)^{36} - 1 \right] = \frac{12000}{0.00417} \ge 0.1615$$

$$\therefore A = 1200 \text{ x } 38.729 = \text{Rs. 4,64,748}$$

.....

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Example 6: Mr. Nagori invested certain principal for 3 years at 8% interest compounded half yearly. If he received Rs.72957.5 at the end of 3^{rd} year, find the periodic payment he made. $[(1.04)^6 = 1.2653]$

Ans: Given
$$n = 3$$
, $r = 8$, $p = 2 \Rightarrow i = \frac{8}{2 \times 100} = 0.04$
Now, $A = \frac{P}{i} \left[(1+i)^{np} - 1 \right]$
∴ 72957.5 = $\frac{P}{0.04} [(1+0.04)^6 - 1] = \frac{P}{0.04} \times 0.2653$
∴ 72957.5 = $P[6.6325]$
∴ $P = \frac{72957.5}{6.6325} = 11000$

Thus, the periodic payment is Rs. 11,000

4.4 SINKING FUND

The fund (money) which is kept aside to accumulate a certain sum in a fixed period through periodic equal payments is called as **sinking fund**.

We can consider an example of a machine in a factory which needs to be replaced after say 10 years. The amount for buying a new machine 10 years from now may be very large, so a proportionate amount is accumulated every year so that it amounts to the required sum in 10 years. This annual amount is called as *sinking fund*. Another common example is of the *maintenance tax* collected by any Society from its members.

A sinking fund being same as an annuity, the formula to compute the terms is same as that we have learnt in section **2.3.3**

Example 7: A company sets aside a sum of Rs. 15,000 annually to enable it to pay off a debenture issue of Rs. 1,80,000 at the end of 10 years. Assuming that the sum accumulates at 6% p.a., find the surplus after paying off the debenture stock.

Ans: Given P = Rs. 15000, n = 10, r = 6 ⇒ i = 0.06
∴ A =
$$\frac{P}{i} \left[(1+i)^n - 1 \right] = \frac{15000}{0.06} \times \left[(1+0.06)^{10} - 1 \right] = \frac{15000}{0.06} \times 0.7908$$

∴ A = Rs. 1,97,700

Thus, the surplus amount after paying off the debenture stock is = 197712 - 180000 =**Rs. 17712**.

Example 8: Shriniketan Co-op Hsg. Society has 8 members and collects Rs. 2500 as maintenance charges from every member per year. The rate of compound interest is 8% p.a. If after 4 years the society needs to do a work worth Rs. 100000, are the annual charges enough to bear the cost?

Ans: Since we want to verify whether Rs. 2500 yearly charges are enough or not we assume it to be *P* and find its value using the formula:

$$A = \frac{P}{i} \left[\left(1 + i \right)^n - 1 \right]$$

Here $A = \text{Rs. 100000}, n = 4, r = 8 \implies i = 0.08$

$$\therefore P = \frac{A \ge i}{(1+i)^n - 1} = \frac{100000 \ge 0.08}{(1+0.08)^4 - 1} = 22192$$

Thus, the annual payment of all the members i.e. 8 members should be Rs. 22192.

: the annual payment per member = $\frac{22192}{8}$ = Rs. 2774

This payment is less than Rs. 2500 which the society has decided to take presently. Thus, the society should increase the annual sinking fund.

4.5 EQUATED MONTHLY INSTALLMENT (EMI)

Suppose a person takes a loan from a bank at a certain rate of interest for a fixed period. The equal payments which the person has to make to the bank per month are called as *equated monthly installments* in short EMI.

Thus, EMI is a kind of annuity with **period of payment being monthly** and the present value being the sum borrowed.

We will now study the method of finding the EMI using reducing balance method and flat interest method.

(a) Reducing balance method:

Let us recall the formula of finding the present value of an annuity. $p \begin{bmatrix} 1 \end{bmatrix}$

$$PV = \frac{P}{i} \left[1 - \frac{1}{\left(1+i\right)^{np}} \right]$$

The equal periodic payment (P) is our EMI which is denoted it by E. The present value (*PV*) is same as the sum (*S*) borrowed.

Also the period being monthly p = 12, $i = \frac{r}{1200}$ as we are interested in

finding monthly installments and *n* is period in years. Substituting this in the above formula we have:

$$S = \frac{E}{i} \left[1 - \frac{1}{\left(1+i\right)^{12n}} \right]$$

Thus, if S is the sum borrowed for n years with rate of interest r % p.a. then the *EMI* is calculated by the formula:

$$E = \frac{S \ge i}{1 - \frac{1}{(1+i)^{12n}}}$$

(b) Flat Interest Method:

Here the amount is calculated using Simple Interest for the period and the EMI is computed by dividing the amount by total number of monthly installments.

Let S denote the sum borrowed, r denote the rate of interest and n denote the duration in years, then as we know the amount using simple interest formula is $A = S\left(1 + \frac{nr}{100}\right)$. The total number of monthly installments for duration of **n** years is **12n**. Hence the *EML* is calculated as

duration of n years is 12n. Hence the *EMI* is calculated as

$$\boldsymbol{E} = \frac{A}{12n}$$

Example 9: Mr. Sudhir Joshi has taken a loan of Rs. 10,00,000 from a bank for 10 years at 11% p.a. Find his EMI using (a) reducing balance method and (b) Flat interest method.

Ans: Given S = Rs. 1000000, n = 10, $r = 11 \implies i = \frac{11}{1200} = 0.0092$

(a) Using flat interest method:

$$A = S\left(1 + \frac{nr}{100}\right) = 100000\left(1 + \frac{110}{100}\right) = 2100000$$

Thus, $E = \frac{A}{12n} = \frac{2100000}{120} = 17,500$... (1)

(b) Using reducing balance method:

Now,
$$E = \frac{S \times i}{1 - \frac{1}{(1+i)^{12n}}} = \frac{1000000 \times 0.0092}{1 - \frac{1}{(1+0.0092)^{120}}} = 13797.65$$

 $\therefore E =$ Rs. 13,798 approximately (2)

Comparing (1) and (2), we can see that the EMI using flat interest method is higher than by reducing balance method.

Example 10: Mr. Prabhakar Naik has borrowed a sum of Rs. 60,000 from a person at 6% p.a. and is due to return it back in 4 monthly installments. Find the *EMI* he has to pay and also prepare the amortization table of repayment.

Ans: Given S = Rs. 60,000; n = 4 months;

$$r = 6\% \implies i = \frac{6}{1200} = 0.005$$

Now, $E = \frac{S \times i}{1 - \frac{1}{(1+i)^n}} = \frac{60000 \times 0.005}{1 - \frac{1}{(1+0.005)^4}} = \frac{300}{0.01975}$
 $\therefore E = \text{Rs. 15,187.97}$

Now, we will prepare the *amortization table* i.e. the table of repayment of the sum borrowed using reducing balance method.

In the beginning of the 1st month the outstanding principal is the sum borrowed i.e. Rs. 60000 and the EMI paid is Rs. 15187.97

The interest on the outstanding principal is $0.005 \times 60000 = \text{Rs.} 300 \dots (1)$ Thus, the principal repayment is $15187.97 - 300 = \text{Rs.} 14887.97 \dots (2)$ The outstanding principal (**O**/**P**) in the beginning of the 2^{nd} month is now 60000 - 14887.97 = 45112.03.

Note:

- (1) is called the *interest part* of the EMI and (2) is called as the *principal part* of the EMI.
- As the tenure increases the interest part reduces and the principal part increases.

Month	O/P	EMI	Interest Part	Principal Part
	<i>(a)</i>	<i>(b)</i>	$(c) = (a) \ge i$	(<i>b</i>) - (<i>c</i>)
1	60000	15187.97	300	14887.97
2	45112.03	15187.97	225.56	14962.45
3	30141.02	15187.97	150.75	15037.22
4	15111.80	15187.97	75.56	15112.41

This calculation can be tabulated as follows:

In the beginning of the 4th month the outstanding principal is Rs. 15111.80 but the actual principal repayment in that month is Rs. 15112.41. This difference is due to rounding off the values to two decimals, which leads the borrower to pay 61 paise more!!

Example 11: Mr. Shyam Rane has borrowed a sum of Rs. 100000 from a bank at 12% p.a. and is due to return it back in 5 monthly installments. Find the *EMI* he has to pay and also prepare the amortization table of repayment.

Ans: Given S = Rs. 100000; n = 5 months; $r = 12\% \text{ p.a.} = \frac{12}{12} = 1\% \text{ p.m} \Rightarrow i = 0.01$ Now, $E = \frac{S \times i}{1 - \frac{1}{(1+i)^n}} = \frac{100000 \times 0.01}{1 - \frac{1}{(1+0.01)^5}} = \frac{1000}{0.0485343} = 20603.98$

The amortization table is as follows:

Month	O/P	EMI	Interest Part	Principal Part
	<i>(a)</i>	<i>(b)</i>	$(c) = (a) \ge i$	(<i>b</i>) - (<i>c</i>)
1	100000	20603.98	1000	19603.98
2	80396.02	20603.98	803.96	19800.02
3	60596	20603.98	605.96	19998.02
4	40597.98	20603.98	405.98	20198
5	20399.98	20603.98	204	20399.98

Check your progress

- 1. An overdraft of Rs. 50,000 is to be paid back in equal annual installments in 20 years. Find the installments, if the interest is 12% p.a. compounded annually. $[(1.12)^{20} = 9.64629]$
- **2.** A man borrows Rs. 30,000 at 6% p.a. compounded semi-annually for 5 years. Find the periodic payments he has to make.
- 3. What periodic payments Mr. Narayanan has to make if he has borrowed Rs. 1,00,000 at 12% p.a. compounded annually for 12 years? $[(1.12)^{12} = 3.896]$
- **4.** Find the future value of an immediate annuity of Rs. 1200 at 6% p.a. compounded annually for 3 years.
- 5. Find the future value of an immediate annuity of Rs. 500 at 8% p.a. compounded p.m. for 5 years.
- 6. Find the accumulated value after 2 years if a sum of Rs. 1500 is invested at the end of every year at 10% p.a. compounded quarterly.
- 7. Find the accumulated amount of an immediate annuity of Rs. 1000 at 9% p.a. compounded semi-annually for 4 years.
- 8. Find the future value of an immediate annuity of Rs. 2800 paid at 10% p.a. compounded quarterly for 2 years. Also find the interest earned on the annuity.
- **9.** Find the sum invested and the accumulated amount for an ordinary annuity with periodic payment of Rs. 2500, at the rate of interest of 9% p.a. for 2 years if the period of payment is (a) yearly, (b) half-yearly, (c) quarterly or (d) monthly.
- 10. Find the sum invested and the accumulated amount for an ordinary annuity with periodic payment of Rs. 1500, at the rate of interest of 10% p.a. for 3 years if the period of payment is (a) yearly, (b) half-yearly, (c) quarterly or (d) monthly.
- **11.** Mr. Banerjee wants to accumulate Rs. 5,00,000 at the end of 10 years from now. How much amount should he invest every year at the rate of interest of 9% p.a. compounded annually?
- **12.** Find the periodic payment to be made so that Rs. 25000 gets accumulated at the end of 4 years at 6% p.a. compounded annually.
- **13.** Find the periodic payment to be made so that Rs. 30,000 gets accumulated at the end of 5 years at 8% p.a. compounded half yearly.
- **14.** Find the periodic payment to be made so that Rs. 2000 gets accumulated at the end of 2 years at 12% p.a. compounded quarterly.
- **15.** Find the rate of interest if a person depositing Rs. 1000 annually for 2 years receives Rs. 2070.
- **16.** Find the rate of interest compounded p.a. if an immediate annuity of Rs. 50,000 amounts to Rs. 1,03,000 in 2 years.

- Find the rate of interest compounded p.a. if an immediate annuity of Rs. 5000 amounts to Rs. 10400 in 2 years.
- **18.** What is the value of the annuity at the end of 5 years, if Rs. 1000 p.m. is deposited into an account earning interest 9% p.a. compounded monthly? What is the interest paid in this amount?
- **19.** What is the value of the annuity at the end of 3 years, if Rs. 500 p.m. is deposited into an account earning interest 6% p.a. compounded monthly? What is the interest paid in this amount?
- **20.** Mr. Ashish Gokhale borrows Rs. 5000 from a bank at 8% compound interest. If he makes an annual payment of Rs. 1500 for 4 years, what is his remaining loan amount after 4 years?
- (**Hint**: find the amount using compound interest formula for 4 years and then find the accumulated amount of annuity, the difference is the remaining amount.)
- **21.** Find the present value of an immediate annuity of Rs. 10,000 for 3 years at 6% p.a. compounded annually.
- **22.** Find the present value of an immediate annuity of Rs. 100000 for 4 years at 8% p.a. compounded half yearly.
- **23.** Find the present value of an immediate annuity of Rs. 1600 for 2 years at 7% p.a. compounded half yearly.
- **24.** A loan is repaid fully with interest in 5 annual installments of Rs. 15,000 at 8% p.a. Find the present value of the loan.
- **25.** Mr. Suman borrows Rs. 50,000 from Mr. Juman and agreed to pay Rs. 14000 annually for 4 years at 10% p.a. Is this business profitable to Mr. Juman?

(Hint: Find the *PV* of the annuity and compare with Rs. 50000)

- **26.** Mr. Paradkar is interested in saving a certain sum which will amount to Rs. 3,50,000 in 5 years. If the rate of interest is 12% p.a., how much should he save yearly to achieve his target?
- 27. Mr. Kedar Pethkar invests Rs. 10000 per year for his daughter from her first birthday onwards. If he receives an interest of 8.5% p.a., what is the amount accumulated when his daughter turns 18?
- **28.** Dr. Wakankar, a dentist has started his own dispensary. He wants to install a machine chair which costs Rs. 3,25,000. The machine chair is also available on monthly rent of Rs. 9000 at 9% p.a. for 3 years. Should Dr. Wakankar buy it in cash or rent it?
- **29.** A sum of Rs. 50,000 is required to buy a new machine in a factory. What sinking fund should the factory accumulate at 8% p.a. compounded annually if the machine is to be replaced after 5 years?
- **30.** The present cost of a machine is Rs. 80,000. Find the sinking fund the company has to generate so that it could buy a new machine after 10 years, whose value then would be 25% more than of today's price. The rate of compound interest being 12% p.a. compounded annually.

- **31.** Mr. Mistry has to options while buying a German wheel alignment machine for his garage: (a) either buy it at Rs. 1,26,000 or (b) take it on lease for 5 years at an annual rent of Rs. 30,000 at the rate of interest of 12% p.a.. Assuming no scrap value for the machine which option should Mr. Mistry exercise?
- **32.** Regency Co-op. Hsg. Society which has 50 members require Rs. 12,60,000 at the end of 3 years from now for the society repairs. If the rate of compound interest is 10% p.a., how much fund the society should collect from every member to meet the necessary sum?
- **33.** Mr. Lalwaney is of 40 years now and wants to create a fund of Rs. 15,00,000 when he is 60. What sum of money should he save annually so that at 13% p.a. he would achieve his target?
- **34.** If a society accumulates Rs. 1000 p.a. from its 200 members for 5 years and receives 12% interest then find the sum accumulated at the end of the fifth year. If the society wants Rs. 13,00,000 for society maintenance after 5 years, is the annual fund of Rs. 1000 per member sufficient?
- **35.** How much amount should a factory owner invest every year at 6% p.a. for 6 years, so that he can replace a mixture-drum (machine) costing Rs. 60,000, if the scrap value of the mixture-drum is Rs. 8,000 at the end of 6 years.
- **36.** If a society accumulates Rs. 800 p.a. from its 100 members for 3 years and receives 9% interest then find the sum accumulated at the end of the third year. If the society wants Rs. 2,50,000 for society maintenance after 3 years, is the annual fund of Rs. 800 per member sufficient?
- **37.** Mr. Kanishk wants clear his loan of Rs. 10,00,000 taken at 12% p.a. in 240 monthly installments. Find his EMI using reducing balance method.
- **38.** Using the reducing balance method find the EMI for the following:

Loan amount (in Rs.)	Rate of Interest (in % p.a.)	Period of Loan (in years)
i) 1000	6	5
ii) 50000	6	10
iii) 8000	7	6
iv) 12000	9	10
v) 1000	9.5	10
vi) 1100000	12.5	20

- **39.** Mr. Vilas Khopkar has taken a loan of Rs. 90,000 at 11% p.a. Find the EMI using (a) reducing balance method and (b) Flat interest method, if he has to return the loan in 4 years.
- **40.** Find the EMI using reducing balance method on a sum of Rs. 36,000 at 9%, to be returned in 6 monthly installments.
- **41.** Find the EMI using reducing balance method on a sum of Rs. 72,000 at 12%, to be returned in 12 installments.
- **42.** Mr. Sachin Andhale has borrowed Rs. 10,000 from his friend at 9% p.a. and has agreed to return the amount with interest in 4 months. Find his EMI and also prepare the amortization table.
- **43.** Mr. Arvind Kamble has borrowed Rs. 30,000 from his friend at 14% p.a. If he is to return this amount in 5 monthly installments, find the installment amount, the interest paid and prepare the amortization table for repayment.
- **44.** Mrs. Chaphekar has taken a loan of Rs. 1,25,000 from a bank at 12% p.a. If the loan has to be returned in 3 years, find the EMI, Mrs. Chaphekar has to pay. Prepare the amortization table of repayment of loan and find the interest she has to pay.
- **45.** A loan of Rs. 75,000 is to be returned with interest in 4 installments at 15% p.a. Find the value of the installments.
- **46.** A loan of Rs. 60,000 is to be returned in 6 equal installments at 12% p.a. Find the amount of the installments.
- **47.** Find the sum accumulated by paying an EMI of Rs. 11,800 for 2 years at 10% p.a.
- **48.** Find the sum accumulated by paying an EMI of Rs. 1,800 for 2 years at 12% p.a.
- **49.** Find the sum accumulated by paying an EMI of Rs. 12,000 for 3 years at 9% p.a.
- **50.** Find the sum accumulated by paying an EMI of Rs. 11,000 for 8 years at 9.5% p.a.

(9) Period	Sum Invested	Accumulated Amount
Yearly	5000	5225
Half-yearly	10000	10695.5
Quarterly	20000	21648
Monthly	60000	65471

(2) 3517

(5) 36555.65 **(6)** 13104

(1) 6694

Hints & Solutions to Check your progress

(4) 3820.32(8) 24461

(3) 16,144

(7) 9380

(10)		
Period	Sum Invested	Accumulated Amount
Yearly	4500	4965
Half-yearly	9000	10203
Quarterly	18000	20693
Monthly	54000	62635

(11) 32910 (12) 5715 (13) 2498.72 (14) 225 (15) 7% (16) 6% (17) 8% (18) 75424, 15424 (19) 19688, 1688 (20) 4719 (21) 26730 **(22)** 673274.5 **(23)** 5877 (24) 59890.65 (25) 44378, Yes **(26)** 97093.4 **(27)** 393229.95 (28) 283021.25, take it on rent (30) 17698.42 (31) 108143.28 < 126000, Mr. Mistry (29) 12523 should use the second option. (32) 16245 (33) 18530 (**35**) 7454.86 (**36**) 2,62,248; yes (34) 1270569.47, not sufficient (37) 11,011

C	38	3	
•	~	•	

Loan amount (in Rs.)	Rate of interest (in % p.a.)	Period of Loan (in yrs.)	EMI (in Rs.)
i) 1000	6	5	19
ii) 50000	6	10	555
iii) 8000	7	6	136
iv) 12000	9	10	152
v) 1000	9.5	10	13
vi) 1100000	12.5	20	12498

(39) 2326, 2700 (40)

(40) 6158.48 (41) 6397.11

(42)

Month	O/P	EMI	Interest Part	Principal Part
	<i>(a)</i>	<i>(b)</i>	$(c) = (a) \ge i$	(<i>b</i>) - (<i>c</i>)
1	10000	2547.05	75	2472.05
2	7527.95	2547.05	56.45	2490.6
3	5037.35	2547.05	37.78	2509.27
4	2528.08	2547.05	18.96	2528.09

(43)

Month	O/P	EMI	Interest Part	Principal Part
	<i>(a)</i>	(b)	$(c) = (a) \ge i$	(<i>b</i>) - (<i>c</i>)
1	30000	6212.23	351	5861.23
2	24138.77	6212.23	282.42	5929.81
3	18208.96	6212.23	213.04	5999.19
4	12209.77	6212.23	142.85	6069.38
5	6140.39	6212.23	71.84	6140.39

(45) 19339.57	(46) 16353	(47) 3,12,673.60
(48) 48552.24	(49) 4,93,832.6	(50) 15,72,727

SHARES AND MUTUAL FUNDS

Unit Structure :

- 5.0 Objectives
- 5.1 Introduction
 - 5.1.1 Bonus Shares
 - 5.1.2 Splitting of shares
- 5.2 Mutual Funds
- 5.3 Systematic Investment Plan (SIP)

5.0 OBJECTIVES:

After going through this chapter you will be able to:

- Define a share, face value, market value, dividend, equity shares preferential shares, bonus shares.
- Understand the concept of Mutual fund.
- Calculate Net Income after considering entry load, dividend, change in Net Asset Value (N.A.V) and exit load.
- Understand the Systematic Investment Plan (S.I.P).

5.1 INTRODUCTION

In day-to-day life we hear about shares, share market etc. Here we will see, exactly what these terms deal with .

When a group of persons plan to establish a company, they form a company under the companies Act 1956. Now this company is an established company. The people who establish this company are called promoters of the company. These promoters can now raise a certain amount of capital to start (run) the company. They divide this required amount into small parts called shares.

A share is the smallest unit of capital of a company. Stock is the American term for share. Usually a share is of value Rs. 100 /- or Rs.50/- or Rs. 10 /- or Rs. 5/- or Rs. 2/- or Rs./- 1. This value is called the face value of the share. These shares are sold to the public. (usually face value is Rs. 10/-, unless otherwise specified). This sale is called the Initial Public Offer (IPO) of the company.

The company issues share certificates to the persons from whom it accepts the money to raise the capital. Persons who have paid money to form the capital are called share holders. Now-a- days the shares are not in the form of paper, but in the electronic dematerialised (Demat) form, hence the allotment of shares is done directly in the demat account, without a certificate.
Face value or nominal value or Par value is the value printed on the share certificate. Since shares exist in electronic demat form, we can say that the face value is the value stated in the I.P.O. subscription form.

The shareholders enjoy the profits (if any) of the company, after providing for the taxes and other reserve funds. This is called as **dividend.**

Types of shares ; Mainly the shares are of two types i) Preference shares and ii) Equity shares or common shares or ordinary shares .

i) Preference shares : These shares have a priority over the equity shares. From the profits made by a company, a dividend at a fixed rate is paid to them first, before distributing any profit amount to the equity shareholders. Also, if and when the company is closed down then while returning of the capital, these shareholders get a preference. Again, preference shares are mainly of two types:

- a) Cumulative Preference shares: In case of loss or inadequate profit, The preference shareholders are not paid their fixed rate of dividend, then the dividend is accumulated in the subsequent years to these shareholders & is paid preferentially whenever possible.
- **b)** Non-cumulative Preference shares: As in the case of cumulative preference shares, here the unpaid dividends do not accumulate.

ii) Equity shares : These are the shares for whom the dividend and the return of capital is paid after paying the preference shareholders. In case of equity shares , the rate of dividend is not fixed and it is decided by the Board of Directors .

Share Market

Shareholders are allowed to buy or sell shares like commodities. Selling or buying a share for a price higher than its face value is legal. The share prices are allowed to be subject to the market forces of demand and supply and thus the prices at which shares are traded can be above or below the face value.

The place at which the shares are bought and sold is called a share market or stock Exchange and the price at which a share is traded is called its Market Price (MP) or the Market value. If the market price of a share is same as its face value, then the share is said to be traded at Par.

If M.P. is greater than face value of a share , then the share is said to be available at a premium or above par and is called premium share or above par share.

If M.P. is lower than face value of a share , then the share is said to be available at a discount or below par & the share is called a discount share or below par share .

The purchase and sale of shares can take place through brokerage firms and depositary Participant (DP). e.g. Sharekhan.com, Kotak Securities

ltd, ICICI direct.com etc. They charge a commission on the purchase and sale of shares, which is called as a brokerage. The brokerage is charged as a percentage of the M.P. of the share. Normally it is below 1%.

5.1.1 Bonus Shares

Sometimes, when a company's free reserves are high, it may choose to capitalize a part of it by converting it into shares. This is done by issuing bonus shares to existing shareholders. These bonus shares are issued free of cost. The ratio of bonus shares to the existing shares is fixed.

Getting bonus shares increases the number of shares of shareholders. But since this applies to all the shareholders in a fixed ratio, hence the percentage of a shareholder's ownership of the company remains same as before.

Now, we will study some examples based on the above concepts:

Example 1

Mr. Prashant invested Rs. 75,375/- to purchase equity shares of a company at market price of Rs. 250 /- through a brokerage firm, charging 0.5% brokerage. The face value of a share is Rs. 10/-. How many shares did Mr. Prashant purchase?

Solution: Brokerage per share = $250 \times \frac{0.5}{100} = 1.25$

 \therefore cost of purchasing one share = 250+1.25 = 251.25

$$\therefore \text{ Number of shares purchased} = \frac{75375}{251.25} = 300$$

Example 2

Mr. Sandeep received Rs. 4,30,272 /- after selling shares of a company at market price of Rs. 720 /- through Sharekhan Ltd., with brokerage 0.4%. Find the number of shares he sold.

Solution: Brokerage per share = $720 \times \frac{0.4}{100} = 2.88$ selling price of a share = 720 - 2.88 = 717.12 \therefore Number of shares sold = $\frac{430272}{717.12} = 600$

Example 3

Ashus Beauty World ' has issued 60,000 shares of par value of Rs. 10/- each. The company declared a total dividend of Rs. 72,000 /- . Find the rate of dividend paid by the company.

Solution: Face value of 60,000 shares = $60,000 \times 10 = 6,00,000$ Rate of Dividend = $\frac{\text{Total Dividend}}{\text{Face value of } 60,000 \text{ share}} \times 100$

$$= \frac{72000}{600000} \ge 100 = 12$$

 \therefore The rate of dividend paid by the company is 12%

Example 4

The capital of ABC Company consists of Rs. 15 lakhs in 6 % cumulative preference shares of Rs. 100 each and Rs. 30 lakhs in equity shares of Rs.10/- each. The dividends on cumulative preference shares for earlier year was not paid . This year , the company has to distribute profit of Rs. 3 lakh after keeping 20 % as reserve fund. Find the percentage rate of dividend paid to the equity shareholders.

Solution: Reserve fund $= \frac{20}{100} \times 300000 = \text{Rs. } 60,000/\text{-}$ Profit to be distributed = 3,00,000 - 60,000 = 2,40,000

Annual dividend for 6 % cumulative preference shareholders

$$=\frac{6}{100} \times 1500000 = 90,000$$

This needs to be paid for 2 years (last year & current year) as the preference shares are cumulative & last year's dividend was not paid.

.: Total Dividend paid to Preference shareholders $= 2 \times 90000 = 1,80,000/-$

Now, dividend to be distributed to the equity shareholders :. Rate of dividend = $\frac{60,000}{30,00,000} \times 100 = 2$

- \therefore The rate of dividend to the equity shareholders is 2 %

Example 5

Mr. Dinesh bought some shares of a company which had a face value of Rs.100 /-. The company declared a dividend of 15 % but Mr. Dinesh's rate of return on investment was only 12%. At what market price did he purchase the shares? There was no brokerage involved.

Solution:

Dividend on one share = $\frac{\text{Rate of Dividend}}{100}$ x face value of one share $= 15 \times 100 = \text{Rs}. 15/ \therefore \text{ Rate of Return on investment} = \frac{\text{Dividend on one share}}{\text{purchase price of 1 share}} \times 100$ $\therefore 12 = \frac{15}{\text{purchase of 1 share}} \times 100$ \therefore purchase price of 1 share = $\frac{15}{12}$ x 100 = 125

Example 6: Comparison of two stocks

Mr. Subu invested Rs. 20,000 /- in Rs. 100/- shares of company A at the rate of Rs. 125/- per share . He received 10 % dividend on these shares. Mr. Subu also invested Rs. 24,000/- in Rs. 10/- shares of company B at Rs.12/- per share and he received 15 % dividend. Which investment is more beneficial?

Solution : <u>Company A</u>

Rate of return = $\frac{\text{Dividend on one share}}{\text{purchase price of 1 share}} \times 100 = \frac{10}{125} \times 100 = 8\%$ 125

Company B

Rate of return = $\frac{1.5}{12}$ x 100 = 12.5 %

Investment in company B is more profitable.

Example 7

Ms. Ashma Mehta bought 300 shares of a company of face value Rs. 100 /- each at a market price of Rs. 240 /- each . After receiving a dividend at 8 %, she sold the shares at Rs . 256 /- each. Find her rate of return on investment. There was no brokerage involved.

Solution: Difference in the market price = 256-240 = 16

Dividend on 1 share = $\frac{\text{Rate of dividend}}{100} x$ face value of 1 share = $\frac{8}{100} x 100=8$

Rate of Return on Investment

$$= \frac{(\text{Price change}) + (\text{Dividend on 1 share})}{\text{purchase price of 1 share}} = \frac{16+8}{240} \times 100$$
$$= \frac{2400}{240} = 10$$

 \therefore The rate of return on investment was 10 %.

5.1.2 Splitting of shares:

Sometimes companies split the face value of a share & break it up into smaller units . For e.g. a Rs. 100 /- share can be split into 10 shares each of face value Rs. 10 /- or a Rs. 10/- share can be split into two shares of face value Rs. 5/- each . Usually this does not affect a shareholder's wealth . However , it can make selling of a part of the holdings easier.

Example 8

Mr. Joshi purchased 30 shares of Rs. 10/- each of Medi computers Ltd. on 20th Jan. 2007, at Rs. 36/- per share. On 3rd April 2007, the company decided t split their shares from the face value of Rs. 10/- per share to Rs. 2/- per share. On 4th April 2007, the market value of each share was Rs. 8/- per share. Find Mr. Joshi's gain or loss, if he was to

sell the shares on 4th April 2007? (No brokerage was involved in the transaction).

Solution: On 20th Jan 2007 Purchase cost of 30 shares = $30 \times 36 = 1080/-$

On 3rd April 2007, each Rs. 10/- share became 5 shares of Rs. 2/- each.

:. No. of shares = $30 \ge 5 = 150$ On 4th April 2007, market value of 150 shares was @ Rs. 8 each = $150 \ge 8 = 1200$:. His gain = 1200-1080 = 120/-

Example 9

Rahul purchased 500 shares of Rs. 100 of company A at Rs. 700 /-. After 2 months, he received a dividend of 25 % . After 6 months, he also got one bonus share for every 4 shares held. After 5 months, he sold all his shares at Rs. 610/- each. The brokerage was 2% on both, purchases & sales. Find his percentage return on the investment.

Solution: For purchase:

Face value = Rs. 100 /- , No. of shares = 500, market price = Rs.700/-Dividend = 25%, brokerage = 2%

Purchase price of one share = $700 + \frac{2}{100} \times 700 = 714$

:. Total purchase = $500 \times 714 = \text{Rs.}3,57,000/\text{-}$

Dividend = $\frac{25}{100}$ of 100 i.e. Rs. 25 /- per share \therefore Total dividend = 500 x25 = Rs. 12,500 /-Now, bonus shares are 1 for every 4 shares.

$$\therefore \text{ No. of bonus shares} = \frac{1}{4} \times 500 = 125$$

$$\therefore \text{ Total No. of shares} = 500 + 125 = 625$$

For sales,

No. of shares = 625, market price = 610, Brokerage 2%

Sale price of one share = 610 - 2% of 610 = 597.8

 \therefore Total sale value = sale price of one share x No. of shares

$$= 597.8 \times 625 = \text{Rs.} 3, 73,625/-$$

Net profit = sale value + Dividend - purchase value

$$= 3,73,625 + 12500 - 3,57,000$$

= Rs . 29,125/- .
:. % gain = $\frac{29,125}{3,57,000}$ x 100 = 8.16

= 8.16

EXERCISE :

1) Mr. Amar invested Rs 1,20,480/- to buy equity shares of a company at market price of Rs . 480 /- at 0.4 % brokerage. Find the No. of shares he purchased.

Ans: 250

2) Aditi invested Rs. 19,890 /- to purchase shares of a company with face value of Rs.10/- each , at market price of Rs. 130/- . She received dividend of 20 % as well Afterwards , she sold these shares at market price of Rs. 180/- . She had to pay brokerage of 2 % for both purchase and sales of shares. Find her net profit.

Ans: No. of shares =150, sales = 26460, Dividend = 300, purchase = 19,890, profit= 6870

3) Amol wants to invest some amount in company A or company B, by purchasing equity shares of face value of Rs. 10 /- each, with market price of R. 360/- and Rs. 470/- respectively. The companies are expected to declare dividends at 20 % and 45% respectively. Advise him on the choice of shares of company. **Ans**: company B is a better choice.

4) Find the percentage gain or loss if 200 shares of face value Rs. 10/were purchased at Rs . 350/- each and sold later at Rs. 352 /- , the brokerage being 0.5 % on each of the transaction .

Ans: -0.43 % i.e. a loss of 43 %

5) Find the number of shares if the total dividend at 8% on the shares with face value Rs.10/- was Rs. 240. **Ans** :- 300

5.2 MUTUAL FUNDS

In the previous unit shares, we have studied how one can transact in shares. Now, we will study what are the mutual funds and how they function.

An investor can invest money directly in shares or he can invest his money through mutual funds. Mutual funds are managed by large financial services with a professional team of fund Managers & research experts.

Mutual fund is a pool of money, drawn from investors .The amount collected is invested in different portfolios of securities, by the fund managers and the profits (returns), proportional to the investment, are passed back to the investors.

At a given time, the total value is divided by the total number of units to get the value of a single unit a given time. This is called Net Asset Value (NAV).

 $\therefore \text{ NAV} = \frac{\text{Net Assets of the scheme}}{\text{Total No. of units outstanding}}$

or NAV =<u>Total Assets-liabilities</u> Total No. of units outstanding

There are mixed or hybrid funds which invest in both debt and equity. The offer documents give the guidelines / constraints under which the fund managers would operate. e.g. investment in equity 80% to 100%, investment in money markets 0% to 20% etc.

In India , the mutual funds are governed by SEBI (Securities and Exchange Board of India) .There are different companies , called the ' Fund Houses ' (like SBI or Reliance or HDFC) which float different mutual funds. Each such fund is called a 'scheme', e .g. HDFC has a scheme ' HDFC Tax saver ' etc.

Like IPO of a company's share , a mutual fund scheme starts by having a N.F.O. (New Fund Offer) . Investors can invest by purchasing Units of the mutual funds .Usually a unit is of Rs. 10/- . A share is the smallest unit of a company's capital , whereas in mutual funds , even a fraction of a unit can be purchased after the N.F.O .

Let us study the following example to understand this concept:

Example 10

A mutual fund 's scheme shows the following on 01/01/2007

Total value of securities	Rs. 1500 crores
(Equity, Bonds etc.)	
Cash	Rs. 100 crores
Liabilities	Rs. 200 crores
Total No. of units outstanding	Rs. 100 crores

 $\therefore \text{ NAV} = \frac{\text{Rs. 1500 crores} + \text{Rs. 100 crores} - \text{Rs. 200 crores}}{100 \text{ crores}}$

 $= \text{Rs.} \frac{1400 \text{ crores}}{100 \text{ crores}} = \text{Rs.} 14 \text{ per unit}.$

The NAV of a mutual Fund scheme is calculated and disclosed to the puble for evey working day. The NAV changes daily. Investors try to invest when NAV is low and sell the units and get profits when the NAV is high.

Most mutual fund schemes are not traded at stock market. Thus, investor purchases as well as sells the units to AMC i.e. Asset management company, This sale is called redemption of units.

Basically funds are of two types :-

1) close ended funds 2) open ended funds.

1) Close ended mutual funds :- These are offered with a fixed date of maturity and can be purchased from mutual fund companies during a specific period. The investor can get the amount after expiry date of the fund. If an investor wants to exit before the maturity date, he can

sell the units on the stock exchange at a discount or through a buyback option by the fund .

2) Open ended funds : These have no fixed date of maturity and the units can be sold or repurchased at any time .The no. of units & its capital changes daily .

Entry load & Exit load : Some mutual fund schemes collect a charge when investors purchase or redeem units . These are usually percentage of NAV . The charge levied while purchasing a unit is called the entry load & the charge collected on redemption is called exit load .

Usually , the debt funds have not loads $\ .$ When there are no charges while purchasing or selling of units , these funds are called No $\ Load$ Funds $\ .$

Mutual Funds can be broadly categorised into two types : 'Dividend ' funds which offer a dividend and 'Growth ' funds which do not offer a dividend .

In mutual unds, the dividend given has no direct relation to the profit earned. The mutual fund invests the money in different shares that may or may not give a dividend at different times & different rates. The fund manager may at any arbitrary point, decide to give a part of the units' value back to the investors. This is called dividend.

For a growth fund, the NAV does not come down due to dividends. It moves up or down purely on the basis of the gains or losses of the securities that the fund has invested in.

For a growth fund, the gains per unit are purely from the difference between the redemption price and the purchase price i.e. the total gain is purely the capital gain. For a dividend fund, the total gain is the addition of the capital gain & the dividend.

Capital gain = Amount received after redemption - Amount invested .

Rate of Return = $\frac{\text{Change in NAV + Dividend}}{\text{NAV at the beginning of the period}} \times 100$

(This is for a given period).

Annualised rate of Return = Rate of Return x 365

n

where n is the number of days .

Some important Terms :

i) Assets :- It refers to market value of investment of M.F. in government securities , bonds etc. , its receivables , accrued income & other assets .

ii) liabilities :- It includes all expenses like accrued expenses , payables and other liabilities for the M.F. scheme .

iii) Net Assets :- Total Assets - liabilities

iv) The valuation Date is the date on which NAV is calculated .

Example 11

Mr. Deore invested Rs. 25,000/- to purchase 2,500 uits of ICICI MF -B plan on 4th April 2007 . He decided to sell the units on 14th Nov. 2007 at NAV of Rs. 16.4 /-. The exit load was 2.5 % . Find his profit (Calculations are upto 2 decimal points)

Solution :

No. of units =2500 , purchase cost = Rs. 25,000/-NAV on the date of sale = RS. 16.4/- , exit load =2.5%= of 16.4 = 0.41

∴ selling price of 1 unit = 16.4- 0.41 = 15.99
∴ sale value = 2500 x 15.99
= Rs. 39,975/ ∴ Profit = 39,975 - 25,000
= Rs. 14,975.

Example 12

Ragini invested Rs. 94,070/- in mutual Fund when NAV was Rs. 460 /with entry load of 2.25 %. She received a dividend of Rs. 5/- per unit . She, later sold all units of fund with an exit load of 0.5 %. If her gain was Rs. 1654/-, find NAV at which she sold the units . (Calculations are upto 2 decimal points)

Solution : purchase price of one unit = 460 + 2.25% of 460= 460 + 10.35 = 470.35

No. of units purchased = $\frac{94,070}{470.35}$ = 200

Total dividend = 200 x5 = 1000

Gain = Profit + Dividend

 $\therefore 1654 = Profit + 1000$

 \therefore Profit = 1654- 1000= 654

While selling let NAV of one unit be y

:. sale price of one unit = NAV - exit load = y- 0.5% of y = 0.995 y

 \therefore sale price of 200 units = 200 x 0.995 y= 199 y

Also, profit = Total sale - Total purchase

654 = 199y - 94,070

∴ 199y = 654 + 94,070∴ 199y = 94724∴ y = 476∴ NAV at which she sold units = Rs. 476/-.

Example 13

If a mutual fund had NAV of Rs. 28 /- at the beginning of the year and Rs. 38/- at the end of the year , find the absolute change and the percentage change in NAV during the year .

Solution : NAV at the beginning = Rs. 28/-NAV at the end = Rs. 38/- \therefore Absolute change in NAV = in 38-28 = Rs. 10/-% change = <u>Absolute change</u> x 100 = <u>10</u> x100 = 35.71 % NAV at the beginning 28

Example 14

If NAV was Rs. 72/- at the end of the year, with 12.5 % increase during the year, find NAV at the beginning of the year.

Solution : Let 'x' be the NAV at the beginning of the year . \therefore Absolute change in NAV = 12.5 % of x = $\underline{12.5}$ x x = 0.125 x 100 \therefore NAV at the end of the year = x + 0.125 x = 1.125 x \therefore 1.125 x = 72 \therefore x = $\underline{72}$ 1.125 = 64



Example 15

Rohit purchased some units in open end equity fund at Rs. 16/- . The fund distributed interim dividend of Rs. 5/- per unit , and the NAV of the fund at the end of the year was Rs. 25/- . Find the total percentage return . (Calculations are upto 2 decimal points)

Solution : Total gain = change in NAV + Dividend = (25-16) + 5= 9+5= 14 \therefore Total percentage gain = $\frac{\text{Total gain}}{\text{NAV at the beginning}}$ x 100 = $14 \times 100 = 87.5 \%$

$$=$$
 14 x 100 = 87.5 %
16

Example 16

Mr. Hosur purchased some units in open- end fund at Rs. 30/- and its NAV after 18 months was Rs. 45/- . Find the annualised change in NAV as a percentage .

Solution : change in NAV for 18 months = 45-30 = Rs. 15/- \therefore annualised change = <u>change in NAV</u> x <u>12</u> x 100 NAV at beginning No. of months = $\frac{15}{30} \times \frac{12}{18} \times 100$ = 33.33%

Check your progress :

1) Mr. Kamble purchased 586.909 units of Kotak cash plus retail Growth on 1st June 2007 when the NAV was RS. 20.4461. Its NAV as on 3^{rd} December, 2007 was Rs. 21.1960/-. The fund has neither entry load nor an exit load. Find the amount invested on 1st June 2007 and the value of Mr. Kamble's investment on 3^{rd} December 2007.

Ans. 12,000, 12440.12.

2) Ms . Kannan purchased 113.151 units of 'FT India Prima Plus' on 9th April 2007 and redeemed all the units on 7th Aug 2007 when the NAV was Rs. 35.5573. The entry load was 2.25 % and the exit load was 1 % . If she gained Rs. 483.11, find the NAV on 9th April 2007 . (Calculations are upto 2 decimal points)

Ans . 30.2514

3) Mr. Pandit invested Rs. 10,000/- in Birla Sunlife Equity Fund-Divjdend plan ' on 10/07/2007, when the NAV was Rs. 78.04, and redeemed all the units on 12/11/2007 when the NAV was Rs. 84.54. In the meanwhile, on 31/08/2007, she had received a dividend @ Rs. 10 per unit . Find her total gain and the rate of return considering loads as follows:

The entry load was 2.25 % and the exit load was 0.5 % The number of units were calculated correct upto 3 decimal places.

Ans . Total gain = Rs . 1794.46, Rate of return = 17.94%

4) Given the following information , calculate NAV of the mutual fund :-

No. of units =15000

Market value of investments in Govt . securities = Rs. 20 lakhs Market value of investments in corporate Bonds = Rs. 25 lakhs Other Assets of the fund = Rs. 15 lakhs Liabilities of the fund = 6 lakhs

Ans . Rs. 360/- .

5) Mumtaz purchased 1200 units of TATA BIG Bond- G Rs. 12,000 /- on 14th April 2007 . She sold her units on 9th Dec 2007 at NAV of Rs. 15.36/- . The short term gain tax (STGT) was 10% of the profit . Find her net profit . (Calculations upto 2 decimal points) Ans . profit = 6432 , STGT = 643.2 , Net profit = 5788.8 (profit- STGT) .

5.3 SYSTEMATIC INVESTMENT PLAN (SIP)

In SIP an investor invests a fixed amount (e.g. say 1000/-) every month on a fixed date (e.g. 1st of every month). In general the

minimum amount is Rs. 1000/- per month , in diversified equity schemes . It can be even Rs. 500/- as well in ELSS schemes . If this is done for many months , then each time units are purchased at a different NAV . Over a period of few months, an investor gets the benefit of a phenomenon called 'Rupee cost Averaging'.

Rupee-cost- averaging :- If NAV increases , the no. of units decreases & if NAV decreases , the no. of units purchased increases . Thus on the whole , it lowers the average cost of units because indirectly ,the investor buys more units when NAV prices are low & he buys less units , when NAV prices are high . It is called Rupee-cost-Averaging .

Consider the following example :-

Mr. Shaikh keeps Rs. 5000/- on 3rd of every month for 4 months as follows :-(Calculations are correct to 2 points of decimal)

Month	Amount (in Rs.)	NAV	No. of units he gets
1	5000	109.48	5000/109.48=45.67
2	5000	112.36	5000/112.36=44.50
3	5000	108.14	5000/108.14 = 46.24
4	5000	105.62	5000/105.62=47.34
Total	20,000		183.75

 \therefore Avg price of units = 20,000 / 183.75 = 108.84

If Mr. Shaikh would have invested the entire amount of Rs. 20,000/-0n 3rd of first month only, with NAV Rs. 109.48/-, the no. of units purchased would have been 20,000/109.48 = 182.68

Thus he gained more units and average price of units also was Rs.108.84 instead of Rs.109.48 which was NAV on 3rd of the first month

If SIP is followed for a long period of time, it can create wealth to meet a person's future needs like housing, higher education etc.

Now, we will study the following examples to understand SIP .

Example 17

Mr. Patil invested in a SIP of a M.F., a fixed sum of Rs. 10,000/- on 5th of every month, for 4 months. The NAV on these dates were Rs. 34.26, 46.12, 39.34 and 41.85. The entry load was 2.25 % through out the period. Find the average price, including the entry load, using the Rupee-cost-Averaging method. How does it compare with the Arithmetic mean of the prices? (Calculations are correct to 4 digits decimal)

Solution :

Month	NAV	Entry load = 2.25%	Total price	No.ofunits=1000/ Total price
1	34.2600	0.7708	35.0308	285.4627
2	46.1200	1.0377	47.1577	212.0544
3	39.3400	0.8851	40.2252	248.6006
4	41.8500	0.9141	42.7916	233.6906
TOTAL			165.2053	979.8083

By using Rupee-cost-Averaging method :-

Avg Price = $\underline{\text{Total amount}}$

Total No. of units

 $= \frac{40,000}{979.8083} = 40.8243$

A.M. of price = $\frac{\text{Total price}}{4} = \frac{165.2053}{4} = 41.3013$

 \therefore Avg. price using Rupee-cost- Averaging method is less than A.M. of prices .

Example 18

Mr. Desai invested Rs. 5000/- on 1st of every month for 5 months in a SIP of a M.F. with NAV's as 48.15, 52.83, 41.28, 35.44 & 32.65 respectively .There was no entry load charged. Find the average price, Mr. Desai paid using the Rupee-cost-Averaging method. After 6 months ,he sold all his units, when NAV was Rs. 51.64 with contingent deferred sales charge (CDSC) as 2.25 %. Find his net gain. (Calculations are correct to 2 digits decimal)

solution. consider the following table.	Solution :	consider	the following	table :-
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Month	Amount (in Rs.)	NAV	No. of units
1	5000	48.15	5000/48.15=103.84
2	5000	52.83	5000/52.83=94.64
3	5000	41.28	5000/41.28=121.12
4	5000	35.44	5000/35.44=141.08
5	5000	32.65	5000/32.65=153.14
TOTAL	25000		613.82

Avg . price of units $= \frac{25000}{613.820} = 40.73$

For selling :

selling price of one unit = 51.64 - 2.25% of 51.64 = 50.48 Total sales = 50.48 x 613.82= 30,991.77

 \therefore Net gain = 30,991.77 - 25,000 = Rs. 5991.77.

Check your progress :

1) Mr. Thomas started a SIP in 'HDFC long term advantage Fund '. On the 10th July , Aug and Sept 2007 he invested Rs. 1000/- each at the NAVs Rs. 44.100, Rs. 43.761/-, s. 45.455 respectively . The entry load was 2.25% . Find his average acquisition cost per unit upto 3 decimal places . (Calculations are up to 3 decimal points).

Ans. Rs. 45.427/- .

2) Maneeshad Rs. 20,000/- on 2nd of every month for 5 onths in a SIP of a mutual fund, with NAVs as Rs. 53.12, Rs. 56.26, Rs. 48.86, Rs.50.44 and Rs. 54.62 respectively. The entry load was 2.25 % throughout this period. Find average price, including the entry load, using the Rupee-cost -Averaging method and compare it with Arithmetic mean of prices.

(Calculate up to 2 decimal points)

Ans. 53.70, 53.84.